Optimization of Orbital Transfer of Electrodynamic Tether Satellite by Nonlinear Programming

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Abstract: Aiming at removing multiple space debris in the Low Earth Orbit (LEO) with Electrodynamic Tether (EDT), this work applies an optimization method of orbit transfer using a nonlinear programming (NLP). EDT can provide an energy effective performance for orbital transfer as well as a long-life operation, however, orbital transfer using EDT has not been sufficiently studied because of the difficulty of control only by electric current in tether. In this work, based on a simplified mission scenario, a control problem to minimize a semimajor axis of tethered satellite's orbit in a certain period of time is solved by an NLP method. Optimal orbital transfers are compared for different initial inclinations, and effects of a couple of orbital perturbations, an atmospheric drag and the Earth's oblateness (J2) effect, are finally discussed.

Nomenclature

a	=	semimajor axis of satellite's orbit
e	=	eccentricity of satellite's orbit
i	=	inclination of satellite's orbit
Ω	=	right ascension of ascending node of satellite's orbit
ω	=	argument of perigee of satellite's orbit
v	=	true anomaly of satellite's orbit
μ	=	gravitational constant of the Earth
f_r	=	acceleration in the radial direction of satellite's orbit
f_θ	=	acceleration in the transverse direction of satellite's orbit
f_h	=	acceleration in the orbit normal direction of satellite's orbit
B_x	=	the Earth's magnetic field in the radial direction of satellite's orbit
B_y	=	the Earth's magnetic field in the transverse direction of satellite's orbit
B_z	=	the Earth's magnetic field in the orbit normal direction of satellite's orbit
θ	=	in-plane attitude angle of satellite
ϕ	=	out-of-plane attitude angle of satellite
χ_0	=	initial value of the state vector for a nonlinear programing method
χ_f	=	final value of the state vector for a nonlinear programing method
μ_m	=	dipole strength for the Earth's magnetic field
I	=	electric current in tether
L	=	length of tether

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m	=	total mass of satellite system
$\boldsymbol{a}_{drag}$	=	acceleration vector of the atmospheric drag in the Euler-Hill coordinate system
C_D	=	coefficient of drag
A/m	=	area mass ratio of a satellite
ρ	=	the density of the Earth's atmosphere
v_{rel}	=	velocity relative to the rotating Earth's atmosphere
$\boldsymbol{v}_{rel}$	=	velocity vector relative to the rotating Earth's atmosphere
ω_e	=	rotating speed of the Earth
U_2	=	gravity potential of the J2 terms
J_2	=	coefficient of the second-order zonal harmonics in the Earth's gravity field
r_e	=	radius of the Earth
ϕ	=	terrestrial latitude
$a_{J2,r}$	=	J2 acceleration in the radial direction of the Earth
$a_{J2,\phi}$	=	J2 acceleration in the terrestrial latitude direction of the Earth
$a_{J2,\lambda}$	=	J2 acceleration in the terrestrial longitude direction of the Earth
$a_{J2,x}$	=	J2 acceleration in the radial direction of satellite's orbit
$a_{J2,y}$	=	J2 acceleration in the transverse direction of satellite's orbit
$a_{J2,z}$	=	J2 acceleration in the orbit normal direction of satellite's orbit

I. Introduction

Since humankind has advanced into space, a large number of artificial objects have been exhausted, and today, there are more than 16,000 space debris in earth orbit which are certainly tracked. Each of debris has a potential to damage spacecraft in earth orbit. Therefore, It is urgently necessary to reduce the number of debris for the long-term sustainability of outer space activities. NASA shows that the future environment in Low Earth Orbit (LEO) could be stabilized with a removal rate of 5 objects per year.[1] Active Debris Removal (ADR) is a way to remove large objects using a space craft in orbit. However, ADR sometimes needs a large amount of energy to achieve a deorbit or a transfer to the other harmless orbit. Therefore, it is desirable for a debris removal system to adopt high efficient propulsion device. In this study, an electrodynamic tether (EDT) is treated. EDT can provide energy effective performance for an orbital transfer as well as a long-life operation. In the past orbital transfer using EDT has not been sufficiently studied mainly because of the difficulty of control only by an electric current in tether. Aiming at removing multiple debris objects in LEO, this work applies a nonlinear programing (NLP) method to achieve an orbit transfer using an EDT system, compares for different initial inclinations, and finally discusses effects of a couple of orbital perturbations, an atmospheric drag and the Earth's oblateness (J2) effect.

