A new thrust measurement method for evaluating higher frequency variation by applying acceleration measurement to null-balance method

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Akira Kakami[1], Kenji Kashihara[2], Shota Takeshida[3], and Yasuyuki Yano[4]

University of Miyazaki, Miyazaki, 889-2192, Japan

Abstract: We propose a new thrust stand measuring thrust variation beyond the resonant frequency. The proposed method is null-balance thrust measurement method with acceleration based compensation. A controller monitors pendulum motion and adjusts a solenoid actuator in such a way that the pendulum deflection angle is kept constant. Spontaneously, acceleration and solenoid-actuator current was measured during active control. Thrust is directly determined from the current and acceleration because spring constant and damping factor originating from wires and the thrust stand can be ignored due to null-position active control. The prototype thrust stand successfully measured thrust with its variation in the frequency range from 0 to 85 Hz, whereas conventional null-position method yielded 410-% overestimation at a resonant frequency of 30 Hz. Beyond 90 Hz, the proposed method showed errors in sensitivity and phase shift. The deterioration in accuracy is caused by the translational flexibility in torsional hinge. Measurement of translational spring constant of the torsional hinge and numerical simulation with a pendulum fixed with a hinge having rotational and translational flexibility yielded a correlation between frequency and accuracy, which is similar to that of the experiment.

Nomenclature

c	=	Damping coefficient of pendulum for rotational motion
c_{h}	=	Damping coefficient of torsional hinge for translational motion
E	=	Driving voltage for solenoid actuator
E	=	Thrust to current ratio of solenoid actuator
i	=	Solenoid actuator driving current
I_{G}	=	Moment of inertia around center of gravity
I_m	=	Moment of inertia around rotating axis of torsional hinge
k	=	Torsional spring constant of torsional hinge
k_{h}	=	Translational spring constant of torsional hinge
l_{a1}	=	Length between torsional hinge and accelerometer 1
l_{a2}	=	Length between torsional hinge and accelerometer 2
l_{G}	=	Length between torsional hinge to center of gravity of thrust stand pendulum

[1] Associate professor, Department of Mechanical Design Systems Engineering, akira.kakami@gmail.com.
[2] Graduate student, Department of Mechanical Systems Engineering.
[3] Former undergraduate student, Department of Mechanical Systems Engineering.
[4] Staff, Faculty of Engineering, yasuyuki@cc.miyazaki-u.ac.jp.

l_s	=	Length between torsional hinge to solenoid actuator
l_t	=	Length between torsional hinge to thruster
L	=	Inductance of solenoid actuator
R	=	Resistance of solenoid actuator
T	=	Thrust
T_{cur}	=	Thrust determined from actuator-driving current using the null balance method
T_{pro}	=	Thrust determined with proposed method
T_{ref}	=	Reference thrust (force exerted with a calibrator)
x_{h}	=	Displacement originating from the translational flexibility of torsional hinges
θ	=	Angular displacement of pendulum
Δ	=	Amplitude

I. Introduction

Pendulum-type thrust stands have been utilized for evaluating performance of space propulsion devices[1-3]. Thrust is evaluated using pendulum displacement, since angular displacement of rigid body pendulums is proportional to external force, i.e. thrust, as long as displacement is sufficiently small. Whereas the thrust stand, which has relatively simple structure, evaluates thrust accurately, it has difficulty in measuring time variation in thrust because of resonance. Below one third or one tenth of the resonant frequency of the pendulum, displacement presents instantaneous thrust without delay. At higher frequency, pendulum displacement exhibits lags from varying thrust, and its dependence on frequency.

The null balance method is also applied to thrust measurement[4-8]. This method is basically equivalent to a classical balance scale. An actuator is continuously adjusted to null pendulum displacement, and thrust is determined from control variables: actuator-driving current or voltage. Digital control, whose parameters are flexibly adjustable, is applicable to the method, and accordingly, enables flexible adjustment in sensitivity and frequency response. Nevertheless, resonance yielded errors in thrust evaluation; the thrust stand showed overestimation in the vicinity of the resonant frequency, and underestimation in relatively high frequency domain[4]. From these points of view, the conventional thrust measurement device is not able to appraise high frequency thrust variation.

