Facility Effects on Hall Thruster Performance Through Cathode Coupling

IEPC-2015-309／ISTS-2015-b-309

Presented at Joint Conference of 30th International Symposium on Space Technology and Science,
34th International Electric Propulsion Conference and 6th Nano-satellite Symposium
Hyogo-Kobe, Japan
July 4–10, 2015

R. Spektor[*], W. G. Tighe[†]
The Aerospace Corporation, El Segundo, CA 90245, USA

P. H. Stoltz[‡], K. R. C. Beckwith[§]
Tech-X Corporation, Boulder, Co 80303, USA

An investigation of a phenomenon commonly described as a facility effect, where thrust of an HCT is observed to decrease non-linearly with the decreasing background pressure in a vacuum chamber is presented. A simple 1-D numerical model describing such a thrust decrease, caused by the variation of the cathode coupling voltage with the magnetic field and neutral density, was developed. It was found that some of the experimentally observed performance trends can be qualitatively replicated by selecting the model parameters appropriately. With no magnetic field, or when the cathode is immersed in a field dominated by the parallel component, the model shows that thrust is largely independent of the background pressure. Thrust decrease can be observed with the model when the perpendicular component of magnetic field is present. General scaling laws are apparent from the model. 1) Magnitude of the facility effect increases with the discharge current. 2) Magnitude of the facility effect is very sensitive to and scales inversely with the magnitude of the perpendicular magnetic field.

Nomenclature

V_d	= HCT discharge voltage	j	= Discharge (total) current density
V_c	= Cathode coupling potential	j_e	= Electron current density
I_d	= HCT discharge current	j_i	= Ion current density
T	= Thrust	E	= Electric field
n_b	= Background gas neutral number density	B	= Magnetic field
n_n	= Neutral number density	β	= Ionization coefficient rate for Xe
v_n	= Neutral velocity	σ	= Electron conductivity
n_e	= Plasma number density	ν_{ei}	= Eelectron-ion collision rate
v_e	= Electron velocity	ν_{en}	= Electron-neutral collision rate
v_i	= Ion velocity	ν_i	= Ion collision rate
ω	= Electron cyclotron frequency	m_e	= Electron mass
ρ	= Plasma density	m_i	= Ion mass
P	= Plasma pressure	U	= First ionization energy
P_e	= Electron pressure	k_b	= Boltzmann constant
T_e	= Electron temperature	q	= Unit charge

[*]Laboratory Manager, Propulsion Science Department, rostislav.spektor@aero.org
[†]Senior Scientist, Propulsion Science Department
[‡]Vice President, Beam-Plasma Interactions
[§]Senior Research Scientist

I. Introduction

The problem of the Hall Current Thruster (HCT) performance and plume divergence as a function of the vacuum chamber pressure has recently emerged as an issue of utmost importance for both fundamental research and applied science. The problem has been termed "facility effects". For a long time it has been known that the background gas in a vacuum chamber can be ingested by an HCT through the front face, ionized, and then accelerated back out of the thruster, thus producing thrust in excess of the true vacuum measurement. Furthermore, the interaction between the accelerating ions and the background neutral gas can alter the plume properties. Both of these issues were addressed during adoption of the HCT technology.[1–3] Maximum acceptable vacuum chamber pressure guidelines were developed in order to minimize the effects of background pressure. For example, in order to minimize facility effects on thruster performance, the guideline recommended to operate HCTs below $5 \cdot 10^{-5}$ Torr.[1] This number was determined using a simple ingestion model, by calculating the ratio of the effective mass flow rate of the background gas through the face of the thruster to the feed mass flow rate. The recommendation called for the ingested mass flow rate to be less than the thruster mass flow rate uncertainty. Even though this maximum pressure recommendation was developed specifically for the SPT-100, it was widely adopted without any further justification for other, more powerful HCTs. The dominant experimentally observed facility effect on the SPT-100 performance was to produce an instability that led to significant thrust degradation at a pressure above $3{\cdot}10^{-5}$ to $8{\cdot}10^{-5}$ Torr, depending on the thruster condition.[1]

Subsequent work on facility effects had mainly focused on plume properties of the HCTs as a function of the vacuum chamber background pressure.[4,5] The few papers that investigated facility effects on the HCT performance seemed only to confirm the original findings.[6,7] A complicating factor was that most laboratory tests with the HCTs were performed with the thrusters operated in the constant flow mode, letting the discharge current (thus total power) vary. A more realistic test is to operate a thruster in the constant power mode, adjusting the flow rate to maintain constant power, even when other parameters, such as magnetic field, are varied.

A more recent and closer re-examination of the old SPT-100 data revealed troubling trends, which were not previously appreciated.[8] For example, even when operated below the recommended pressure, the SPT-100 data revealed a decrease in performance with decreasing background pressure.[2,8] Furthermore, the decrease was non-linear with a sharper slope toward lower pressure. Nether the magnitude nor the shape of the trend curve could be explained by the simple ingestion model. The resulting discussion stimulated a much deeper examination of the pressure effects on the HCT performance.[9–17] The issue is of practical importance since even the workhorse SPT-100 was shown to produce 4% less thrust. In addition, the facility effects issue points to the fundamental lack of understanding of some of the HCT physics.

In this paper we propose a theoretical framework that attempts to explain the observed performance degradation with pressure. We rely on simple, tested ideas but combine them in a way that has not been attempted before. While we keep the fundamental physics in mind, the final acceptance metric is whether or not the theory can replicate, however imperfectly, all observed experimental trends. The rest of the paper is structured as follows. In section II we describe a theoretical approach and derive the governing set of equations. In section III we describe a simplified version of the model and in section IV we present the results of such an approach. The simplified equations allow us to understand the underlying mechanisms behind the experimental findings and serve as a springboard for a more sophisticated MHD model. We discuss the results of the model in section V and make concluding remarks and propose avenues for the future work in section VI.

