Numerical Study on Diffusion Effects in Magnetoplasmadynamic Arcjet Thrusters

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In the present study, the effects of momentum and thermal diffusion on numerical prediction of the operation of self-field (both co-axial and 2D) Magnetoplasmadynamic Arcjet thrusters is investigated using a single fluid Argon model. The plasma regime in the thruster is modeled either as fully or partially ionized perfect gas flow. The flow and electromagnetic field equations (derived from Maxwell's equations), which govern the plasma flow, are discretized using a finite volume approach. The validated numerical code is used to simulate 2D and electromagnetic thrust equivalent axisymmetric MPD Arcjet thrusters. The diffusion effects are prominent where thermal thrust contributes significantly to the thrust. Diffusion is seen to lower the predicted specific impulse for both fully ionized and partially ionized plasma models. However, the effect is more significant for the partially ionized plasma model.

Nomenclature

2D	=	two-dimensional
Δx	=	grid size in x-direction
α	=	co-efficient to specify the amount of current through cathode tip
β	=	electron Hall parameter
ε_0	=	permittivity of free space (=8.854×10^{-12} farad/m)
$\boldsymbol{\mu_0}$	=	magnetic permeability of free space (=$4\pi\times 10^{-7}$ henry/m)
κ	=	Boltzmann constant (=1.38×10^{-23} J/K)
ρ	=	density
σ	=	electrical conductivity
ω_e	=	gyro frequency of electrons
τ_e	=	collision time of electrons
$\mathbf{B}$	=	self induced magnetic field
B_0	=	self induced magnetic field at inlet
d	=	diameter
e	=	charge of a proton (=1.602×10^{-19} C)

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e_t = total specific energy of the fluid
E = electric field
F = thrust
g_0 = gravitational acceleration on earth (=9.81 m/s^2)
I = input current
I_{sp} = specific impulse
j = current density
E = inlet mass flow rate
M = Mach number
MPDT = Magnetoplasmadynamic Thruster
n = outward normal vector
n_e = number density of electrons
p = pressure
R_{gas} = specific gas constant
r,θ,x = cylindrical coordinates
t = time
T = temperature
v = velocity vector (u, v components)
W = width of the channel in 2D MPDT
x,y,z = Cartesian coordinates
K = Heat conduction coefficient
μ = viscosity coefficient
$\overline{C}$ = Average thermal velocity
Q = collision cross section
$\overline{\overline{\tau}}$ = stress tensor
q = heat flux
α = degree of ionization

Subscripts
a = anode
c = cathode
coax = axisymmetric
em = electromagnetic
e = electron
ii = ion-ion
in = ion-neutral
nn = neutral-neutral

Introduction

Magnetoplasmadynamic (MPD) Arcjet thrusters are electromagnetic class of plasma propulsion thrusters that converts electrical energy into plasma kinetic energy. The plasma rocket engine is accomplished by having both electric and magnetic fields perpendicular to each other, across a plasma. The resulting current density '***J***' flowing in the direction of the applied field '***E***' gives rise to a '***JxB***' Lorentz force, per unit volume, which accelerates the plasma out of rocket. Here '***B***' is the induced magnetic field from the current flowing in the electrodes. The associated reaction force accelerates the rocket in the direction opposite to the plasma flow.

There have been various analytical, experimental and numerical studies that bring out the effect of Diffusion terms on thrusters performances[1-3] (Thrust and I_{sp}).These studies report an unexpected low values of thrust and hence I_{sp} ,due to the detrimental effects of viscous drag. On the other hand, DiCapua[4] and Villani [5] have shown from order of magnitude analysis that viscous dissipation is probably a second order phenomenon with little effect on the thrusters performance. This article presents some of the interesting observations resulting from a comparative study on 2D and electromagnetic thrust equivalent axisymmetric thruster geometry at various operating conditions.

An outline of formulation and numerical methodology used to develop the 2D and axisymmetric MPD thruster model is presented concisely in following sections for completion Further details on the numerical model and code validation can be found in Ref. [6,7].

Formulation

The plasma flow in the thruster is modelled based on some typical assumptions and by using generic governing equations to simulate important physical processes in the thruster. Argon (propellant) is assumed to be partially and fully ionised with single level of ionisation, as it enters the chamber. It is considered to be thermally non conducting and quasi-neutral (n_i=n_e) plasma throughout the system of operation. Further, the plasma flow is two-dimensional (x-y plane) and the magnetic field acts only in third (z) direction for 2D-MPDT. In axisymmetric MPDT, the flow is axisymmetric (x-r plane) and magnetic field acts only in the azimuthal (θ) direction.

