Stabilizing Low Frequency Oscillation with Two Stages Filter in Hall thrusters

IEPC-2015-284／ISTS-2015-b-284

Presented at Joint Conference of 30th International Symposium on Space Technology and Science
34th International Electric Propulsion Conference and 6th Nano-satellite Symposium,
Hyogo-Kobe, Japan
July 4 – 10, 2015

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Abstract: **The filter is the most usually method to suppress low frequency discharge current oscillation in Hall thrusters. The only form of the filter in actual use involves RLC networks, which servers the purpose to reduce the level of conducted electromagnetic interference returning to the PPU, which is designed as a filter. Recently, the role of the filter in the oscillation control was introduced. It is comments that the filter regulates the voltage across itself according to the variation of discharge current so as to decrease its fluctuation in the discharge circuit, which is designed as a controller. Therefore, a kind of two stages filter is proposed to maintain these two purposes: filter and controller and the detailed design method is discussed.**

I. Introduction

Low frequency oscillation is one of the most important problems of Hall thrusters in its discharge process. The large amplitude of low frequency discharge current oscillation in the range of 10~100kHz may have an adverse effect on the power processing unit (PPU), lowers the efficiency and specific impulse, and shortens the operating life of a Hall thruster. The suppression of current oscillation is an essential challenge to be solved for the practical applications of the thrusters, not only to determine the effects of such oscillation on the thrusters operation, reliability, and lifetime, but also to control electromagnetic compatibility with the other equipments installed in a satellite.

From the birth of Hall thrusters to now, researchers have done a lot of work on the mechanism, characteristics, and methods of instability control of low frequency oscillation. This oscillation mode was identified as a "loop" or"circuit" oscillation due to it is sensitive to the circuit of the system in the 1970's in Russian literature[1]. Numerous models were proposed to study the mechanism of low frequency oscillations. The most representative models are the Fife's predator-prey model[2] and Boeuf's breathing type model[3]. Due to lack of uniformity in the initial theoretical study, researchers struggle for seeking more meticulous methods based on dynamics of charged particles. Noguchi et al. proposed a one-dimensional linear perturbation model and found that the low frequency oscillations have several characteristics of an ionization cycle[4]. Yamamoto et al. studied the low frequency oscillations in the linear type and anode layer type thruster and comments that low frequency oscillations are caused by the ionization instability[5]. In recent years, Barral et al. have been doing a great deal of research on improving of low frequency oscillation model, analyzing its physical mechanism and stabilizing

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theory[6],[7],[8],[9]. Because of the negative effects of the low frequency oscillations, researchers tried all kinds of possible ways to mitigate low frequency oscillations. A filter, sometimes also called matching network, is mainly a component applied to reduce the discharge current low frequency oscillations in the range of 10–100 kHz; it is also an important component in the space mission of a Hall thruster. Previous studies indicated that the low frequency oscillations are sensitive to parameters of filter and can be mitigated to an acceptable level with proper values. A filter unit is always involved between the thrusters and the power supply. The traditional filter consists of an inductor and a capacitor, sometimes a resistor connected between the inductor is also applied. It can be seen that this type filter is a low-pass filter, which aimed to isolate the interfering signal from the thrusters to the power supply. Recently, the role of the filter in the oscillation control was introduced by Yu et al and Barral et al[10,11,12]. It is comments that the filter regulates the voltage across itself according to the variation of discharge current so as to decrease its fluctuation in the discharge circuit, which is designed as a controller. Therefore, the matching network between the thrusters and power supply has two functions, which is filter and controller. However, it is very difficult to mix the function of filter and controller in one network, because the aim and function of these two parts is quite different. This maybe the reason of that there are not design method of the matching network until now. The parameters are always given through try and fail method in the practice. Therefore, it comes to our mind why we separate the matching network into two stages. The first stage is filter, which aimed to isolate the interfering signal from the thrusters to the power supply. This second stage is controller, which provides a regulating voltage to decrease the low frequency discharge current oscillation. The design method of two stages filter is proposed in this paper.

The general organization of this paper is as follows: Sec. II discusses the design of filter stage, Sec. III discusses the design of controller stage, and Sec. IV is the conclusions.

