Study on Hollow Cathode Model and Inner Pressure

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Abstract: A 2D numerical model of hollow cathode is presented and compered with experiments specific to inner gas pressure. The hollow cathode model includes plasma discharge, gas flow, and heat transfer processes. Two orifice diameter sizes are simulated and experimented. The calculated peak plasma density and fractional ionization are especially large in the smaller orifice due to electron-ion coulomb collision frequency. For cathode orifice smaller than 0.5mm, the extremely high inner pressure which exceeds 10kPa can be explained by high electron pressure resulting from the high fractional ionization. As the Knudsen number in the orifice is around 0.1 in the model, slip boundary condition should be used.

I. Introduction

THe hollow cathode is an important component of electric propulsion system, such as Hall thruster and Ion thruster, and is always a research focus because of its importance and developing difficulties. With the development of all-electric propulsion satellite, the electric propulsion becomes more and more popular, which increases the demands for hollow cathode research. The hollow cathode is used to provide electrons for propellant ionization and beam neutralization. It works at very high temperature and is required to have a very long life. As a result, both experimental and numerical methods have been used by many institutions to do the research. However, there are still many areas not researched, such as the extremely high inner pressure of hollow cathode with small orifice. The neutral gas provides the basic particles background for the hollow cathode plasma, therefore the accuracy of the model about gas pressure is of great significance for understanding of cathode plasma and choice of gas flow rate. The Jet Propulsion Laboratory has built a relatively complete hollow cathode model,[1,2] and revealed the effect of viscosity on the neutral flow for cathode with a large orifice which has an inner pressure about 1kPa.[3] For cathode with orifice smaller than 0.5mm, the inner pressure can be several kilo Pascal to larger than 10 kilo Pascal, which cannot be easily simulated. In this article, a 2D axisymmetric numerical model of hollow cathode is presented, and compared with experiments especially for neutral flow and pressure inside the hollow cathode. The result shows that such cathode can have very high fractional ionization, resulting in a large electron pressure that cannot be neglected.

II. Numerical Model

The 2D axisymmetric hollow cathode model is established under COMSOL Multiphysics, including plasma discharge, gas flow, and heat transfer processes. The geometry of the model is shown in Fig. 1. Two sizes of orifice diameter are simulated, 0.5mm and 0.35mm. The plasma model is based on continuity hypothesis, and solves the collision reactions and conservation equations for particles. The model includes electron, Xe, Xe^+, excited state Xe

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atoms, and elastic collision, excitation, de-excitation, ionization and recombination reactions between these articles. The electron number density and energy density are solved by continuity and transport equations, Eqs. (1)-(4),

$$\frac{\partial n_e}{\partial t}+\nabla\cdot\mathbf{\Gamma}_e=R_e \tag{1}$$

$$\mathbf{\Gamma}_e=-n_e\mu_e\boldsymbol{E}-D_e\nabla n_e \tag{2}$$

$$\frac{\partial n_\varepsilon}{\partial t}+\nabla\cdot\mathbf{\Gamma}_\varepsilon+E\cdot\mathbf{\Gamma}_e=R_\varepsilon \tag{3}$$

$$\mathbf{\Gamma}_\varepsilon=-n_\varepsilon\mu_\varepsilon\boldsymbol{E}-D_\varepsilon\nabla n_\varepsilon \tag{4}$$

where n, Γ, R, t represent density, flux, source term, time, respectively, and subscripts e, ε represent electron and electron energy, respectively.