II. Theoretical Considerations

Various theories have been proposed to explain the observed thrust degradation with pressure. Proposed explanations range from the plasma plume interaction with the conducting walls of the vacuum chamber[16] to the extra thrust generation in the far field of the the thruster[11] and to the acceleration of the ions produced by the charge exchange collisions (CEX) inside the acceleration zone.[10] Experimental evidence indicates that two properties are predominantly affected by the variation in the background pressure: plume divergence and ion beam energy.[14,15,17] Plume divergence increases at lower pressure, potentially indicating that the acceleration zone moves further downstream of the thruster, allowing the electric field to bow out to provide an increased transverse component of acceleration to the ions, thus reducing the thruster performance.

Reduction in the ion beam energy is an indication that either the ions are born at a lower potential or that the potential drop decreases with lower pressure. Experimental evidence points to the latter explanation since the measured plume plasma potential downstream of the thruster increases at lower pressure.

Ion beam energy is determined in part by the difference between the applied discharge voltage V_d, which is typically held constant, and the cathode coupling potential V_c, as shown schematically in Fig. 1(a). The potential at which ions are produced is somewhat lower than the anode potential, that difference will be ignored for the sake of simplicity. It should also be noted that V_c is only a notional, albeit useful, concept since the distance from the cathode at which it should be measured is not well defined. Cathode coupling voltage has been the subject of multiple studies[18–25] and was shown to be sensitive to four main factors: cathode flow rate, discharge current, cathode location with respect to the thruster, and magnetic field topology. Recent studies have shown that the background pressure can also affect V_c.[14,15] These studies specifically investigated the background pressure, cathode location, and mass flow rate effects on cathode coupling as contributors to the facility effects.

In this paper we propose a theory that explains how the cathode coupling voltage varies as a function of these parameters. Performance trends determined by facility effects can be qualitatively reproduced by noting that thrust is proportional to the discharge current times a square root of the beam voltage: $T \sim I_d\sqrt{V_d - V_c}$. We will not attempt to calculate the absolute value of thrust with our simple model since those values are highly dependent on the specifics of the geometry and configuration. Instead, the purpose of this work is to understand the underlying physical mechanisms responsible for the observed performance trends. Contributions of plume divergence to facility effects are also not included.

Steady state plasma discharge in the near-cathode region can be described by a set of coupled plasma fluid equations

$$
\begin{aligned}
\nabla \cdot n_n \vec{v}_n &= -\beta n_e n_n, & \text{(1a)}\\
\nabla \cdot n_e \vec{v}_i &= \beta n_e (n_n + n_b), & \text{(1b)}\\
\nabla \cdot n_e \vec{v}_e &= \beta n_e (n_n + n_b), & \text{(1c)}\\
\nabla \cdot n_e \vec{v}_i \vec{v}_i &= \frac{q n_e}{m_i}\left(E + v_e \times B\right) - 0.5 n_e \nu_i \vec{v}_i + \beta n_e (n_n + n_b)\vec{v}_n, & \text{(1d)}\\
\nabla \cdot \left(\frac{5}{2} T_e \vec{j_e} + \kappa_e \nabla T_e\right) &= \vec{E}\cdot\vec{j_e} - 3 n_e k_b \frac{m_e}{m_i}(\nu_{ei} + \nu_{en}) T_e - q\beta n_e (n_n + n_b) U, & \text{(1e)}\\
\vec{j_e} &= \sigma\left(\vec{E} + \frac{1}{q n_e}\nabla n_e k_b T_e\right), & \text{(1f)}
\end{aligned}
$$

where σ is the electron conductivity, β is the ionization coefficient for Xe, $U = 12.1298$ eV is the first ionization energy for Xe, and n_b is the neutral background number density, which is assumed to be homogenous. The other variables in equation set (1) are: n_n and v_n – the neutral number density and velocity, n_e, v_e, v_i – are the plasma number density, electron and ion velocities, T_e is the electron temperature, E and B are the electric and magnetic fields, j_e is the electron current density, νs are the collision frequencies between various species, m_e and m_i are the electron and ion masses, k_b is Boltzmann constant, and q is the unit charge. The background neutral density influences these equations by affecting the ionization (ion and electron conservation equations), as well as through the electron-neutral collision term in the momentum and energy conservation equations, and the generalized Ohm's law equation. Expressions for various rates used in the calculation are given in the Appendix.

Boundary conditions at the cathode are determined by prescribing the neutral and plasma densities (based on the mass flow rate and the discharged current) and the electron temperature. Mass flow rate and the discharge current are fixed for each simulation. Neutral density inside the cathode orifice can be computed using a procedure described in Appendix B of Ref. 26 that leads to Eq. (B-12) therein. Similarly, the plasma density inside the cathode orifice can be determined using Eq. (6.5-4) in Ref. 26. Furthermore, we reduce each density by an order of magnitude to account for the decrease between the orifice and the cathode exit as shown in Fig. 6-35 in Ref. 26. At all other boundaries we assume a zero-gradient Neumann condition.

Equation set (1) can be solved for the electric field, which then can be integrated along the spatial coordinate to produce a voltage drop – the cathode coupling voltage. We study facility effects by solving these equations with various background neutral densities. This approach also allows us to study the sensitivity to the discharge current, voltage, magnetic field, mass flow rate, and cathode location. The second and third