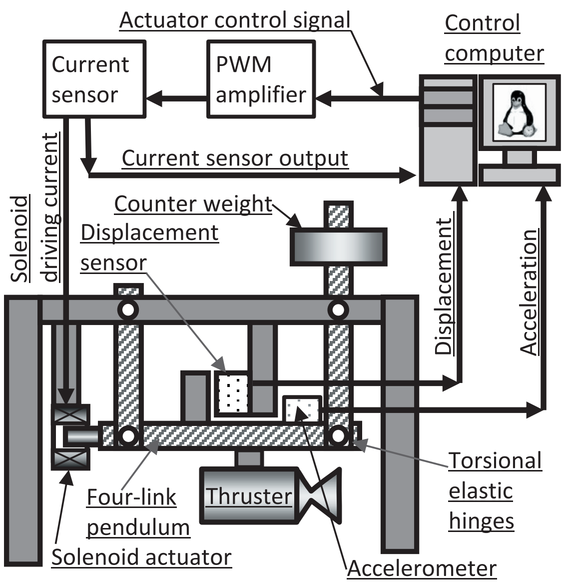


Figure 1. Prototyped thrust stand using null balance method and acceleration compensation.

The authors have proposed to apply acceleration measurement to the null balance method in order to determine higher frequency thrust variation than the resonant frequency[9,10]. The proposed method successfully valuated time variation in thrust in the frequency range from 0 to 75 Hz whereas it yielded a resonant frequency of 20 Hz. Nevertheless, the prototyped thrust stand presented deterioration in accuracy beyond 80 Hz.

In this study, the thrust stand was tested in the frequency range up to 320 Hz to clarify the frequency dependence, which would give the information regarding accuracy deterioration in the higher frequency regime. Then, the cause of the deterioration in accuracy was investigated using simulation and experiment.

II. Proposed method

The Newton equation for a thrust-stand pendulum is expressed as

$$I_m\ddot{\theta} + c\dot{\theta} + k\theta = l_t T + l_s F_0 i \tag{1}$$

For simplicity, $\theta = 0$, which is an equilibrium point of the pendulum where the gravity and the force given by tubes and wires are balanced, is target point of the nulling control. In the case that proportional-integral-derivative (PID) control is applied to determine solenoid actuator control voltage E, the equations regarding driving circuit is described as

$$E = L\dot{i} + iR \tag{2}$$

$$E = k_p\theta + k_d\dot{\theta} + k_I\int\theta dt \tag{3}$$

By selecting appropriate PID variables: k_p, k_d, and k_I, we can assume that the angular velocity and displacement is sufficiently small so that the equation $c\dot{\theta} + k\theta \approx 0$ is valid. Then, Eq. (1) is converted into

$$T = \frac{I_m}{l_t}\ddot{\theta} - \frac{l_s F_0}{l_t} i \tag{4}$$

The equation shows that thrust can be determined using angular acceleration $\ddot{\theta}$ and solenoid-actuator driving current i, and that accuracy is independent of frequency. The second term in the left side in Eq. (4) represents the conventional null balance method. Hence, the first term, which is newly added in the proposed method, is an acceleration-based corrector for the conventional null balance method. In accordance with the modern control theory, Eq. (4) exhibits a sort of disturbance observer.

III. Prototyped thrust stand and calibration procedure

A. Prototyped thrust stand

Figure 1 illustrates a schematic diagram of the prototyped thrust stand. This is a four-link pendulum with four frictionless torsional hinges, in which three stainless-steel tubes were connected through plate-type springs. Pendulum displacement was measured with a displacement sensor with tens of nanometer resolution (Japan System Development, DS2001-TS01). The sensor constituted an inductance-capacitance resonant circuit with an opposing metal plate. The resonant frequency was dependent on the gap between the metal plate and the sensor body. An embedded digital counter evaluated the number of voltage pulses in the resonant circuit while an external signal input was positive, and then output the count using a serial digital output.