Electric potential V is solved by poissen equation (Eq. (5)),

$$\nabla\cdot\left(-\varepsilon_0\nabla\boldsymbol{V}\right)=\rho_v \tag{5}$$

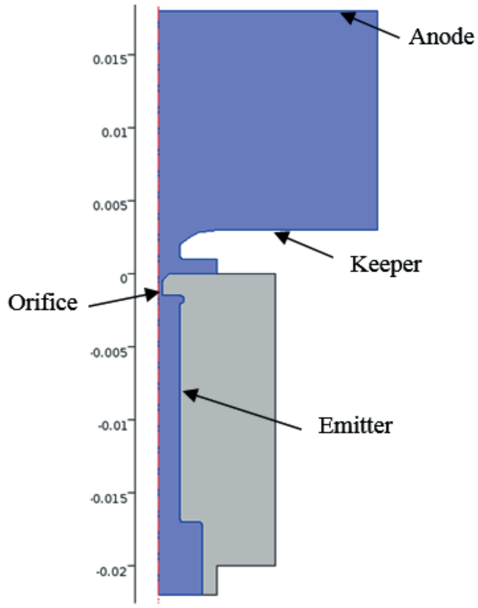

Figure 1. Geometry of the hollow cathode model.

where ε_0 is the vacuum permittivity, and ρ_v is space charge density.

Mass fractions of ion and excited state atoms are solved by continuity and transport equations, Eqs. (6)-(7).

$$\rho\frac{\partial w_k}{\partial t}+\rho\left(\boldsymbol{u}\cdot\nabla\right)w_k+\nabla\cdot\mathbf{\Gamma}_k=R_k \tag{6}$$

$$\mathbf{\Gamma}_k=\rho w_k z_k\mu_k\boldsymbol{E}-\rho D_k\nabla w_k \tag{7}$$

where ρ is mass density of gas mixture, w is mass fraction, $\boldsymbol{u}$ is flow velocity field, Γ is mass density flux, R is volume source term, z is electric charge (in units of elemental charge), μ is mobility, and D is diffusivity. Subscripts k represents k-th kind of heavy particles.

The gas pressure and flow velocity field are solved by continuity and momentum equations, Eqs. (8)-(9),

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\left(\rho\boldsymbol{u}\right)=0 \tag{8}$$

$$\rho\frac{\partial\boldsymbol{u}}{\partial t}+\rho\left(\boldsymbol{u}\cdot\nabla\right)\boldsymbol{u}=\nabla\cdot[-p\mathbf{I}+\boldsymbol{\tau}] \tag{9}$$

where p is gas pressure, $\mathbf{I}$ is identity matrix, and $\boldsymbol{\tau}$ is the viscous stress tensor. The ion temperature is assumed to be equal to neutral temperature, T, which, along with temperature of cathode, is obtained from Eq. (10),

$$\rho C_p\left(\frac{\partial T}{\partial t}+\boldsymbol{u}\cdot\nabla T\right)=\nabla\cdot\left(k\nabla T\right)+Q \tag{10}$$

where C_p is heat capacity, k is thermal conductivity, and Q is plasma heating sources. For heat transfer in solid, $\boldsymbol{u}$ and Q are both equal to 0. Mass density ρ is solved by Eq. (11), in which the electron pressure is included,

$$p=\frac{\rho RT}{M}+n_e eT_e \tag{11}$$

where R is universal gas constant, M is molar mass of Xe, and T_e is electron temperature in V, which is assumed to obey the Maxwell distribution. Simulation results with and without electron pressure are compared in section IV.

On the surface of electrodes, electrons are set to be absorbed, while ions and excited atoms become ground state atoms. Thermionic emission flux of the emitter, J, is defined according to the Richardson-Dushman equation, eq. (12),

$$\boldsymbol{J} = DT^2 \exp(\frac{-e\phi_0}{k_B T}) \exp[(\frac{e}{k_B T})\sqrt{\frac{e\boldsymbol{E}}{4\pi\varepsilon_0}}] \tag{12}$$

where D is the material-specific modification, Φ_0 is the work function of material, and k_B is Boltzmann constant.

Anode and Keeper are given constant currents in simulation. Inlet of cathode is given a mass flow, and the boundary of plume region keeps a given pressure. For gas flow process, velocity boundary condition has two types, no slip (eq. (13)) and slip (eq. (14)),

$$u = 0 \tag{13}$$

$$u = \frac{2-a_v}{a_v}\frac{\lambda\tau_n}{\mu} \tag{14}$$

where u is gas velocity on wall, a_v is tangential momentum accommodation coefficient, μ is the fluid's dynamic viscosity, λ is the molecules' mean free path, and τ_n is the tangential shear stress.