II. Mission Scenario

In this work, the following three mission scenarios are supposed: 1) An ADR satellite rendezvouses with a space debris in earth orbit and captures it. Then, the satellite lowers its altitude with the debris. When the satellite reaches its altitude enough to decay the debris into atmosphere, it releases debris. And then, the satellite raises its altitude to approach next target debris, 2) An ADR satellite rendezvouses with a space debris in earth orbit and captures it. Then, the satellite changes its orbit and captures another debris. After the repeating of this operation, the satellite lowers its altitude with several debris. When the satellite reaches an altitude enough to decay the debris into atmosphere, it releases all of the debris. 3) An ADR satellite rendezvouses with a space debris in earth orbit and captures it. The satellite attaches a removal device such as a small EDT which can decay debris. After releasing the debris with the removal device, the satellite changes its orbit and captures another debris

This paper focuses on the first scenario. To examine the feasibility of the mission scenario, as a first step, a simplified one to achieve an orbital transfer between two sets of the orbital elements is treated.

III. Dynamics of EDT Satellite

The electric current generated in tether is restricted by density of the Earth's ionospheric plasma and physical or mechanical property of the tether. The electromagnetic force generated by EDT system is relatively small and the system behaves like a low-thrust spacecraft. However, there is a fundamental difference between EDT satellite and low-thrust spacecraft. The Lorentz force in an EDT system affects only in the normal direction to the tether as well as to the magnetic field.

Since an orbit of EDT satellite changes slowly, it can be expressed by a Gaussian form of variational equations.[2]

$$\dot{a}=(2a^2/h)[e\sin v f_r+(p/r)f_\theta] \tag{1}$$

$$\dot{e}=(1/h)\{p\sin(v)f_r+[(p+r)\cos v+re]f_\theta\} \tag{2}$$

$$\dot{i}=(r\cos(\omega+v)/h)f_h \tag{3}$$

$$\dot{\Omega}=\{r\sin(\omega+v)/(h\sin i)\}f_h \tag{4}$$

$$\dot{\omega}=(1/he)\left[-p\cos v f_r+(p+r)\sin v f_\theta\right]-[r\sin(\omega+v)\cos i/(h\sin i)]f_h \tag{5}$$

$$\dot{v}=h/r^2+(1/eh)\left[p\cos v f_r-(p+r)\sin v f_\theta\right] \tag{6}$$

where $h=\sqrt{\mu a(1\text{-}e^2)}$, $p=a(1-e^2)$, $r=p/(1+e\cos v)$.
The Earth's magnetic field vectors in Euler-Hill frame are defined as follows[3]:

$$B_x=-2\left(\mu_m/r^3\right)\sin(\omega+v)\sin i \tag{7}$$

$$B_y=\left(\mu_m/r^3\right)\cos(\omega+v)\sin i \tag{8}$$

$$B_z=\left(\mu_m/r^3\right)\cos i \tag{9}$$

And the acceleration vectors are expressed such that.

$$\boldsymbol{f}=\begin{Bmatrix} f_r \\ f_\theta \\ f_h \end{Bmatrix}=\begin{Bmatrix} IL/m(B_z\sin\theta\cos\phi-B_y\sin\phi) \\ IL/m(B_x\sin\phi-B_z\cos\theta\cos\phi) \\ IL/m(B_y\cos\theta\cos\phi-B_x\sin\theta\cos\phi) \end{Bmatrix} \tag{10}$$

IV. Orbital Perturbations

Spacecraft in earth orbit is affected by orbital perturbations. In this study, an atmospheric drag and the Earth's oblateness (J2) effect are considered. The effect of the perturbations could be expressed simply by adding the perturbation acceleration vector to Eq. (10).