Pendulum displacement is nulled using a solenoid actuator by adjusting driving current. Angular acceleration $\ddot{\theta}$ was valuated using a piezoelectric type accelerometer and an amplifier with a gain of 50. Solenoid actuator driving current was measured using a Hall-effect current sensor.

B. Control systems

A discrete Kalman filter, which was implemented with GNU Scientific Library (GSL) [11] to manipulate matrices and vectors, determined θ, $\dot{\theta}$, and $\int\theta dt$ from measured values of the displacement sensor. Solenoid-actuator driving voltage was calculated using Eq. (3). Programs for Kalman filter and control system run in a Linux-based personal computer with Real-Time Application Interface (RTAI) [12], a real time Linux variant, in order to execute control for nulling pendulum displacement in an accurate period of 1 ms. For evaluating 320-Hz class thrust variation, measured values from the accelerometer and current sensor were also monitored with the computer in the period of 100 μs.

C. Calibration

Replacing coefficients in Eq. (4), thrust is expressed as

$$T = \alpha\ddot{\theta} + \beta i \tag{5}$$

Coefficients α and β needed to be calculated with calibration. We prototyped a calibrator using a load cell and solenoid actuator, and conducted *static* and *dynamic* calibration. In static calibration, procedure of which was the same as conventional procedure, constant force was exerted to the thrust stand pendulum through the load cell. In the case that appropriate PID variables for the nulling control of the pendulum is stable, β can be calculated from the correlation between calibration force and i because $\ddot{\theta}$ is negligible.

For dynamic calibration, the solenoid actuator was driven with biased sinusoidal current to provide sinusoidally-varying biased thrust. Simultaneously, pendulum acceleration $l_a\ddot{\theta}$ and i were measured to determine α using β, which had already evaluated with static calibration.

IV. Experimental results

A. Static calibration results

Figure 2 shows the dependence of current sensor output, which represents actuator-driving current i on reference thrust T_{ref}. Error bars present a standard deviation of time-varying values during measurement. As illustrated in Fig. 2, current sensor output voltage, i.e. current i is linearly decreased with reference thrust. For current sensor, positive value indicates that the actuator produces a force in the same direction as thrust. The null balance method produces negative force so that pendulum displacement keeps constant.

Static calibration yields a linearity correlation between i and T_{ref} with a coefficient of determination R^2 of 0.9981 when a static thrust was exerted. From the result, the thrust stand accurately measured static thrust.

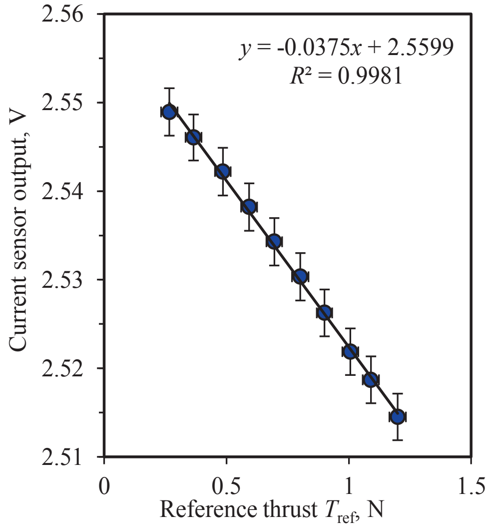


Figure 2. Correlation between reference thrust and current sensor output voltage for actuator driving current. Error bars shows standard deviation.

B. Dynamic calibration results

Figure 3 shows the dependence of amplitude ratio of evaluated thrusts to reference thrust $\Delta T_{\mathrm{pro}}/\Delta T_{\mathrm{ref}}$ and $\Delta T_{\mathrm{cur}}/\Delta T_{\mathrm{ref}}$ on frequency. The ideal value of amplitude ratios are 1, which shows no discrepancy between evaluated and reference thrusts for the corresponding frequency component.