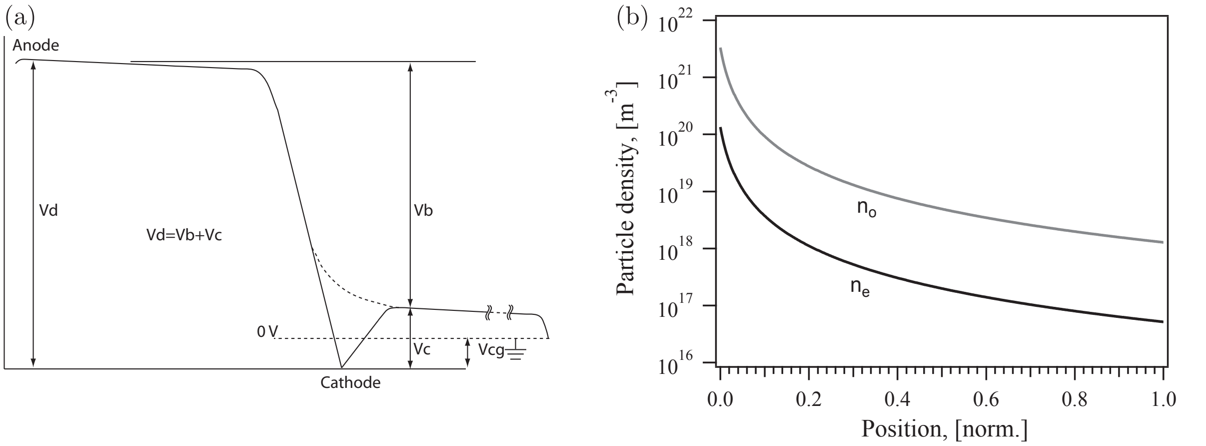

Figure 1. Panel (a) shows schematically the potential differences between the various parts of the HCT plasma plume. Panel (b) shows the neutral and plasma number density profiles as a function of the normalized distance away from the cathode. The profiles were produced using Eq. (2) with $\theta = 45$ and particle densities at the cathode derived for $I_d = 15$ A operation.

equations in the set above lead to the algebraic current density conservation law: $j = qn_e(v_i - v_e) = j_i - j_e$, where the total current density j (discharge current density) is fixed. This simplification allows a computation of the electron current without solving the differential mass conservation equation for electrons. However, even with this simplification, equation set (1) cannot be solved analytically, even in one dimension.

We, therefore, take two approaches to the problem. First, we simplify the equations such that they can be solved with minimal computer aid and yet still keep most salient features relevant to our study. We then qualitatively compare the results of this simple model to the experimental trends and establish the underlying physical mechanisms. Next, we use computer simulation software to solve the problem numerically with high fidelity in order to achieve a better quantitative agreement with the data.

III. Simple Model

In order to simplify equation set (1) further we assume a 1-D geometry with the cathode orifice at the left and plasma plume expanding to the right, and use Mathematica© software to solve the equations. Early attempts at solving Eqs. (1) did not produce convergent results. We found that convergence strongly depended on the plasma density profile. The 1-D framework does not account for the plasma expansion in 3-D, resulting in an artificially low plasma density decay as a function of distance. Various successful simple cathode plume models made an assumption that neutrals as well as plasma expand at a constant angle away from the cathode.[20,27] We therefore simplify our model further by following that approach. We write the ion and neutral density evolutions as

$$n_{n,e} = n^0_{n,e} \frac{r^2}{(r + x \tan\theta_{n,e})^2}, \tag{2}$$

where r is the cathode orifice radius, which is assumed to be 0.5 mm for convenience, x is the coordinate along the cathode plume and θ is the angle of expansion. This enables us to simplify the system of differential equations by removing the ion and neutral mass conservations from Eqs. (1). The remaining ion momentum and electron energy conservation equations can now be solved numerically with the appropriate boundary conditions. For all reported simulations we assumed the electron temperature of $T_e = 2.3$ eV at the cathode exit, which is consistent with experimentally measured values.[20,26]

The angle of neutral expansion has been measured to be $\theta_n = 45$ degrees, while the best fit to plasma expanding out of a cathode operated with an HCT at a discharge current of 20 A was found to be $\theta_e =$ 11.5 degrees.[20] However, for this work we assumed for simplicity that plasma and neutrals expand at 45 degrees. Effects of varying the plasma expansion angle are only briefly discussed here. We plot typical plasma and neutral density profiles as a function of the normalized distance away from the cathode calculated from Eq. (2) in Fig. 1(b). The profiles were obtained by assuming a discharge current of 15 A.

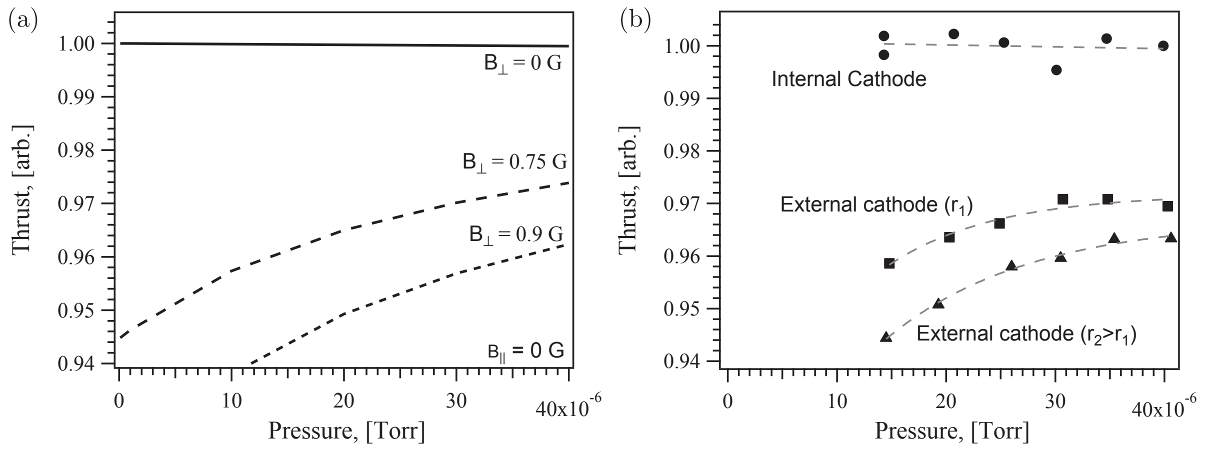

Figure 2. Panel (a) shows normalized thrust as a function of pressure for various values of perpendicular magnetic field derived from the simple model. Panel (b) shows normalized thrust as a function of pressure measured for H6 thruster and various cathode locations. Data taken from Ref. 14.