A. Atmospheric Drag

Atmospheric drag in LEO is caused by relative velocity between spacecraft and the Earth's atmosphere. The acceleration vector is expressed as follows[4]:

$$\boldsymbol{a}_{drag}=-\frac{1}{2}C_D\frac{A}{m}\rho v_{rel}\boldsymbol{v}_{rel} \tag{11}$$

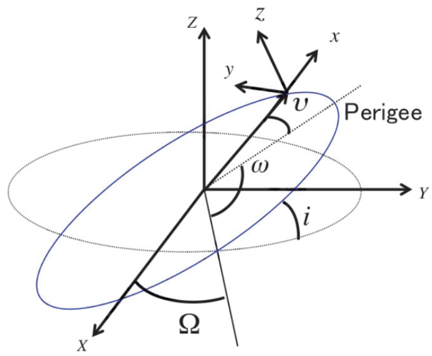

Figure 1. Euler-Hill frame relative to the inertial Earth coordinate system.

C_D is the dimensionless quantity, which reflects spacecraft's susceptibility to drag forces. It was set as C_D=2.2. *A*/*m* is the area mass ratio of spacecraft, and it was set based on a profile of an EDT system described in Section VI, such that A/m = 9.8×10^{-8}km^2/kg. Several models are available to express the density of the atmosphere. In this study a simple one called an exponential model[4] was selected. The velocity vector relative to the rotating atmosphere in the Euler-Hill coordinate system is expressed as follows:

$$\boldsymbol{v_{rel}} = \begin{Bmatrix} \dot{r} \\ \dot{v}r - \omega_e r \cos i \\ \omega_e r \sin i \cos(\omega + v) \end{Bmatrix} \tag{12}$$

B. The Earth's Oblateness (J2) Effect

Because the Earth is a non-spherical central body, gradient of the gravity potential occurs. This gradient yields a disturbing acceleration. The Earth's oblateness effect called a J2 effect is the most influential, and its potential is expressed as follow:

$$U_2 = -\frac{1}{2}\frac{\mu}{r} J_2 \left(\frac{r_e}{r}\right)^2 \left(3\sin^2\phi\right) - 1 \tag{13}$$

J2 is the coefficient of the second-order zonal harmonics in the Earth's gravity field, whose value is 1.0826267×10^{-3}. The acceleration vector of the J2 perturbation in spherical coordinates is expressed as follows:

$$\begin{Bmatrix} a_{J_2,r} \\ a_{J_2,\phi} \\ a_{J_2,\lambda} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{3\mu J_2 {r_e}^2}{2r^4}\left(1 - 3\sin^2\phi\right) \\ -\dfrac{3\mu J_2 {r_e}^2}{2r^4}\left(2\cos\phi\sin\phi\right) \\ 0 \end{Bmatrix} \tag{14}$$

Translate from sphere coordinates to Euler-Hill frame.

$$\begin{Bmatrix} a_{J_2,x} \\ a_{J_2,y} \\ a_{J_2,z} \end{Bmatrix} = \begin{Bmatrix} a_{J_2,r} \\ \dfrac{\cos(\omega+v)\sin i}{\cos\phi} a_{J_2,\phi} + \dfrac{\cos i}{\cos\phi} a_{J_2,\lambda} \\ \dfrac{\cos i}{\cos\phi} a_{J_2,\phi} + \dfrac{\cos(\omega+v)\sin i}{\cos\phi} a_{J_2,\lambda} \end{Bmatrix} \tag{15}$$

V. Optimal orbital transfer using a Nonlinear Programming

Generally, optimization of orbital transfer deals with a large number of variables. Instead of taking an analytical approach which requires to derive Jacobian and Hessian matrices from the original variational equations, this study chooses a numerical approach using a Nonlinear programming (NLP) method.[5] In order to optimize orbital transfer of EDT satellite, a Sequential Quadratic Programming (SQP) method was used in the framework of an NLP. The Hermite-Simpson method was also introduced to discretize the orbital elements during the transfer time, and the total number of computational grid was selected as 300.