II. Design of filter stage

As one of the matching network between the thruster and power supply, the function of a filter is to protect the power supply. That is to obtain a stable load for the power supply, and therefore, the power supply can operated in the normal electrical state. Another demand is that the filter should have low insert impedance. In a manner of speaking, the design goal is to ensure the stabilization of the current or the voltage on the power supply side, even if the current is fluctuant on the thruster side. The filter can be seen as a two-port network, the power supply provides the power in constant and the thruster consumes it with the frequency of 20-40kHz. Thus, for the point view of energy conservation, the filter needs a capacitor large enough to store energy and release to a fluctuating load. Connecting the capacitor between the positive and negative of the power supply directly is clearly unreasonable. Therefore, we need to choose a component to connect between the capacitor and the power supply, which aimed to charge the capacitor. This chosen component should have low D.C. impedance since it is connect in the discharge circuit. Also, the component should have high A.C. impedance. The reason can be discussed as follows: the thruster consumes the power with pulsating mode, which would cause the fluctuation of capacitor voltage. The voltage difference between the port of power supply and the port of thruster would draw a fluctuant current of power supply and we don't expect that situation to occur. Thus, the component should have high A.C. impedance to suppress this kind of current fluctuation. Obviously, the component meeting the above criteria is inductor. So, the filter stage consist a inductor and a capacitor. On the side of the thruster, we use specific property of charge storage to stabilizing the discharge voltage, and on the side of power supply, we use inductor to achieve the constant-current. In order to avoid the resonance of LC circuit and the electromagnetic interference after the extinction of thrusters, a resistor is connected parallel between the inductor. That is the usually configuration of the filter unit as shown in Figure 1.

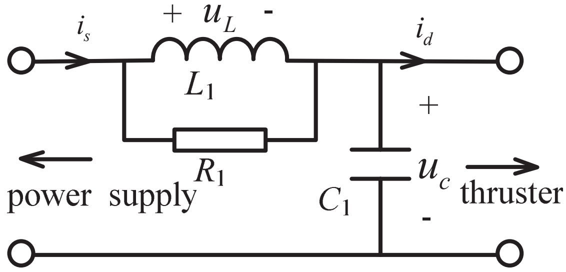

Figure 1. Configuration of the filter stage

According to the analysis above, the inductor in the filter stage should be big enough to achieve a small current fluctuation of the power supply. On the other hand, the capacitor should be big enough to ensure a small voltage fluctuation of the thrusters. According to Fourier analysis, a fluctuant signal can be expressed as summation of different frequency component. Therefore, the thruster current, i_s, can be expressed as follows,

$$i_s(t) = \sum_i^n I_s^{(i)} \sin(2\pi f_i t + \varphi_i) \tag{1}$$

where $I_s^{(i)}$ is the amplitude of the fluctuant current with frequency of f_i and phase angle of φ_i. If we are temporarily not able to consider the existing of the resistor, the inductor voltage can be expressed as,

$$u_L(t) = \sum_i^n 2\pi f_i L_1 \cdot I_s^{(i)} \cos(2\pi f_i t + \varphi_i)\,. \tag{2}$$

According to the electric circuit theory, the capacitor voltage can be expressed as,

$$u_C(t) = \int \frac{1}{C_1}(i_d(t) - i_s(t))dt\,. \tag{3}$$

Also, we introduce the Fourier analysis method and therefore the equation 3 can be written as,

$$u_C(t) = \int \frac{1}{C_1} \sum_i^n (I_d^{(i)}(t)\sin(2\pi f_i t + \theta_i) - I_s^{(i)}(t)\sin(2\pi f_i t + \varphi_i))dt\,, \tag{4}$$

$$u_C(t) = \sum_i^n \frac{1}{2\pi f_i C_1}(I_s^{(i)}(t)\cos(2\pi f_i t + \varphi_i) - I_d^{(i)}(t)\cos(2\pi f_i t + \theta_i))\,. \tag{5}$$

According to Kirchhoff's law and taking the requirement of a small thruster voltage fluctuation into account, the capacitor voltage should small than the inductor voltage fluctuation. Thus, we obtained that,

$$|u_C(t)| = \left|\sum_i^n \frac{1}{2\pi f_i C_1}(I_s^{(i)}(t)\cos(2\pi f_i t + \varphi_i) - I_d^{(i)}(t)\cos(2\pi f_i t + \theta_i))\right| \le u_L(t) \tag{6}$$

Substituting equation (2) into (6), it can get,

$$\left|\sum_i^n \frac{(I_s^{(i)}(t)\cos(2\pi f_i t + \varphi_i) - I_d^{(i)}(t)\cos(2\pi f_i t + \theta_i))}{2\pi f_i C_1}\right| \le \sum_i^n 2\pi f_i L_1 \cdot I_s^{(i)} \cos(2\pi f_i t + \varphi_i)\,. \tag{7}$$