The magnetic field produced by an HCT at a typical cathode location is not negligible and can be as high as a few tens of Gauss. Furthermore, magnetic field topology in the near-cathode region is highly dependent on the HCT configuration. For example, cathode location with respect to the magnetic field separatrix was discussed by Sommerville and King.[23, 24] For our simple model we assume a constant magnetic field along the line of the electron motion (not to be confused with cathode axis) as well as perpendicular to it. In practice the self-induced magnetic field can play a significant role in the dynamics of the cathode near-field discharge. The current flowing through a small cathode orifice can produce a field as high as the externally applied magnetic field and we include the effects of the self-induced magnetic field in the MHD numerical simulation. However, because the field is azimuthal it plays no role in our 1-D model.With the stated restrictions the generalized Ohm's law can be written in 1-D as:

$$E_x = j_{ex} \cdot \frac{m_e \nu_e}{n_e q^2} \left(1 + \frac{\omega_y^2}{\omega_x^2 + \nu_e^2} \right) - \frac{1}{q n_e} \frac{\partial}{\partial x} (n_e k_b T_e), \tag{3}$$

where $\omega = qB/m_e$ is the electron cyclotron frequency, subscript x refers to the direction of the electron motion (away from the cathode), and the y direction is perpendicular to that. The electron collision frequency is a sum for the electron-neutral and electron-ion collision frequencies, $\nu_e = \nu_{en} + \nu_{ei}$. The electric field, obtained from Eq. (3), is integrated along the x axis for distance L, which is assumed to be 5 cm, to determine the cathode coupling voltage. Since we are interested in qualitative trends only, the actual length is not critically important. Thrust is then estimated by taking a square root of the difference between the discharge voltage, assumed to be 300 V, and the coupling voltage. All subsequent figures the plotted thrust values were obtained in this way and normalized by the thrust value at the highest pressure on each plot. Thus, each graph indicates the thrust reduction fraction as a function of the background chamber pressure – the facility effect. We consider a background pressure variation from 0 Torr – the ideal vacuum condition, to $4 \cdot 10^{-4}$ Torr – a vacuum condition typically observed in many vacuum chambers while operating a high power HCT.

IV. Simple Model Results

Figure 2(a) shows the normalized thrust obtained from the simple model as a function of the background pressure for a thruster operated at 15 A and 300 V and various values of perpendicular magnetic field. Note that in this figure we normalized all thrust values to the highest pressure thrust with $B_\perp = 0$ in order to compare the results with the available data, which is shown in Fig. 2(b). In this and the following figures the perpendicular component of the magnetic field $B_\perp$ corresponds to B_y in the text, while the parallel component $B_{||}$ corresponds to B_x. It is remarkable that according to this simple model, with zero magnetic field, thrust shows no dependence on the background pressure, i.e. no facility effect. As the perpendicular

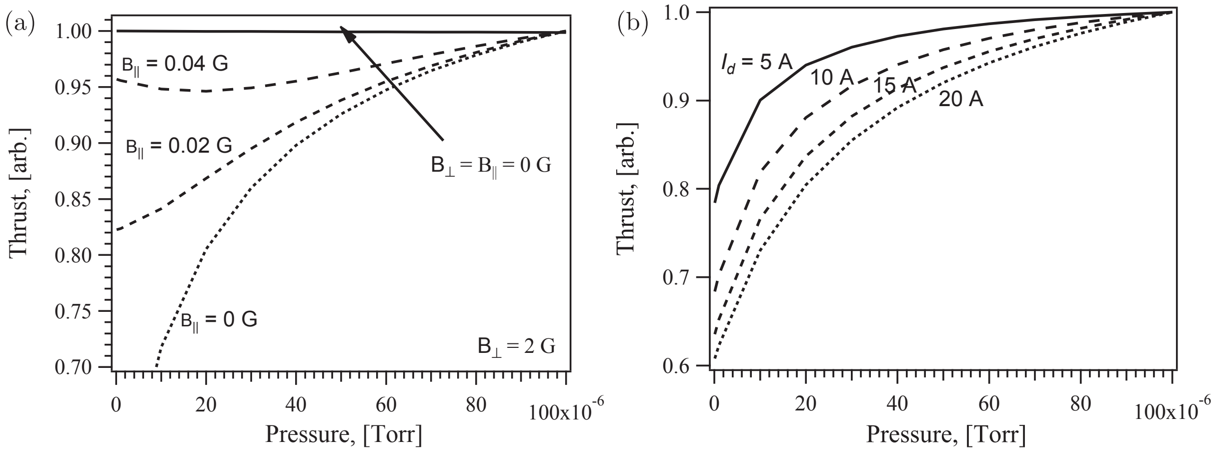

Figure 3. Panel (a) shows normalized thrust as a function of parallel magnetic field strength. Thrust dependence on pressure reduces with increased $B_{||}$. Panel (b) shows normalized thrust as a function of discharge current with non-zero $B_{\perp}$. Thrust dependence on pressure increases with increased current.

magnetic field increases, thrust drops for a given pressure. Also, for a given magnetic field value a non-linear dependence appears with an increasing slope toward the lower pressure − qualitatively similar to the experimental observations. An example of such an experimental observation with the H6 thruster is shown in Fig. 2(b). Here thrust data from Ref. 14 are shown normalized in a similar manner as the model. The data shows that when the cathode is placed at the HCT centerline ($B_{\perp} = 0$) thrust is not affected by the background pressure, as the model suggests. The parallel component of magnetic field can be large at the thruster centerline but, as will be shown later, it plays no role in the electron transport as long as $B_{\perp}$ is small. When the cathode is placed externally to the thruster, facility effects are experimentally observed as a non-linear drop in thrust at lower pressure. The non-linear trend varies with the cathode location. The model hints that such a variation may be caused by the differences in the local magnetic field strength and its direction, as shown by Fig. 2(a). For some thrusters it was reported that thrust slightly increased as the background pressure was reduced.[17] That trend can also be replicated by the simple model with the right combination of the perpendicular and parallel magnetic fields.