Anode temperature is assumed to be 450K, and keeper temperature is 700K. The cathode is heated by ions and electrons, and cooled by emitted electrons.[4] Exterior surfaces of cathode lose heat by radiation.

III. Experimental Setup

The hollow cathode for experiment is shown in Fig. 2. Two orifice diameter sizes, 0.5mm and 0.35mm, are experimented. The insert is made of LaB_6, 15mm long when orifice diameter is 0.5mm, 7mm long when orifice diameter is 0.35mm, and enclosed in a graphite sleeve to avoid the diffusion of boron into metal materials. The keeper orifice has a diameter of 3 mm and is 1 mm away from the cathode orifice. Besides, a planar anode is used, which is 15 mm away from the cathode.

A 0.8m-diameter by 1.5m-long vacuum system is used for cathode operation. Xenon is the propellant. No current was applied to the keeper when the cathode was discharging. A barometer was used to measure the gas pressure upstream of the cathode.

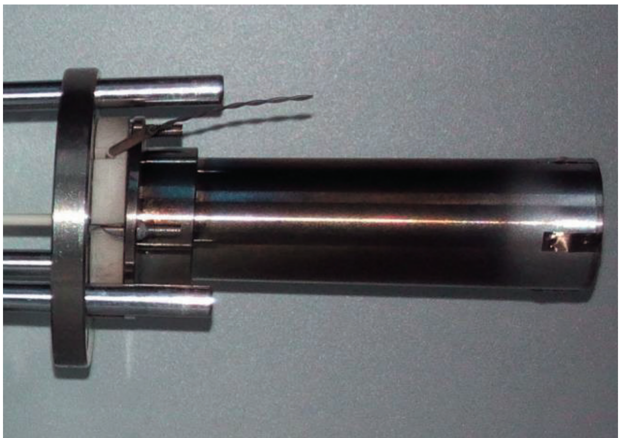

Figure 2. Hollow cathode for experiment.

IV. Results

A. Plasma Parameters

The plasma parameters with electron pressure and slip boundary condition are shown in Fig. 3-7. The discharge current is 4A, gas flow rate is 4sccm for 0.5mm orifice, 3sccm for 0.35mm orifice, and a_v is 0.5. Fig. 3 is the plasma density inside the cathode. For the cathode with larger orifice, the peak plasma density is about 8×10^{21} 1/m^3, large than that reported in Ref. 5. Whereas in the cathode with a smaller orifice, the peak density is about 3×10^{22} 1/m^3, resulting in a very large fractional ionization that will be discussed later. Plasma potential and electron temperature are shown in Fig. 4. It should be noted that the plasma parameters in the plume are simulated by fluid model. The gas temperature, which is also ion temperature, is shown in Fig. 5. The temperature is very high so that it makes a great contribution to the viscosity and pressure.

Which should be paid special attention to is the plasma fractional ionization shown in Fig. 6. The value of fractional ionization in the small orifice can be as high as 63%, because the coulomb collision frequency is very large as shown in Fig. 7, which greatly impedes electron transportation. Simultaneously, the ionization rate is also great because of the high electron density, which corporately contributes to the high plasma density. Higher plasma density results in greater impediment to electron transportation and greater ionization rate. Finally the plasma density reaches a very big value. At the same time, the gas pressure decreases rapidly through the orifice. Thus the high fractional ionization is actualized.