If the design goals is that the power supply current fluctuation is on part per α of the thruster current fluctuation, the inductor and capacitor of filter stage must be satisfied with,

$$L_1 C_1 \ge \left| \frac{\sum_i^n \frac{1}{2\pi f_i}(1-\alpha)\cos(2\pi f_i t + \delta_i)}{\sum_i^n 2\pi f_i \cdot \cos(2\pi f_i t + \varphi_i)} \right| \tag{8}$$

where δ_i is vector resultant angle of power supply current and thruster current. It is unreasonable to consider waves of all frequency to set the parameters of the inductor and capacitor. Taken only the frequency of low frequency oscillation, f_0, into account, the equation (8) can be simplified to,

$$L_1 C_1 \ge \left| \frac{(1-\alpha)\beta}{4\pi^2 f_0^2} \right|. \tag{9}$$

Here, the parameter β represent the effects of phase angle δ_i and φ_i. Obviously, it is difficult to obtain the value of the parameter β, but we can estimate the range of its value. φ_i is the phase angle of the inductor current and δ_i is vector resultant angle of power supply current and thruster current. According to the analysis of the phase relationship in Ref 13, the phase-angle difference of δ_i and φ_i is equal to 180°. Thus, $\beta = \frac{\cos\delta_0}{\cos\varphi_0}$ and its value is around -1. For example, we want to insure that the ripple current of power supply is a tenth of the thruster current fluctuation, that is, the parameter α equal to 10. The frequency of low frequency oscillation is assumed to be 20 kHz. Under these assumption, the product of the inductance and capacitance should be more than or equal to 5.7×10^{-10}. If we choose the inductance to be 0.1 mH, the capacitance should be more than 5.7 μF. If the equation (9) is met, the inductance and the capacitance are optional. However, the value of the inductor and capacitor should also take some factors into consideration, such as the DC power dissipations, the volume and weight. After the determination of inductance and capacitance, it can be seen that a network of LC would have a resonant peak in its frequency characteristic. This would induce undesired electromagnetic interference. Thus, a resistor is usually connected parallel between the inductor. Under normal conditions, the resistance is in the range of 50-200Ω. Excessive resistance will cause the damping coefficient to be too small and the electromagnetic interference will increase. Undervalued resistance will weaken the effects of the inductor and cause the increasing of power supply current fluctuation. Thus, the resistance of the resistor is often given based on the experiments.

III. Design of controller stage

From the view of control theory, the discharge circuit of a Hall thruster can be seen as a feedback control system which consists of a thruster, a discharge power supply, and a filter as shown in Fig. 2. In this system, the Hall thruster is controlled object; the discharge power supply voltage is reference signal and the filter is controller. So, we name this stage of filter as controller stage. The controller stage filter regulates the voltage across itself according to the variation of discharge current so as to affect the electric field distribution in the discharge channel, and therefore decrease the discharge current fluctuation in the discharge circuit. The simplest component which can provide a varying voltage with the change of current is the inductor. However, the magnetized plasma of a Hall thruster exhibits varied oscillations ranging from kHz to GHz. High-frequency current oscillation will cause an undesirable high amplitude oscillation of inductor voltage. Thus, we wish the controller stage have enough gain in the low band, but decaying the signal in high-frequency band. The required frequency response characteristics would looks like that shown in Figure 3. According to the control theory, it is a typical second order system which can be realized by the network (controller stage) shown in Figure 4. The transfer function can be expressed as,

$$\Phi(s) = \frac{K\omega^2 s}{s^2 + 2\omega\zeta s + \omega^2}\ , \tag{10}$$

where $K = L_2 , \omega = \frac{1}{\sqrt{L_2 C_2}} , \zeta = \frac{1}{2R_2}\sqrt{\frac{L_2}{C_2}}$. Obviously, the parameters of controller stage relate to the required gain of the system. If we suppose the minimum gain is g_0 with the low frequency oscillation frequency of f , it must satisfied that

$$20\lg\left|\frac{K\omega_n^2 j\omega}{(j\omega)^2 + 2\omega_n\zeta \cdot j\omega + \omega_n^2}\right| = 20\lg\left|\frac{2\pi f R_2 L_2 j}{2\pi f L_2 j - 4\pi^2 f^2 R_2 L_2 C_2 + R_2}\right| \geq g_0\ . \tag{11}$$

Rewritten the equation (10), we can get,

$$\frac{2\pi f R_2 L_2}{\sqrt{(R_2 - 4\pi^2 f^2 R_2 L_2 C_2)^2 + (2\pi f L_2)^2}} = 10^{\frac{g_0}{20}}\ . \tag{12}$$

Considering the turnover frequency of the system is

$$f_z = \frac{1}{2\pi\sqrt{L_2 C_2}}\ , \tag{13}$$

equation (12) can be expressed as,

$$\frac{(1-(\frac{f}{f_z})^2)^2}{(2\pi f L_2)^2} - \frac{1}{R_2^2} = 10^{-\frac{g_0}{10}}\ . \tag{13}$$

The resistor affect only the gain near the turnover frequency, thus we ignore the effects of resistor temporarily. Then it can be achieved,

$$L_2 = 10^{\frac{g_0}{20}} \cdot \frac{1-\left(\frac{f}{f_z}\right)^2}{2\pi f} \tag{14}$$