We note that magnetic field values used to generate curves in Fig. 2(a) seem to be an order of magnitude too small. One possible explanation for this discrepancy with the experimental results is that the model does not take into account anomalous electron transport, which can play a significant role in the cathode plume physics.[20,28] Without anomalous transport, values of $B_{\perp}$ on the order of tens of Gauss significantly reduce plasma conductivity. Large values of coupling voltage are needed to sustain the current, on the order of 100s of Volts. Rather than including an ad-hoc anomalous transport term in order to achieve realistic magnetic field values, we chose to use the simple model as is, and only perform qualitative, comparative analysis. Another possible explanation that may only partly account for the discrepancy in the magnetic field magnitude determined by the model is that the assumption of a constant angle expansion is not correct. Figure 4(a) shows that the concavity of the thrust versus pressure curve strongly depends on the expansion angle θ_e as well as magnetic field. Future analysis should incorporate a more realistic approximation, or solve the electron mass conservation equation explicitly. Finally, a small amount of the parallel component of the magnetic field may also increase the value of the perpendicular component necessary to produce the facility effects.

At first it may seem that magnetic field in the direction parallel to the electron flow should not play a role in determining plasma conductivity. That is indeed the case when there is also no perpendicular magnetic field. However, with both components of magnetic field present the electron motion becomes inherently three dimensional and the parallel magnetic field plays a significant role. We will discuss this further in Section V. Figure 3(a) demonstrates the facility effect sensitivity to the parallel component of magnetic field. The lower dashed curve in the figure was created with $B_{\perp} = 2$ G and $B_{||} = 0$ G. The curve shows a significant dependence on the background pressure. As the parallel component of magnetic field increases, the sensitivity to pressure diminishes until it completely disappears when $B_{||} \approx 0.1 B_{\perp}$. This potentially explains why thrusters with the center mounted cathodes seem to be immune to facility effects.[14,15]

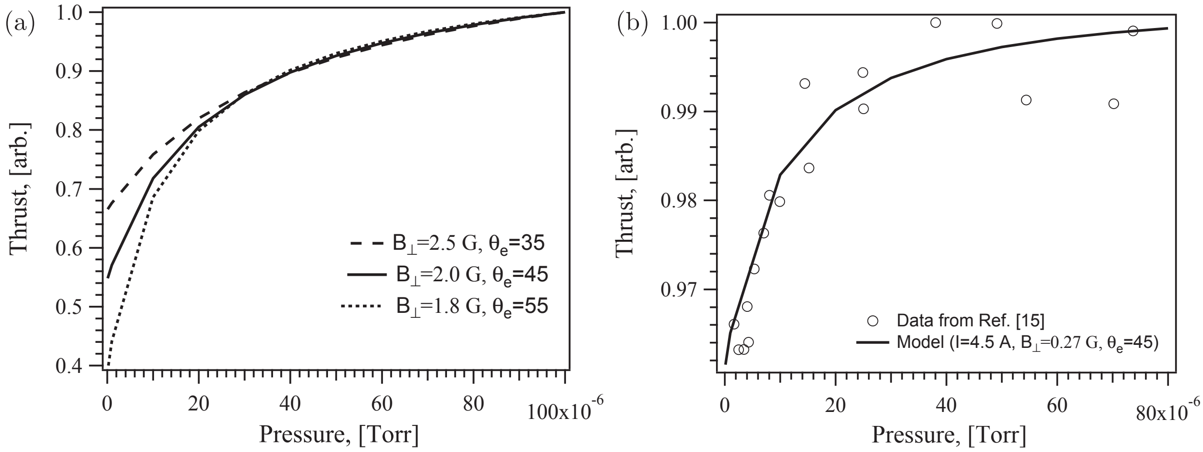

Figure 4. Panel (a) shows normalized thrust derived from the simple model for various combination of magnetic field and plume divergence. Panel (b) shows normalized thrust measured for SPT-100 and reported in Ref. 15, shown by open circles, and a simple model fit to the data, solid curve.

Hofer and Anderson[14] reported that facility effects increase with power, more specifically with current. To investigate that trend using our model we plotted normalized thrust for various discharge current levels and the same discharge voltage. To produce these curves we assumed that the magnetic field and mass flow rate scale with the discharge current – a realistic assumption for most HCTs. Figure 3(b) shows that the model can qualitatively replicate the observed dependence – thrust degradation increases with the current. The results also show that the concavity of the curve increases as the discharge current decreases. The practical implication of this observation is that some thrusters may require thrust measurements down to lower pressure in order to observe facility effects. This question is relevant to the discussion of the minimum facility pressure requirement needed to produce space-like performance.

A survey of the published facility effects data indicates that the amount of thrust decrease with pressure as well as the shape of the trend curve is a very sensitive function of the operating parameters for a given thruster. It is not surprising that different thrusters exhibit various degrees of facility effect sensitivity. So far we have shown that our simple model can replicate general trends observed with the cathode location (or magnetic field topology) as well as the thruster current. Is the model flexible enough to fit a specific curvature for an actual thruster? Figures 4(a) and 3(b) show that the curvature of the thrust vs. pressure curve can be controlled slightly by selecting the plasma expansion angle, magnetic field, and current. In Fig. 4(b) we show normalized thrust for the SPT-100 versus background pressure as reported by Diamant *et al.* in Ref. 15 (open circles) and a simple model curve fit. The best fit for the discharge current of 4.5 A was obtained with the plasma expansion angle of 45 degrees and $B_y = 0.27$ G. We once again note that the magnitude of the perpendicular magnetic field needed to fit the data is too low to be realistic, however, that is not surprising given the stated limitations of the simple model.