VI. Numerical Results

Based on the dynamics of an EDT system and the optimization technique stated in the previous sections, a numerical simulation was conducted. Note here that this study does not concern the librational motion of the EDT's attitude on orbit, that is, $\theta = \phi = 0$ deg. And the range of the control input (the electric current in tether) is limited to $-4 \le I \le 4$ A. The total mass of the EDT satellite is 400 kg and the tether length is 15km. A schematic view of the EDT satellite is shown in Figure 2.

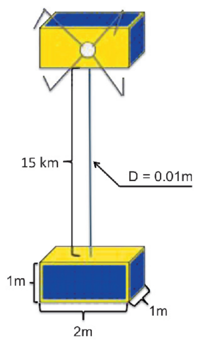

Figure 2. Conceptual diagram of the EDT satellite

A. Minimization of the Semimajor Axis

Considering the scenario described in Section II, a minimization problem for the semimajor axis of the EDT system was treated. As stated in the previous section, the EDT should reach an orbit in order to decay captured debris into atmosphere, at the same time, retain an orbital location to smoothly move on to next mission. For the optimization problem, final time was fixed at t_f = 13 hour. The initial orbit and final orbital elements were set as follows:

$$[a_0, e_0, \Omega_0, \omega_0, \upsilon_0] = [7080km, 0.02, 30\deg, 50\deg, 0\deg]$$

$$[e_f, \Omega_f, \omega_f] = [0.02, 30\deg, 50\deg]$$

Also, initial and final values of the inclination were set such that $i_{0 \text{ or } f}$ = 25, 45, 65, 95 deg. Note that the final value of the true anomaly (v_f) cannot be restricted in this optimization problem.

First, we look at the case without perturbations. Figure 3 shows time histories of the orbital elements computed by the NLP method. These results were obtained by using a MATLAB function (in the Optimization Toolbox) called "fmincon." All orbital elements show periodical changes in accordance with the EDT's orbital motion. Inversely proportional to the inclination, the semimajor axis decreases. These results are caused from magnitude of the acceleration vectors generated by Lorentz force in the EDT. These results are caused from magnitude of the acceleration vectors generated by Lorentz force in the EDT. As shown in Eqs. (7) to (10), small inclination generates large acceleration vector in the transverse direction of the EDT's orbit when $\theta = \phi$ =0, that lead to increase the semimajor axis.

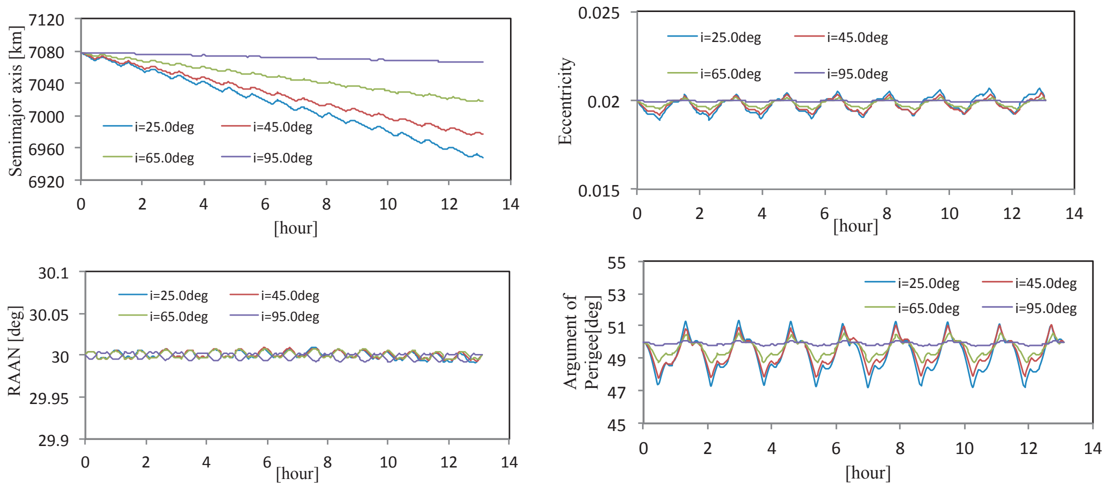

Figure 3. Optimal orbit transfer of the EDT satellite.