The performance prediction of an MPD Arcjet thruster is accomplished by solving integral form of compressible mass, momentum and energy conservation equations with the electromagnetic body forces and Joule (Ohmic) heating included as source terms in momentum and energy equations[6, 7.] Maxwell's equations together with Ohm's law, neglecting Hall Effect and ion slip, and Ampere's law, neglecting displacement current[8], are used to determine magnetic field and current density. The above stated governing equations are presented below

Mass conservation equation:

$$\frac{\partial}{\partial t}\int_{\Omega}\rho d\Omega+\oint_{s}\rho\mathbf{v}\cdot\mathbf{n}ds=0 \tag{1}$$

Momentum Equation:

$$\frac{\partial}{\partial t}\int_{\Omega}\rho\mathbf{v}d\Omega+\oint_{s}\rho\mathbf{v}\mathbf{v}\cdot\mathbf{n}ds=-\oint_{s}p\mathbf{n}ds+\int_{\Omega}\mathbf{j}\times\mathbf{B}d\Omega+\oint_{s}\left(\overline{\overline{\tau}}.\vec{n}\right)dS \tag{2}$$

Energy Equation:

$$\frac{\partial}{\partial t}\int_{\Omega}\rho e_t d\Omega+\oint_{s}(\rho e_t+p)\mathbf{v}\cdot\mathbf{n}ds=\int_{\Omega}\mathbf{j}\cdot\mathbf{E}d\Omega-\oint_{s}\left(q.\vec{n}\right)dS+\oint_{s}\left(\overline{\overline{\tau}}.\vec{v}\right).\vec{n}dS \tag{3}$$

Equation of State:

$$p=\rho R_{gas}T \tag{4}$$

Equation of Magnetic flux density:

$$\frac{\partial\mathbf{B}}{\partial t}-\nabla\times(\mathbf{v}\times\mathbf{B})+\nabla\times(\frac{1}{\sigma\mu_0}\nabla\times\mathbf{B})+\nabla\times(\frac{\beta}{\mu_0}(\nabla\times\mathbf{B})\times\mathbf{B})=0 \tag{5}$$

Electrical conductivity for Argon plasma is given by Spitzer-Harm formulation.

$$\sigma=1.5\times10^{-2}\frac{T^{3/2}}{\ln\Lambda} \tag{6}$$

Where Coulomb logarithm $\ln\Lambda$ is $\ln\Lambda=\ln\left[\frac{12\sqrt{2}\pi\varepsilon_0^{3/2}(\kappa T)^{3/2}}{e^3 n_e^{1/2}}\right]$

The electron Hall parameter is taken as[9,10]

$$\beta=\frac{\omega_e\tau_e}{|B|\sigma}=\frac{1}{en_e} \tag{7}$$

Normal viscous stresses is taken from Stokes hypothesis

$$\tau_{xx}=2\mu\left(\frac{\partial u}{\partial x}-\frac{1}{3}\vec{\nabla}.\vec{v}\right) \tag{8}$$

$$\tau_{yy} = 2\mu\left(\frac{\partial u}{\partial x} - \frac{1}{3}\vec{\nabla}.\vec{v}\right) \tag{9}$$

$$\tau_{xy} = \mu\left(\vec{\nabla}.\vec{v}\right) \tag{10}$$

Where μ , coefficient of viscosity is taken as[11]

$$\mu = \mu_n \frac{1-\alpha}{1-\alpha+\alpha\dfrac{Q_{in}}{Q_{nn}}} + \mu_i \frac{\alpha}{\left(1-\alpha\right)\dfrac{Q_{in}}{Q_{ii}}+\alpha} \tag{11}$$

$$\mu_n = \frac{m_i \overline{C_i}}{2Q_{nn}} \tag{12}$$

$$\mu_i = \frac{0.406\left(4\pi\varepsilon_0\right)^2 \sqrt{m_i}\left(k_b T\right)^{\frac{5}{2}}}{e^4 \ln\Gamma} \tag{13}$$

$$Q_{nn} = 1.7\times10^{-18} T^{-\frac{1}{4}} \tag{14}$$

$$Q_{in} = 1.4\times10^{-18} \tag{15}$$

$$Q_{ii} = \frac{e^4 \ln\Gamma}{32\pi\varepsilon_0{}^2 k_b{}^2 T^2} \tag{16}$$

$$\overline{C_i} = \sqrt{\frac{8k_b T}{\pi m_i}} \tag{17}$$

$$\Gamma = 1.24\times10^7 \sqrt{\frac{T^3}{n_e}} \tag{18}$$

Thermal conductivity model for both fully and partially ionized model is taken as[12]

$$K = \frac{1.7142 k_b{}^2 T\sigma}{e^2} \tag{19}$$

All the variables in the above equations are non-dimensionalized with chosen reference quantities and the governing equations in non-dimensional form are used for the numerical solution.