Both the null balance and proposed methods successfully determined thrust in the frequency range no more than 7 Hz. Figure 4 illustrates a time history at 3 Hz. Both methods accurately follow varying reference thrust without phase delay.

In the vicinity of the resonant frequency, the null balance method shows inaccurate valuation. As presented in Fig. 3, amplitude ratio $\Delta T_{\mathrm{cur}}/\Delta T_{\mathrm{ref}}$ increases below the resonant frequency of 30 Hz, reaches the maximum value at the resonant, and afterwards, decreases monotonically. Moreover, the non-negligible phase delay was presented, as shown in Fig. 5. In contrast, the proposed method accurately evaluates thrust with errors below 10% at the frequencies less than 85 Hz. From these results, while the null balance method showed developing errors above one-third of the resonant frequency, the proposed method continued to show accurate thrust.

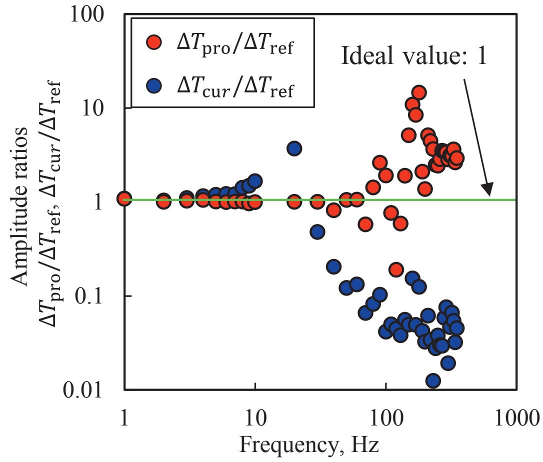


Figure 3. Dependence of ratio of evaluated thrusts to reference value $\Delta T_{\mathrm{pro}}/\Delta T_{\mathrm{ref}}$, ΔT_{cur} / ΔT_{ref} on frequency.

Nevertheless, above 85 Hz, the proposed method yielded errors in amplitude ratio $\Delta T_{\mathrm{pro}}/\Delta T_{\mathrm{ref}}$ with phase delay. Figure 6 illustrates a time history of both evaluated and reference thrusts at 100 Hz. The null balance method only sensed the stationary components of thrust but no varying component. The proposed method responded to time-varying thrust but showed overestimation with non-ignorable phase delay. At 320 Hz, the null balance method evaluated stationary component without responding varying components, and the proposed method clarified stationary components accurately, but showed 200-% errors with a phase delay of approximately 180°

Refocusing on Fig. 3, amplitude ratio $\Delta T_{\mathrm{pro}}/\Delta T_{\mathrm{ref}}$ exhibits frequency dependence that is similar to that for anti-resonant systems. Hence, the prototyped thrust stand possibly had a hidden flexible element, which produces local maximum of $\Delta T_{\mathrm{pro}}/\Delta T_{\mathrm{ref}}$ at 180 Hz. This hidden flexural element will be unveiled in the next section.

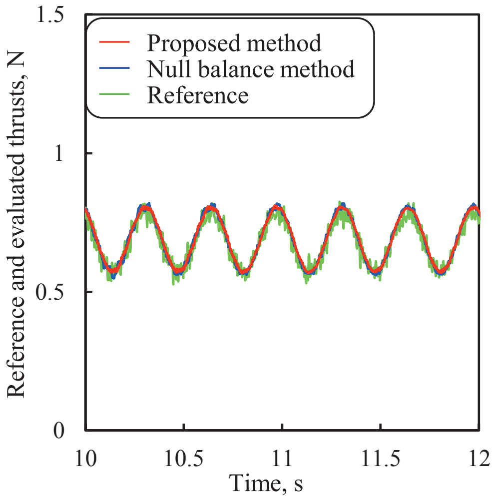


Figure 4. Time history of thrusts T_{ref}, T_{pro}, and T_{cur} for biased sinusoidally-varying reference thrust of 3 Hz in frequency.