In order to improve the fidelity of our findings we attempted to solve equation set (1) numerically in 2-D. However, the computational demands were beyond the currently available capabilities. Instead, we adopted a 2-D MHD approach, which allows tracking the electron and ion temperatures separately. This two-temperature MHD approach has been implemented using the Tech-X Corporation's USim software. The software solves a set of the Euler equations for the neutral gas as well as the two-temperature MHD equations (not shown here), coupled with the generalized Ohm's law. Currently the model can produce realistic plasma and neutral density profiles. We intend to publish full results of the model in an upcoming paper, but will not discuss those results here.

V. Discussion of Results

We now return to Eq. (3) in order to understand the underlying mechanism behind the non-linear thrust decrease. Careful examination of the two terms in this equation reveals that the key factor influencing the shape of the electric field is the ratio of the magnetic field strength in x and y directions and the electron

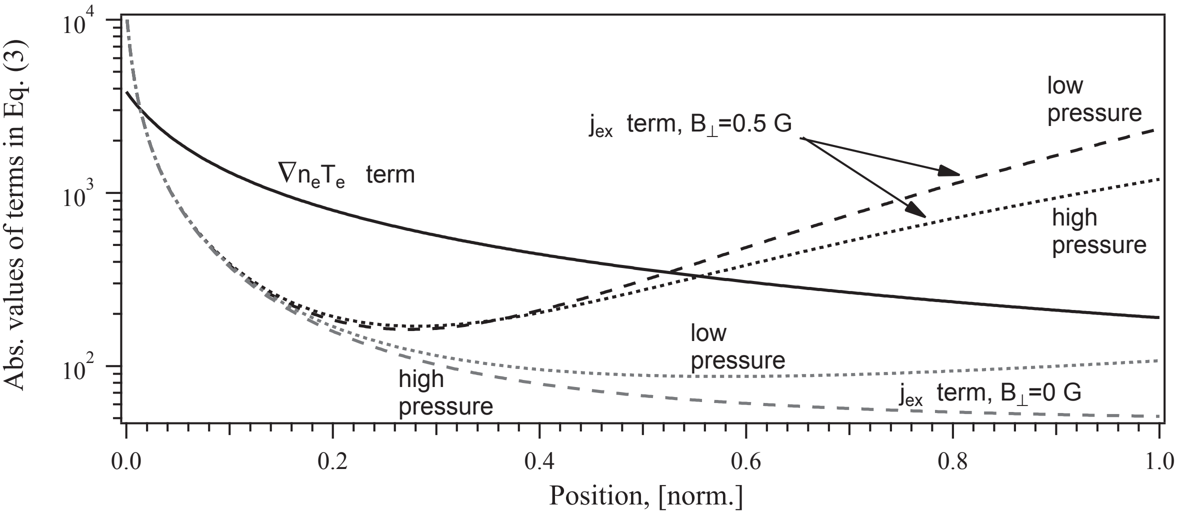

Figure 5. Relative contribution of the resistive and convective terms in Eq. (3). The convective term, labeled $\nabla n_e T_e$ is greater than the resistive term, labeled j_{ex}, when $B_\perp = 0$ at all pressures in the considered range. When $B_\perp > 0$ resistive term can dominate, specifically away from the cathode. Contribution of the resistive term increases as pressure is decreased.

collision frequency. We call this factor the resistivity enhancement ratio since it appears next to the standard plasma resistivity and acts to increase it. The ratio reduces to 0 in the absence of magnetic field, reducing resistivity to the simple familiar expression. It also strongly depends on the background neutral pressure, which enters through the electron-neutral collision frequency, which is a part of ν_e.

Since the cathode coupling voltage V_c is computed as the integral of the electric field along the length of the computational domain, greater values of the electric field in general correspond to lower values of thrust. The value of the electric field is determined by the balance of the two terms in Eq. (3), the resistive term and the electron pressure term. Note that Eq. (3) does not have the ion current term, which is typically present in a more general version of the Ohm's law. The ion current term was verified to be negligible and is thus ignored. The absolute values of the two terms are plotted as a function of the normalized distance away from the cathode exit for a case of $I_d = 20$ A and two cases of magnetic field $B_y = 0.0$ G and $B_y = 0.5$ G in Fig. 5. When the y component of magnetic field is zero the electron pressure term dominates regardless of the neutral pressure, except in a short region immediately adjacent to the cathode, making the coupling voltage nearly independent of the chamber background pressure, as indeed can be seen in Fig. 2(a).

With the non-zero B_y the relative contribution of the resistive term becomes more significant, specifically in the regions away from the cathode, as shown in Fig. 5. The resistive term contribution grows as the background pressure is reduced. Thus, while at high pressure V_c approaches the value determined by the electron pressure term, at low pressure the resistive term plays a large role in determining the cathode coupling voltage. This relative balance explains why thrust, which is proportional to $\sqrt{V_d - V_c}$, saturates as the background pressure increases. At a lower pressure the resistive term, enhanced by the magnetic field plays a larger role in determining thrust, thus producing the observed non-linear behavior.

The x component of magnetic field plays a moderating role. Being in the denominator and additive to the electron collision rate, this term can reduce the overall resistivity. Even relatively small values of B_x can drive the resistivity enhancement ratio to zero, reducing plasma resistivity to the unmagnetized value. Thus, B_x can be viewed as an agent that can help to reduce the cathode coupling voltage and improve performance of the thruster. Furthermore, because B_x changes the functional dependence of the electric field on the electron-neutral collision frequency, it is possible to choose a combination of B_x and B_y such that thrust can slightly increase as the background pressure is lowered. This may at first seem counterintuitive, but such a thrust trend has indeed been observed experimentally, as shown in Fig. 2(b) and also in Ref. 17.