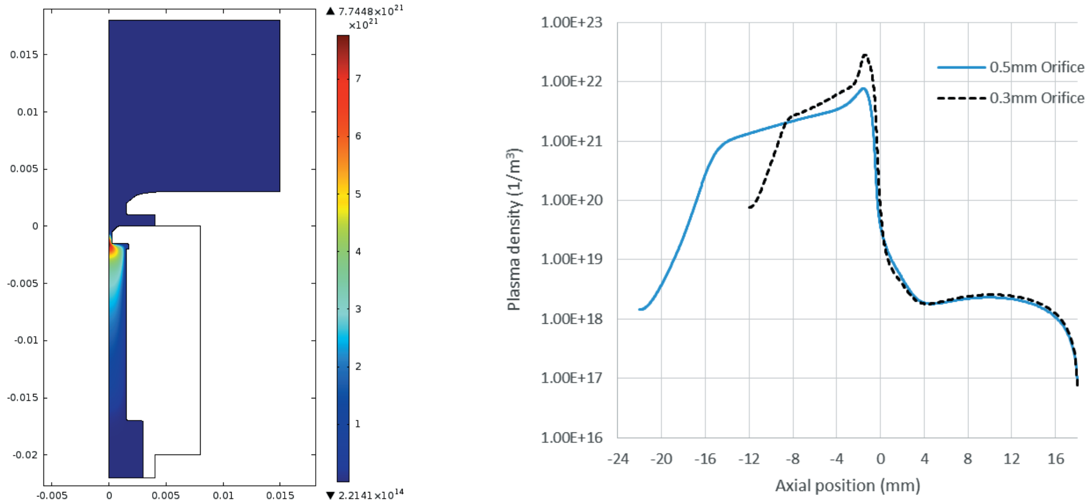

Figure 3. Plasma density inside 0.5mm orifice hollow cathode (left), and comparison of plasma density on axis in cathode with 0.5mm and 0.35mm orifice (right). 0mm is the outlet of orifice.

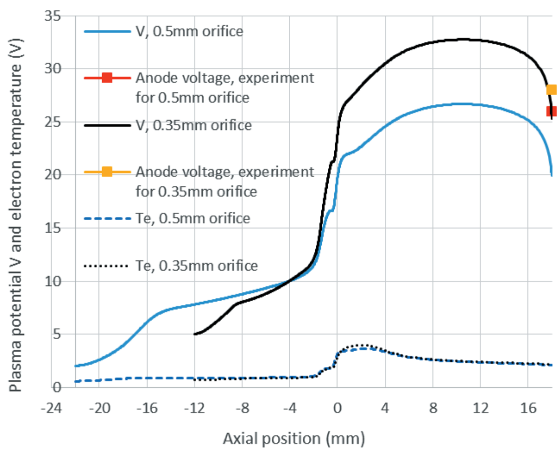

Figure 4. Plasma potential V and electron temperature T_e on axis. 0mm is the outlet of orifice.

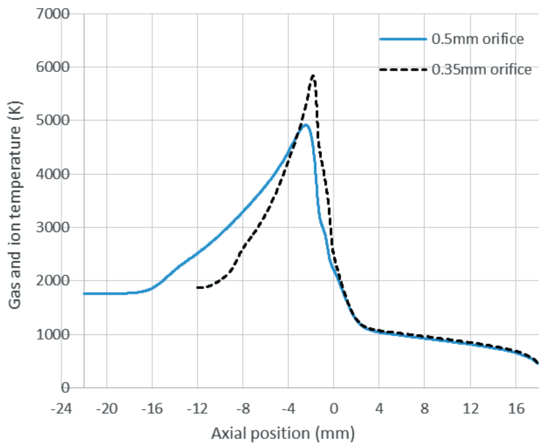

Figure 5. Neutral gas and ion temperature T on axis. 0mm is the outlet of orifice.