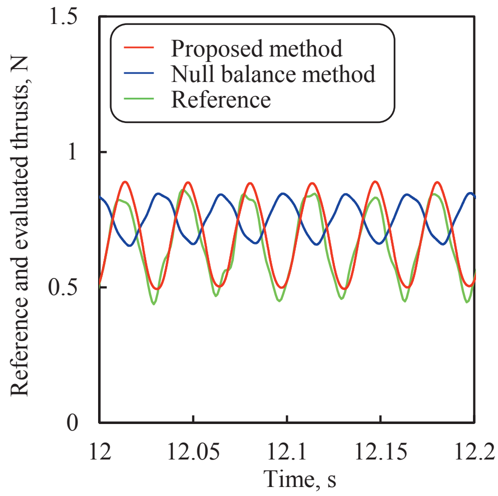


Figure 5. Time history of thrusts T_{ref}, T_{pro}, and T_{cur} for biased sinusoidally-varying reference thrust of 60 Hz in frequency.

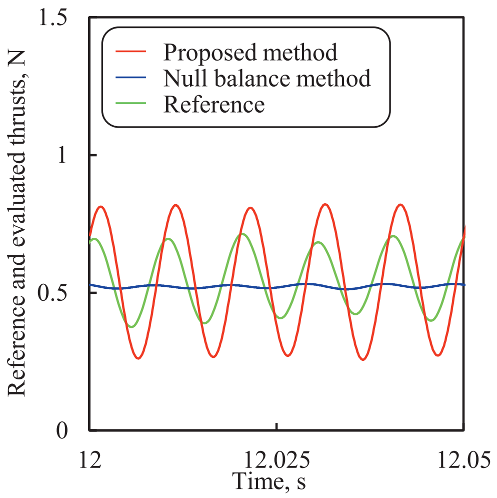


Figure 6. Time history of thrustsT_{ref}, T_{pro}, and T_{cur} for biased sinusoidally-varying reference thrust of 100 Hz in frequency.

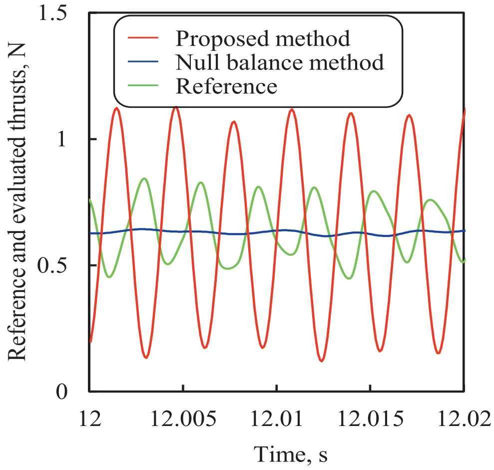


Figure 7. Time history of thrustsT_{ref}, T_{pro}, and T_{cur} for biased sinusoidally-varying reference thrust of 320 Hz in frequency.

C. Causes of deterioration of accuracy in higher frequency regime

The cause of deterioration of $\Delta T_{pro}/\Delta T_{ref}$ beyond 80 Hz is attributable to translational flexibility of the torsional hinges. The torsional hinges, in which three tubes were connected with flat springs, had a translational flexibility. A measurement device was prototyped to determine translational spring constant. The device had a load cell on a linear actuator and laser displacement sensor. The linear actuator exerted a force through the load cell to the center tube of the torsional hinge, the both side tubes of which were fixed on the device. Translational spring constant was evaluated using the correlation between force and displacement. The measurement using the device yielded a spring constant of 7.7×10^5 N/m.