Next, orbital transfer of EDT with a couple of orbital perturbations, an atmospheric drag and the Earth's oblateness (J2) effects were considered. In this case, an optimal orbital transfer using the NLP method could not be successfully achieved. It is conceivable that the differences between the Lorentz force generated in the EDT and the two perturbation forces would cause this issue. The orders of the accelerations by the atmospheric drag and by J2 effect are 10^{-8} and 10^{-5}, respectively, whereas the one by Lorentz force is 10^{-8}. Although the EDT system cannot cancel the effect of J2 perturbation by just applying the original NLP method, the previously obtained time histories of the control input were given to the dynamical model of the EDT with the orbital perturbations in order to see the performance of the control input for the ideal case. Figure 5 shows results of the orbital propagation considering the two orbital perturbations. These results were obtained by the same control input as in Figure 4, and to propagate the orbital elements from their initial values, a MATLAB function called "ode45" was used. As seen in Figure 5 in proportional to the inclination, the oscillation amplitude becomes larger. As far as the semimajor axis is concerned, the ideal control input is still effective for relatively small initial inclinations. On the other hand, for a sun-synchronous orbit, mean change rate of the semimajor axis is 538m/hour. Although it is smaller than for the other initial orbits, it can achieve a major change of semimajor axis after a long transfer time. The behavior of the right ascension of the ascending node (RAAN) is noteworthy. Due to the J2 perturbation, differences between the desired and final values of the RAAN seriously increase inversely proportional to the initial inclinations. From these results, as for a sun-synchronous orbit, near-optimal orbit transfer is expected. It is good notification for debris removal satellite with EDT because there are large numbers of debris in sun-synchronous orbits.

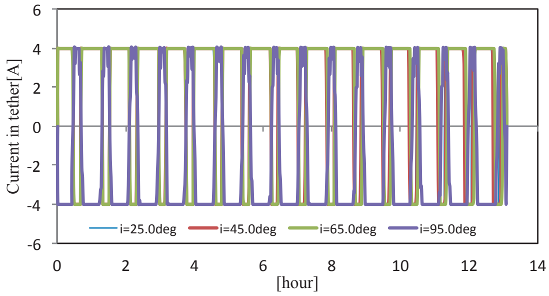

Figure 4. Control input.

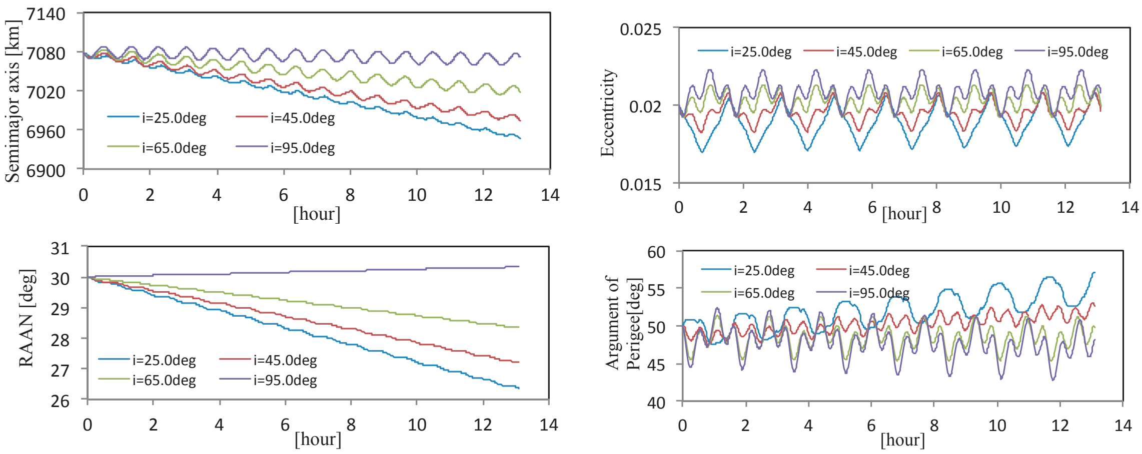

Figure 5. Orbital propagation with atmospheric drag and J2 perturbation.