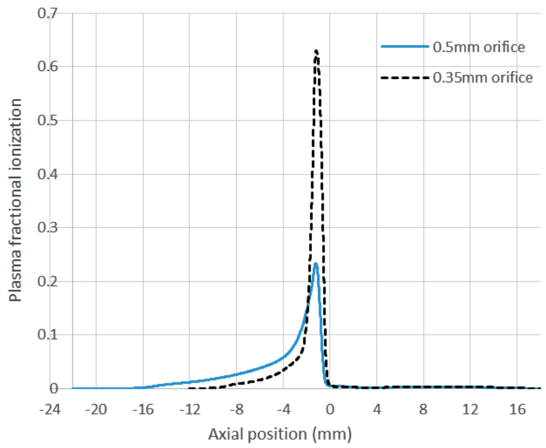

Figure 6. Plasma fractional ionization on axis. 0mm is the outlet of orifice.

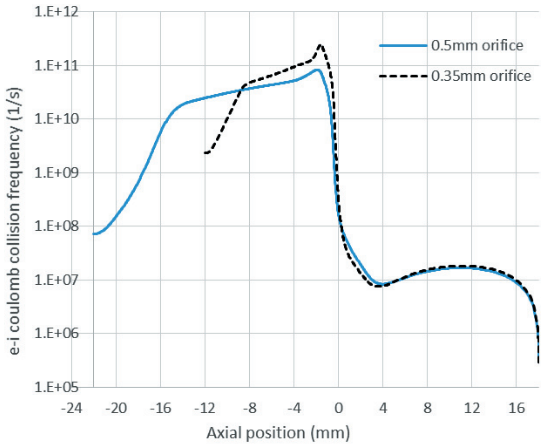

Figure 7. Electron and ion coulomb collision frequency on axis. 0mm is the outlet of orifice.

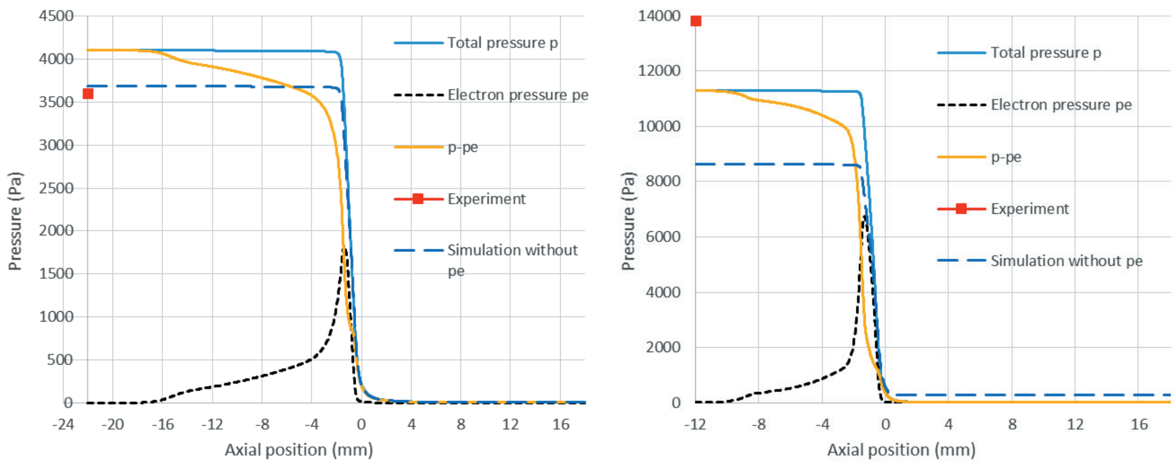

Figure 8. Inner gas pressure *p*, electron pressure p_e, p-p_e, and experiment results along axis. Orifice diameter is 0.5mm (left), and 0.35mm (right). 0mm is the outlet of orifice.

B. Electron Pressure

As shown in Fig. 8, the electron pressure, p_e, plays an important role in the gas pressure inside cathode, especially when the orifice is small. The discharge current is 4A, gas flow rate is 4sccm for 0.5mm orifice, 3sccm for 0.35mm orifice, and a_v is 0.5. In the larger orifice, the electron pressure has a certain value that indeed influences the inner gas pressure, but won't cause a too large error if ignored. While in the smaller orifice, the electron pressure can have a maximum value of 6kPa, so the error will be great without considering the electron pressure. The gas pressure $p = \Sigma n k_B T$, from which the reason why p_e is not negligible comes down to the high fractional ionization and electron temperature in small orifice.