For the purpose of showing that the translational flexibility of the torsional hinge caused anti-resonant-like dependence of $\Delta T_{pro}/\Delta T_{ref}$ on frequency, simulation was conducted using the dynamical model presented in Fig. 8. This model is equivalent to a rigid body pendulum with two masses, i.e. thruster and counter weight, and the torsional hinge that is fixed to the wall using a spring of 7.7×10^5 N/m in spring constant. A variable x_h is a translational displacement of the pendulum at the hinge and is a function of time. The Newton equations having

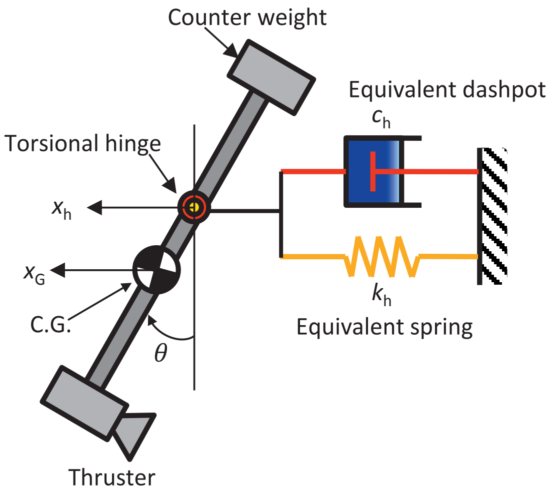


Figure 8. Dynamical model for thrust stand with the torsional hinge that has translational flexibility.

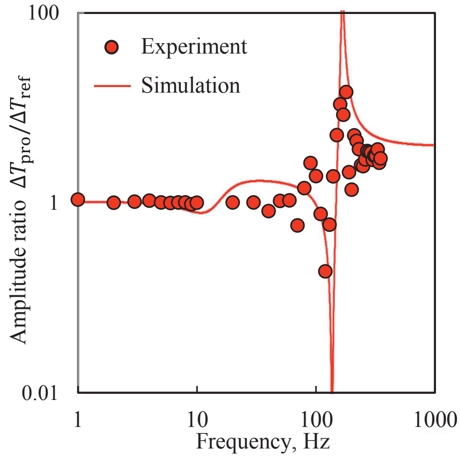


Figure 9. Frequency characteristics of ratio of evaluated thrust to reference value for the proposed method.

dependent variables θ and x_h and an independent variable time t are expressed as second order ordinary differential equations. The differential equations are numerically solved with Scilab.

Figure 9 shows empirical and numerically-calculated dependence of $\Delta T_{pro}/\Delta T_{ref}$ on frequency. Both empirical and numerical simulation results exhibit a local minimum at 140 Hz, a local maximum at 180 Hz, and approaches a constant value of approximately 2.0 in high frequency regime. From the coincident results, translational flexibility deteriorated thrust evaluation beyond 80 Hz. Hence, in the next chapter, we will propose a new method for correcting errors using two-point measurement of acceleration.

V. Improvement by measuring two-point acceleration measurement and adjusting location of center of gravity

A. Proposed improvement and its principle

The experimental results and numerical simulation showed that translational flexibility negatively affected the accuracy of thrust evaluation in the higher frequency regime. In order to extend measureable frequency, we propose the following modification.

By adjusting location and weight of counter weight, the center of gravity of the pendulum including thruster and counter weight is located at the torsional hinge. This adjustment decouples rotational and translational motions in the Newton equation. Pendulum accelerations are measured at two points on the pendulum, as shown in Fig. 9. This allows to evaluate both true rotational and translational accelerations: $\ddot{\theta}$ and $\ddot{x}_h$. This $\ddot{\theta}$ is used in order to determine accurate thrust variation.

The principle can be explained using the Newton equations for traversal and rotational motions:

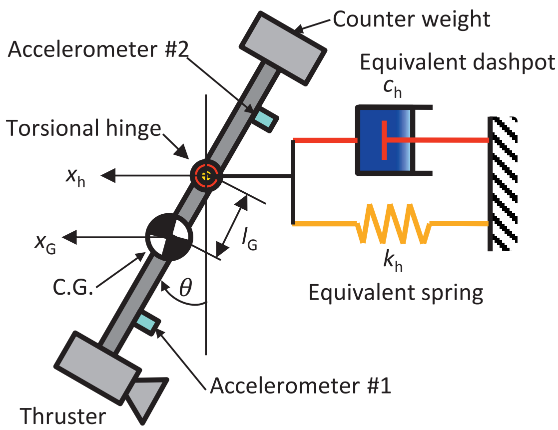


Figure 10. Dynamical model for proposed improvement method using two-point acceleration measurement.