Numerical Methodology

The governing equations taken in non-dimensional form are discretized using a dual-control volume finite volume method. For the flow equations, central scheme with Jameson's artificial dissipation is used to compute convective fluxes. After spatial discretization the equations become ODEs in time. Then they are integrated using an explicit four-stage Runge-Kutta method.

The magnetic field equation is discretized using a central (averaging) type scheme. The gradient terms are computed based on Green's theorem. The spatially discretized ODE is integrated in time using ADI (Implicit Euler) method. Thomas algorithm is used for the solution of tri-diagonal form of equations. The magnetic field equation is solved separately at each flow time step and the electric and magnetic fields are coupled to the flow equation through momentum and energy source terms.

Only top half of the thruster is modeled assuming flow symmetry about the longitudinal axis. Separate boundary conditions are used to obtain the solution of flow and electromagnetic equations. Subsonic/supersonic inflow, no slip wall, centerline symmetry and supersonic outflow are the flow boundary conditions used. Boundary conditions for the electromagnetic equations simulate a constant current at inlet and current conducting electrode surfaces (anode and cathode). Other walls, symmetry plane and exit boundary are considered as non-conducting surfaces.

The convergence of numerical solution is monitored using L2 norm residuals of flow equations and electromagnetic induction equation (at each flow time step). In addition, $L\infty$ norm error in pressure and the difference in net mass flow rate crossing the domain boundaries are also monitored.

Typical thruster parameters such as thrust, its various components (momentum, pressure, thermal and electromagnetic), I_{sp} and input power are calculated from the flow field data. The numerical model of 2D and axisymmetric thrusters are validated separately against available experimental and numerical results in the literature.

Results and Discussions

A 2D thruster, with its dimensions similar to experimental straight-anode thruster[13] is taken for this numerical study. A co-axial MPD arcjet is also simulated, the dimensions of which are chosen considering it generates electromagnetic thrust equal to the 2D thruster[6, 7]. These are taken as the base geometry for this study. Although the experimental thruster consists of only a straight anode, in the computational model geometry a diverging insulator section is fixed at exit to avoid an abrupt expansion region that could cause numerical instability to the flow model. This diverging section also converts any available pressure thrust into momentum component.

The simulations were first carried out for the 2D[13] thruster model with Argon mass flow rate of 2.5g/s, inlet Mach number 0.3 , inlet Argon temperature 5000K . Discharge current was varied from 8kA to 14kA**.**

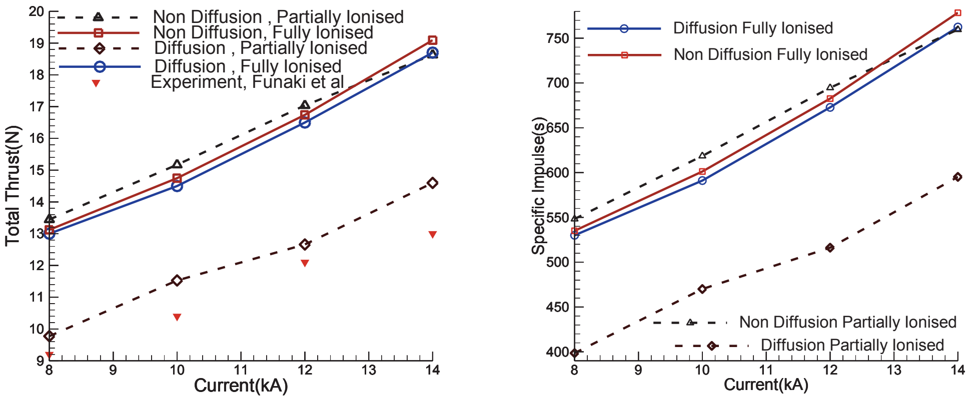

Figure1. a) Total Thrust b) Specific Impulse. For discharge current 8-14kA and mass flow rate of 2.5g/s

One can note that for a given geometry the total thrust (Fig.1(a)) and specific impulse (Fig.1(b)) with diffusion effects considered yield lower values compared to ones obtained using the non-diffusion model. In the simulations with non-diffusive model partial ionisation plasma model improves temperature prediction[7] over that predicted with fully ionized model, however, the thrust predicted is still much higher than the experimentally observed values[13] (see Fig 1(a)). With inclusion of diffusion the partially ionized plasma model predicts thrust much closer to the experimental values and the temperature (not shown here) are also predicted better (lower and closer to those from experiments).