Considering equation (13), we can therefore know that the capacitance must be satisfied,

$$C_2 = 10^{-\frac{g_0}{20}} \cdot \frac{f}{2\pi\left(f_z^2 - f^2\right)} \tag{15}$$

According to the study on the role of resistor in Ref 13, the resistance can be estimated as follows,

$$R_2 = 10^{\frac{g_0}{20}} \cdot \tan\delta\,, \tag{16}$$

where δ is the phase-angle between regulating voltage and discharge current. This phase-angle is estimated in the range of 0-90°, which would affect the variation of the electric field and therefore help to balance the ion production. If we propose g_0 =30 dB, δ =80° , f =20 kHz, and f_z =25 kHz, we can therefore get that the parameters in the controller stage is R_2 =179 Ω, L_2 =0.09 mH, and C_2 =0.447 μF.

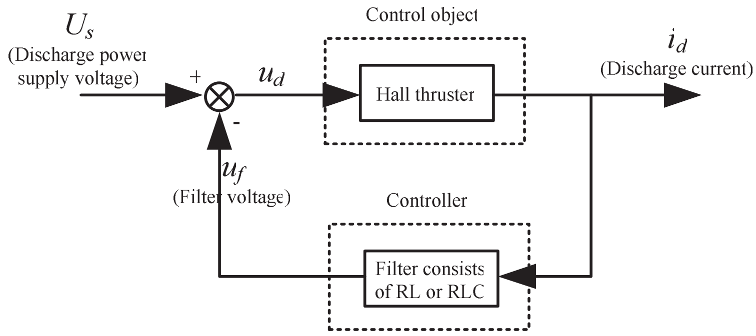

Figure 2. Schematic diagram of feedback control system in Hall thrusters

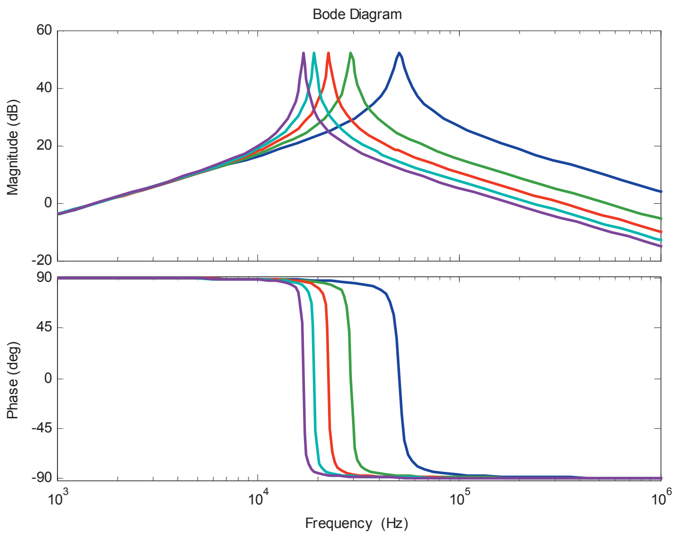

Figure 3 .Schematic diagram of feedback control system in Hall thrusters

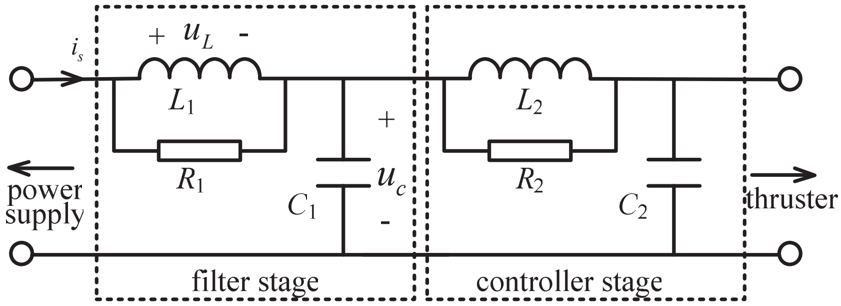

Figure 4 .Schematic diagram of feedback control system in Hall thrusters

IV. Conclusions

Low frequency oscillation is one of the important characteristics of Hall thrusters. The matching network in the circuit (a filter unit) is an important component of the low frequency oscillation suppression. The two functions: filter and controller, of the matching network are discussed. Since it is very difficult to mix the function of filter and controller in one network, a two stage matching network was proposed. The first stage is filter, which aimed to isolate the interfering signal from the thrusters to the power supply. This second stage is controller, which provides a regulating voltage to decrease the low frequency discharge current oscillation. The parameters design method in the two stage matching network is proposed. The work which improves this design method according to the analysis of ionization process is undergoing.

Acknowledgments

Wei Liqiu thans the support of the National Natural Science Foundation of China (No. 51477035), the Fundamental Research Funds for the Central Universities (No. HIT.NSRIF 2015064)

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