$$I_G\ddot{\theta} + c\dot{\theta} + k\theta = l_t T + l_s F_0 i + l_G(k_h x_h + c_h \dot{x}_h) \tag{6}$$

$$m(\ddot{x}_h + l_G\ddot{\theta}) + c_h\dot{x}_h + k_h x_h = T + F_0 i \tag{7}$$

Here, mass and location of counter weights are adjusted so that the center of gravity for the pendulum is placed on the axis of the torsional hinges. The adjustment allows $l_G = 0$, and Eq. (6) is reduced to

$$I_G\ddot{\theta} + c\dot{\theta} + k\theta = l_t T + l_s F_0 i \tag{8}$$

, which is the same form as Eq. (1). Using appropriate setpoint-control parameters so that $c\dot{\theta} + k\theta \approx 0$, thrust is determined using Eq. (4). Nevertheless, the method requires true value of $\ddot{\theta}$, which the sole accelerometer on the pendulum cannot evaluate because it senses the sum of rotational and translational accelerations $\ddot{x}_{\mathrm{h}} + l_a\ddot{\theta}$.

For the purpose of evaluating true value of $\ddot{\theta}$, acceleration is appraised at the two points on the pendulum, as shown in Fig. 9. The measured accelerations are expressed as

$$\ddot{x}_{a1} = \ddot{x}_{\mathrm{h}} + l_{a1}\ddot{\theta} \tag{9}$$

$$\ddot{x}_{a2} = \ddot{x}_{\mathrm{h}} - l_{a2}\ddot{\theta} \tag{10}$$

Rearrangement of the equations for eliminating $\ddot{x}_{\mathrm{h}}$ yields

$$\ddot{\theta} = \frac{\ddot{x}_{a1} - \ddot{x}_{a2}}{l_{a1} + l_{a2}} \tag{11}$$

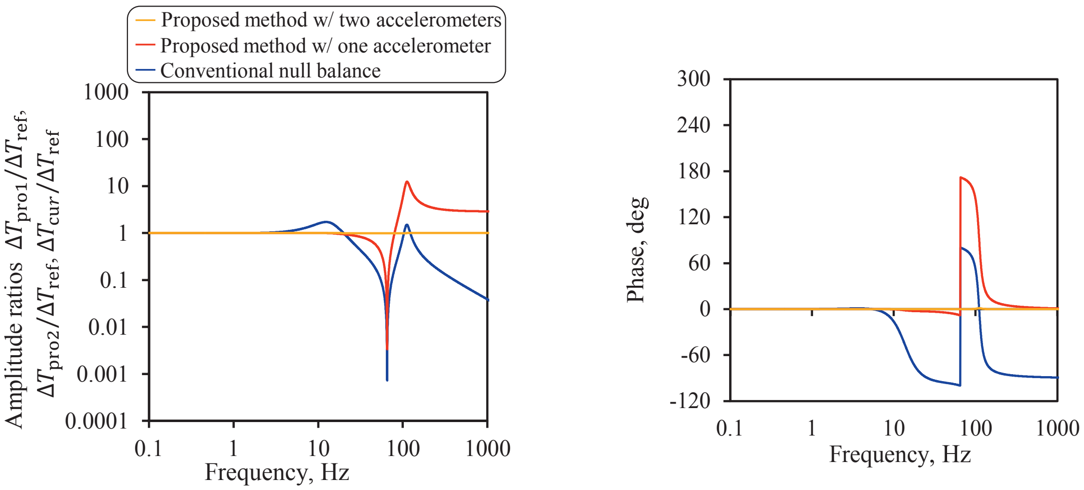
True value of is determined $\ddot{\theta}$ using Eq. (11), and is applicable to Eq. (8) in order to valuate thrust.