It is also noted that the ionization fraction distribution obtained from partially ionized non-diffusion model goes up to 90-95 % and hence performance predictions are almost like those from fully ionized non diffusion model. On the other hand with partially ionized diffusion model, ionization fraction distribution was low at about 75-80 %. This implies that a part of input electrical energy will be utilized for further ionization which in turn will reduce the overall temperature and thus the thermal component of thrust which forms a significant part of total thrust at high mass flow rates.

The co-axial thruster with electromagnetic thrust equal to that of 2D thruster was simulated for mass flow rate of 2.5 g/s and input current of 8 kA. The two thrusters produce an electromagnetic thrust of about 2.38 N for the given current. The performance of this thruster, for the above mentioned conditions and same inlet parameters is presented in a table 1. The thruster qualitative performance is identical to that of the 2D thruster except that the thrust valus are higher.

Co-axial Thruster	Total Thrust(N)	Specific Impulse(s)
Non-diffusion, partially ionized model	15.2	620.40
Diffusion, partially ionized model	10.2	416.32
Non-diffusion, fully ionized model	13.6	551.10
Diffusion, fully ionized model	11.73	478.77

Table 1. Total Thrust and Specific Impulse. For discharge current 8kA and mass flow rate of 2.5g/s

In order to assess and ensure the generality of the trend observed above at other mass flow rates (i.e. varying thruster operation from electromagnetic thrust dominant regime to thermal thrust dominant regime) a parametric study was performed for both 2D and axisymmetric thrusters for discharge current of 10kA and Argon mass flow rate varying from 0.4g/s to 5g/s. The other inlet conditions were maintained.

Figure 2(a) and 2(b) show the total thrust predicted by various models for both the 2D thruster (Fig. 2(a)) and the co-axial thruster (Fig. 2(b)). The electromagnetic thrust component in all the cases depicted in Fig. 2 is 4 N. One can note that the overall trend described in the section above remains the same but the difference in performance prediction between diffusion and non-diffusion models decreases at low mass flow rates where the electromagnetic thrust becomes dominant. It is to be noted that even when electromagnetic thrust component is equal to the thermal thrust component the thrust prediction from partially ionized diffusion model is significantly lower than those predicted by other models. Thus no-diffusion assumption appears to be valid where thermal thrust is negligible compared to the electromagnetic thrust.

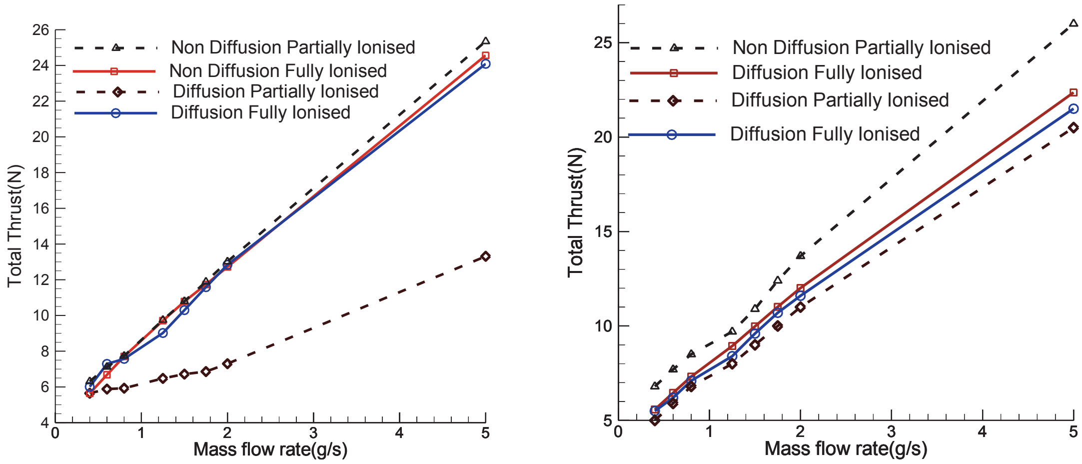

Figure2. Total Thrust for discharge current 10kA and mass flow rate of 0.4g/s-5g/s (a) 2D MPDT (b) Axisymmetric

Conclusion

The self-field MPD thruster was numerically modeled using 2D and axisymmetric thruster models. An existing numerical Eulerian code was modified to compute 2D/axisymmetric viscous compressible flows along with thermal diffusion term in the energy equation. The plasma was modeled both as fully and partially ionized perfect gas undergoing electromagnetic interaction with applied current and induced magnetic fields.

Simulations revealed that the diffusion effects can significantly alter the thruster performance prediction especially under conditions where thermal thrust component is significant. The diffusion effect yield lower maximum temperature and thruster performance compared to the non-diffusion models. The effect of diffusion model is more drastic and realistic with partially ionized plasma model.

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