VI. Conclusions

In this paper we investigated a phenomenon commonly described as the facility effect, where thrust of an HCT is observed to decrease non-linearly with the decreasing background pressure in a vacuum chamber. The amount of that decrease is significantly greater than can be explained by a simple gas ingestion model. To understand the non-linear behavior, we investigated how the cathode coupling voltage is affected by the background pressure in the presence of a magnetic field. We developed a set of equations that describe a plasma flow in the region near a cathode and solved these equations numerically, in a simplified form, to obtain the qualitative trends.

We found that some of the experimentally observed performance trends can be qualitatively replicated by selecting the model parameters appropriately. With no magnetic field, or when the cathode is immersed in a field dominated by the parallel component, the model suggests that thrust is largely independent of the background pressure. This may help to explain why thrusters operated with the cathode inserted in the center of the HCT body do not show performance sensitivity to the background pressure.

Thrust degradation with the decreasing background pressure can be observed with the model, when the perpendicular component of magnetic field is sufficiently large. The values of magnetic field produced by the model that are necessary to observe a thrust decrease are most likely too low. That however, may be a result of the model simplicity. In practice, the strength of the magnetic field needed to observe the facility effect is highly dependent on the thruster design, so no quantitative recommendation can be given based on the simple model. Nevertheless, some general scaling laws are apparent. 1) The thrust loss with pressure is greater at higher values of the discharge current. 2) The magnitude of the facility effect increases with the increasing value of perpendicular magnetic field. This implies that the cathode location should be chosen to minimize B_y. The direction of B_y is not determined with respect to the cathode axis, but rather relative to the direction of the electron flow.

In order to improve fidelity of our results we are developing a more sophisticated 2-D numerical simulation. This simulation will solve the MHD equations in order to determine the electric field profile in the near-cathode region more accurately. A further step toward greater fidelity is to be able to solve the fluid equation set (1). Furthermore, an expansion of the model to include the thruster near-field region may allow us to address the second facility effect issue – the plume divergence.

Appendix

This appendix provides expressions for various rate coefficients used in the calculations. The ionization rate coefficient β for $T_e < 5$ eV is given by the following expression in Appendix E of Ref. 26 (page 475)

$$\beta = 10^{-20}\sqrt{\frac{8qT_e}{\pi m_e}} \cdot \left(3.97 + 0.643T_e - 0.0368T_e^2\right) e^{-U/T_e} \quad [\mathrm{m}^3/\mathrm{s}] \tag{4}$$

The electron-neutral and electron-ion collision frequencies can also be found in Ref. 26 on page 58

$$\nu_{en} = 6.6 \cdot 10^{-19} n_n \left[\frac{0.25T_e - 0.1}{1 + (0.25T_e)^{1.6}}\right] \sqrt{\frac{8qT_e}{\pi m_e}} \quad [\mathrm{s}^{-1}] \tag{5}$$

$$\nu_{ei} = 2.9 \cdot 10^{-12} n_e \left[23 - 0.5 \log\left(10^{-6} n_e / T_e^3\right)\right] / T_e^{1.5} \quad [\mathrm{s}^{-1}] \tag{6}$$

The ion collision rate is assumed to be primarily determined by the CEX cross-section, which can be found in Ref. 29,

$$\nu_i = 10^{-20} n_e v_i \left(181 - 21.2 \log_{10} v_i\right) (U/13.6)^{-1.5} \quad [\mathrm{s}^{-1}] \tag{7}$$

Plasma thermal conductivity κ_e can be found in Refs. 30 and 26 (page 52)

$$\kappa_e = 3.16 \frac{q^2}{m_e} \frac{n_e T_e}{\nu_{ei} + \nu_{en}} \quad [\mathrm{W/meV}] \tag{8}$$

Acknowledgements

The authors would like to express their gratitude to K. D. Diamant for the insightful discussions regarding facility effects. Preparation of this manuscript was supported under The Aerospace Corporation's Sustained Experimentation and Research for Program Applications program.

References

[1]T. Randolph, V. Kim, K. Kozubsky, V. Zhurin, and M. Day. Facility effects on stationary plasma thruster testing. Presented at the 24^{th} International Electric Propulsion Conference, Moscow, Russia, 1993, IEPC-1993-093.

[2]J. M. Sankovic, J.A. Hamley, and T. W. Haag. Performance evaluation of the russian SPT-100 thruster at nasa lerc. Presented at the 24^{th} International Electric Propulsion Conference, Moscow, Russia, 1993, IEPC-1993-094.

[3]J. M. Sankovic and T. W. Haag. Operating characteristics of the russian D-55 thruster with anode layer. Presented at the 30^{th} AIAA/ASME/SAE/ASEE Joint Propulsion Conference (JPC), Indianapolis, IN, July 27-29 1994, AIAA-1994-3011.

[4]M. L. R. Walker. *Effects of Facility Backpressure on the Performance and Plume of a Hall Thruster.* Ph. d., The University of Michigan, 2005.

[5]Y. Azziz. *Experimental and Theoretical Characterization of a Hall Thruster Plume.* Ph. d., Massachusetts Institute Of Technology, June 2007.

[6]J. M. Fife. Numerical simulation of facility effects on Hall thruster performance. Presented at the 26^{th} International Electric Propulsion Conference, Kitakyushu, Japan, October 17-21, 1999, IEPC-1999-097.

[7]R. R. Hofer, P. Y. Peterson, and A. D. Gallimore. Characterizing vacuum facility backpressure effects on the performance of a Hall thruster. Presented at the 27^{th} International Electric Propulsion Conference, Pasadena, Ca, USA, Oct. 15-19, 2001, IEPC-2001-045.

[8]D. Byers and J. Dankanich. A review of hall effect thrusterfacility effects. Presented at The 29^{th} International Electric Propulsion Conference, Ann Arbor, MI, September 20-24, 2009, IEPC-2009-079.