B. Feasibility evaluation using simulation

This proposed improvement method was numerically simulated using Scilab to show that it can estimate thrust accuracy in the wide frequency range from 0 Hz. Dynamical model is almost the same as Fig. 9 except that the center of gravity is placed on the axis of the torsional hinge. Numerically calculated dependence of amplitude ratio $\Delta T_{\mathrm{pro}}/\Delta T_{\mathrm{ref}}$ and phase shift is shown in Fig. 10 for the conventional null balance and proposed methods using one or two point acceleration measurement.

Whereas the ideal value is 1 for the amplitude ratio, the null balance method shows resonance at 10 Hz, and the proposed method using one point acceleration measurement presents anti-resonant like dependence on frequency. In contrast, the proposed method using two-point acceleration measurement yields an amplitude ratio $\Delta T_{\mathrm{pro}}/\Delta T_{\mathrm{ref}}$ of 1 in the frequency range up to 1000 Hz. Theoretically, the proposed method using two-point acceleration will be able to measure thrust with higher frequency.

In practice, however, non-rigidity of the materials for pendulums would induce resonance of 1000-Hz class in frequency, and deteriorate amplitude ratio $\Delta T_{\mathrm{pro}}/\Delta T_{\mathrm{ref}}$ even for the proposed method using two-point acceleration. Hence, the proposed improvement method may extend the measurable frequency up to approximately 300 Hz, which is one-third of the resonant frequency for a flexible beam with mass.



(a) Amplitude ratio（Ideal value is 1） (b) Phase delay (0 is ideal.)

Figure 11. Numerically-evaluated frequency characteristics of thrust evaluated by the proposed improvement method.

VI. Summary

The authors proposed a new thrust measurement method using the null balance method with compensation based on pendulum acceleration. The prototyped thrust stand was exerted with reference thrust in the frequency range from 0 to 320 Hz to investigate measurable frequency range. The following is the summary of the paper.

1. Both the null balance method and proposed method with using single-point acceleration compensation accurately determined thrust below 7 Hz.
2. Whereas the conventional null balance method exhibited overestimation for 30-Hz thrust variation, the proposed method one-point acceleration compensation evaluated thrust with an error of 10 %.
3. The proposed method with one-point acceleration compensation successfully determined thrust variation up to 85 Hz. Nevertheless, above 85 Hz, the proposed method exhibited the frequency dependence that was similar to that of anti-resonance.
4. The deterioration in accuracy of the proposed method was caused by the translational flexibility of the torsional hinges, where three tubes were connected through plate springs. Measurement showed that the torsional hinge had a translational spring constant of 7.7×10^5 N/m.
5. Numerical simulation of motion of the thrust stand with the torsional hinge that had the empirically-evaluated translational spring constant showed the frequency dependence of amplitude ratio $\Delta T_{\mathrm{pro}}/\Delta T_{\mathrm{ref}}$ that was similar to that yielded with the experiments. Hence, deterioration in accuracy of the proposed method was caused by the translational flexibility of the torsional hinge.
6. In order to extend the measurable frequency, we proposed a new improvement method using adjustment of the center of gravity of the pendulum and two-point measurement of acceleration.
7. In the improvement method, the center of gravity of the thrust-stand pendulum is located at the axis of the hinges by adjusting the mass and position of the counterweight. The pendulum accelerations are measured at two points of the pendulum to determine both translational and rotational accelerations: $\ddot{x}_{\mathrm{h}}$ and $\ddot{\theta}$. In the other words, the improvement method decouples rotation and translation in acceleration measurement and the Newton equation.
8. Numerical simulation showed that the improve method using two-point acceleration measurement and position adjustment of center of gravity successfully evaluated thrust variation in the frequency range from 0 to 1000 Hz. Whereas theoretically, the improved method may evaluate thrust frequency in the higher frequency, practically, it may be able to evaluate thrust variation up to 1000 Hz because of non-rigidity of pendulums.

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