The gas fluid functions can also be solved by defining the pressure as massive particles pressure only, thus the functions should include the electric field and collision forces of massive particles, and include the source terms if defining the gas pressure as pressure of neutral atoms. However, this would make the functions unnecessarily complex and imprecise because of the number of dependent assumptions. Solving the fluid functions of whole plasma directly is better.

C. Slip Boundary condition

The influence of fluid boundary conditions and comparison with experiments are shown in Fig. 9. The discharge current is 4A, and gas flow rate is 4sccm for 0.5mm orifice, 3sccm for 0.35mm orifice, with electron pressure effect. The tangential momentum accommodation coefficient a_v has a certain effect on the gas pressure. For the larger orifice, gas pressure when a_v equals to 0.5 is very close to experiment, while when a_v equals to 0.8, the calculated pressure is a little higher. For the smaller orifice, calculated gas pressure is slightly lower in both cases. The no slip boundary condition results in higher simulation pressure value than experiment. The Knudsen number (Kn) is about 0.1-0.2 in the larger orifice, and 0.03-0.1 in the smaller. According to Ref. 6, the gas flow is in slip-flow regime in the smaller orifice, and transition regime in the larger one. This may be the reason why the pressure is a little higher than experiments when orifice is larger. Since the Kn is not too large in the smaller orifice, the slip boundary condition (eq. (14)) can still be used.

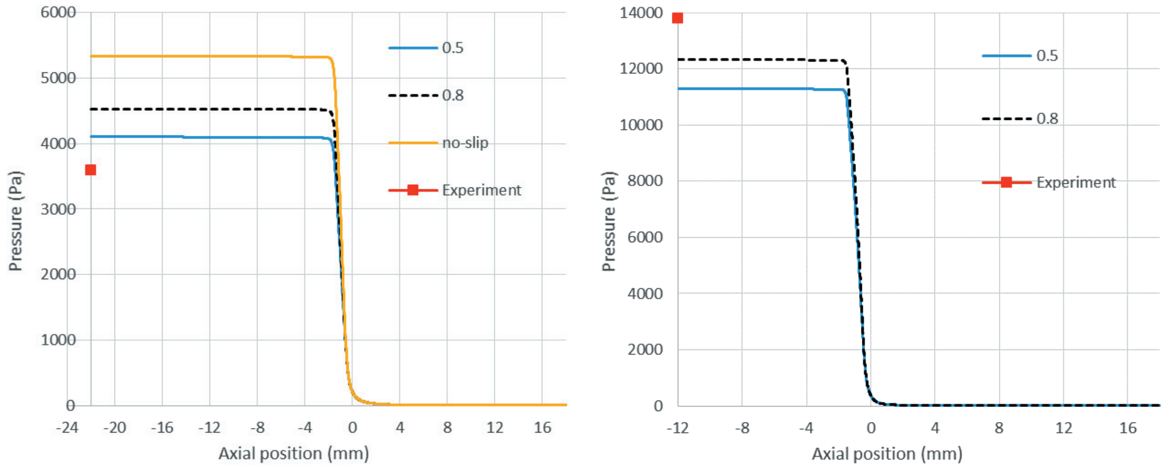

Figure 9. Inner gas pressure with a_v equals to 0.5, 0.8, no slip and experiment results along axis. Orifice diameter is 0.5mm (left), and 0.35mm (right). 0mm is the outlet of orifice.

V. Conclusion

A 2D-axisymmetric hollow cathode model is established and compared to experiments for two orifice diameter sizes. Because of the extremely large coulomb collision frequency between electrons and ions, the calculated plasma density and fractional ionization in small orifice can reach very high values, resulting in electron pressure not negligible when simulating cathode inner pressure. The slip velocity boundary condition should also be employed to simulate cathode inner pressure since the Knudsen number is large enough in the orifice.

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