[9]K. Diamant, R. Spektor, E. J. Beiting, J. Young, and T. J. Curtiss. The effects of background pressure on Hall thruster operation. Presented at the 48^{th} AIAA/ASME/SAE/ASEE Joint Propulsion Conference (JPC), Atlanta, GA, July 29 – August 1 2012, AIAA-2012-3735.

[10]M. W. Crofton and J. E. Pollard. Thrust augmentation by charge exchange. Presented at The 49^{th} AIAA/ASME/SAE/ASEE Joint Propulsion Conference, San Jose, Ca, July 14-17, 2013, AIAA-2013-4131.

[11]M. W. Crofton and I.D. Boyd. Momentum modeling of a Hall thruster plume. Presented at the 33^{rd} International Electric Propulsion Conference, Washington, D.C., USA, 6-10 October 2013, IEPC-2013-374.

[12]H. Kamhawi, W. Huang, and T. Haag. Investigation of the effects of facility background pressure on the performance and operation of the high voltage hall accelerator. Presented at The 50^{th} AIAA/ASME/SAE/ASEE Joint Propulsion Conference, Cleveland, Oh, July 28-30, 2014, AIAA-2014-3707.

[13]W. Huang, H. Kamhawi, R. Lobbia, and D. Brown. Effect of background pressure on the plasma oscillation characteristics of the HiVHAc hall thruster. Presented at The 50^{th} AIAA/ASME/SAE/ASEE Joint Propulsion Conference, Cleveland, Oh, July 28-30, 2014, AIAA-2014-3708.

[14]R. Hofer and J. Anderson. Backpressure effects on the operation of hall thrusters. Presented at The 50^{th} AIAA/ASME/SAE/ASEE Joint Propulsion Conference, Cleveland, Oh, July 28-30, 2014, AIAA-2014-3709.

[15]K. Diamant, R. Liang, and R. Corey. The effect of background pressure on SPT-100 hall thruster performance. Presented at The 50^{th} AIAA/ASME/SAE/ASEE Joint Propulsion Conference, Cleveland, Oh, July 28-30, 2014, AIAA-2014-3710.

[16]J. Frieman, S. King, M. Walker, and V. Khayms. Preliminary assessment of the role of a conducting vacuum chamber in the hall effect thruster electrical circuit. Presented at The 50^{th} AIAA/ASME/SAE/ASEE Joint Propulsion Conference, Cleveland, Oh, July 28-30, 2014, AIAA-2014-3712.

[17]K. D. Diamant, T. J. Curtiss, R. Spektor, E. J. Beiting, V. Hruby, B. Pote, J. Kolencik, and S. Paintal. Performance and plume characterization of the BHT-1500 Hall thruster. Presented at the 34^{rd} International Electric Propulsion Conference, Kobe, Japan, 6-10 July 2015, IEPC-2015-053.

[18]R. R. Hofer, L. K. Johnson, D. M. Goebel, and D. J. Fitzgerald. Effects of an internally-mounted cathode on Hall thruster plume properties. AIAA-2006-4482, Presented at The 42^{nd} AIAA Joint Propulsion Conference, Sacramento, California, 9-12 July 2006.

[19]K. K. Jameson, D. M. Goebel, R. R. Hofer, and R. M. Watkins. Cathode coupling in Hall thrusters. Presented at the 30^{th} International Electric Propulsion Conference, Florence, Italy, 17-20 September 2007, IEPC-2007-278.

[20]K. K. Jameson. *Investigation of Hollow Cathode Effects on Total Thruster Efficiency in a 6 kW Hall Thruster.* Ph.d, UCLA, 2008.

[21]R. R. Hofer, L. K. Johnson, and D. M. Goebel. Effects of internally mounted cathodes on Hall thruster plume properties. *IEEE Trans. Plasma Sci.*, 36(5), 2008.

[22]M. S. McDonald and A. D. Gallimore. Cathode position and orientation effects on cathode coupling in a 6-kw Hall thruster. Presented at the 31^{st} International Electric Propulsion Conference (IEPC), Ann Arbor, MI, USA Sep. 20–24, 2009. IEPC-2009-113.

[23]J. D. Sommerville and L. B. King. Hall-effect thrustercathode coupling, part i: Efficiency improvements from an extended outer pole. *J. Prop. Power*, 27(4):744, July-August 2011.

[24]J. D. Sommerville and L. B. King. Hall-effect thrustercathode coupling, part ii: Ion beam and near-field plume. *J. Prop. Power*, 27(4):754, July-August 2011.

[25]D. M. Goebel, K. K. Jameson, and R. R. Hofer. Hall thruster cathode flow impact on coupling voltage and cathode life. *J. Prop. Power*, 28(2):355, March-April 2012.

[26]D. M. Goebel and I. Katz. *Fundamentals Of Electric Propulsion: Ion And Hall Thrusters.* John Wiley & Sons Inc, Hoboken, NJ, USA, 2008.

[27]I. Katz, I. G. Mikellides, and D. M. Goebel. Model of the plasma potential distribution in the plume of a hollow cathode. Presented at the 40^{th} AIAA/ASME/SAE/ASEE Joint Propulsion Conference (JPC), Fort Lauderdale, Florida, July 11-14 , 2004, AIAA-2004-4108.

[28]I. G. Mikellides, I. Katz, D. M. Goebel, and K. K. Jameson. Evidence of nonclassical plasma transport in hollow cathodes for electric propulsion. *J. Appl. Phys.*, 101(063301), 2007.

[29]S. Roy and B. P. Pandey. Development of a finite element-based hall-thruster model. *J. Prop. Power.*, 19(5):964, September-October 2003.

[30]I. G. Mikellides, I. Katz, D. M. Goebel, and J. E. Polk. Hollow cathode theory and experiment. ii. a two-dimensional theoretical model of the emitter region. *J. Appl. Phys.*, 98(113303), 2005.