B. Comparison of optimal control input and constant input

In order to see the effectiveness of the optimal control input stated in the previous section, an orbital propagation using a constant control input was conducted, and compared with the orbital propagation using the optimal control input. Initial and final values of the orbital parameters were set as follows:

$$[a_0, e_0, i_0, \Omega_0, \omega_0, \upsilon_0] = [7080km, 0.02, 95\deg, 30\deg, 50\deg, 0\deg]$$

$$[e_f, i_f, \Omega_f, \omega_f, v_f] = [0.02, 95\deg, 30\deg, 50\deg, 50\deg]$$

The final time was set at t_f = 13 hour. The control input was set such that I=-4 A during the transfer time. The time history of the control input was given to the EDT satellite's dynamical model with an atmospheric drag and J2 perturbation and all the orbital elements were propagated as in the previous section. Figure 6 shows time histories of some typical orbital elements for the optimal and constant control input. When the constant input was given, the semimajor axis decreases at a rate of 1615 m per hour. It is larger than that of optimal controlled one. It seems that giving the constant input is more effective for orbital transfer. However, the other orbital elements show uncontrolled behaviors. For example, in the case of constant control input, the inclination finally changes to the value such that i_f =94.2deg, while the optimal controlled one retains the same inclination.

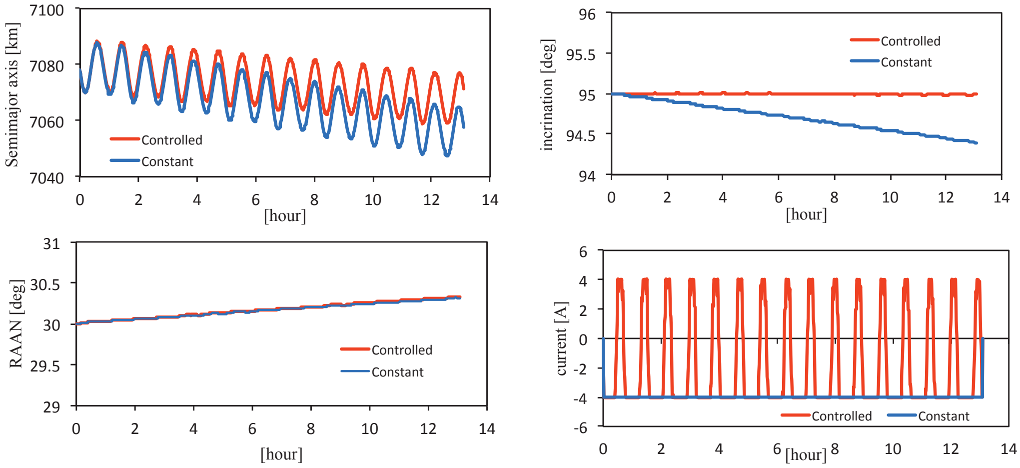

Figure 6. Comparison of optimal control input and constant input.

VII. Conclusions

This study applies a method of orbit transfer using an NLP method aiming at removing multiple space debris in LEO with EDT. Although its system can generate relatively large acceleration in an orbit that has small inclination, the effect of J2 perturbation exists greatly and it is not easily cancelled to achieve optimal orbital transfer. In contrast, the effect of J2 perturbation is quite small and near-optimal orbital transfer of EDT satellite can be expected for a sun-synchronous orbit. As future work, more effective method of an optimal orbital transfer should be derived to capture multiple space debris in different orbits considering the orbital perturbations.

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