

Hybrid simulation of a Magnetic Nozzle plume: exploring magnetized/demagnetized regimes

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Magnetic nozzles (MN) are envisaged as a suitable acceleration system for advanced plasma thrusters. By using an applied divergent magnetic field, the hot plasma of these engines can be efficiently channelled and expanded into vacuum to produce thrust in a very similar manner to a solid de Laval nozzle. The model used here is based on Particle-in-cell (PIC) methods for ions and neutrals, and an anisotropic fluid model for electrons. This work explores the electron magnetization regimes, from the high magnetized and nearly collisionless regime to the low magnetized regime. The demagnetization effects on the plasma 2D supersonic expansion such as beam collimation and divergence, ion detachment and thrust gain are analyzed in depth.

I. Introduction

Some propulsion devices propose the divergent magnetic nozzle as an efficient mechanism to accelerate the plasma, generated upstream, inside the thruster discharge chamber, like the Helicon thruster,^{1,2,3,4} the applied-field magnetoplasma dynamic thruster,⁵ the VASIMR⁶ and the ECR thruster.⁷

The hybrid model used here postulates electrons as an anisotropic fluid, while heavy particles, ions and neutrals, are described by a kinetic description, using a conventional particle-in-cell (PIC) method. This kind of methods have been optimized for several applications; among them it stands out the pioneering work of Fife^{8,9} on the simulation of Hall Effect Thrusters (HET), followed by some improvements of Parra et al.¹⁰ Here, this kind of model is applied to an expanding plasma that flows through a divergent magnetic nozzle. The current model also includes some features described in Ref. 11 for dealing with plasma-wall interaction.

The current work aims to explain how plasma beam collimation, magnetic thrust, and nozzle efficiency behave due to variations on the magnetization regime. Inward ion detachment is observed in Ref. 12. However, that collisionless model only deals with fully magnetized electrons, consequently, the plasma-vacuum edge (delimited by the magnetic field) cannot be overcome by any particle. This is not well justified for partially magnetized electrons which is one of the scenarios to be analysed here, and hopefully, affordable by the hybrid model.

Electrons are considered isothermal over the whole domain of the simulation. Our recent work in Ref. 13 inquires into this issue and concludes that under the expected collisionless regime of a divergent magnetized plasma jet, the isothermal hypothesis early fails. But it can be still applied at the vicinity of the nozzle throat, i.e. near plume. A non-isothermal electron formulation, like the one implemented in Ref. 9, cannot be applied directly: the different boundary conditions and uncertainties on the evolution of the electron distribution function through the expansion makes this problem quite different.

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This article is structured as follows. In a first place, the hybrid model is summarized stressing the key points for its application to the divergent magnetic nozzle. Secondly, a plasma expansion simulation through a divergent magnetic nozzle will be compared for different magnetization regimes. Section IV discusses some specific phenomena regarding the limitations of the current model. The last section is for conclusions and the announcement of future improvements.

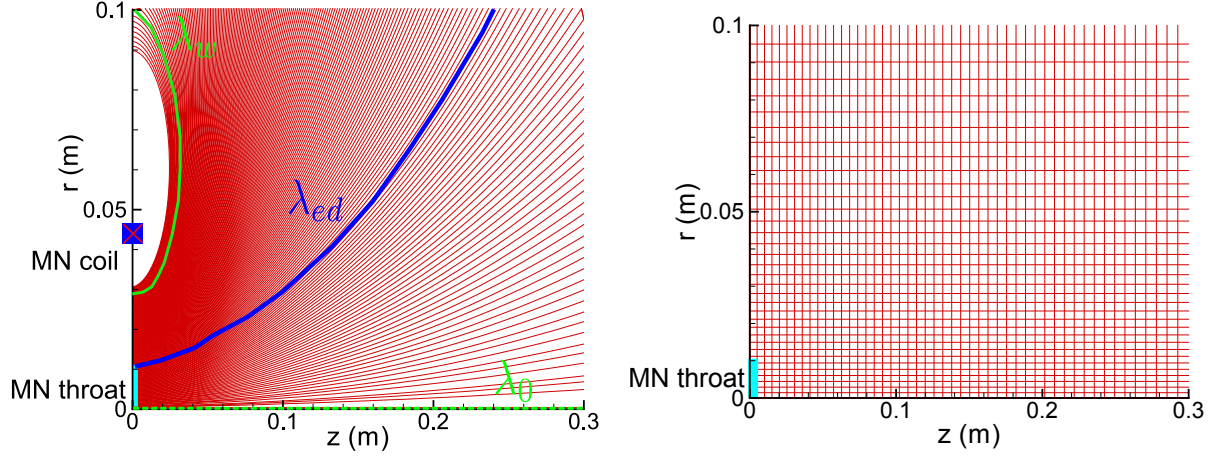


Figure 1. Left: Magnetic mesh for the electron fluid model. The magnetic streamline that defines the nozzle is labelled as “edge” λ_{ed} coloured in blue (plasma-vacuum edge in full fluid models as DIMAGNO¹²), λ_0 overlap the symmetry axis (dashed green line), λ_w is the longest line with both ends intersecting the lateral wall. Right: $r - z$ rectangular mesh for the PIC model. The MN throat panels are depicted in cyan, and its radius is $R = 0.01$ m.

II. The magnetic nozzle hybrid model

An axisymmetric diverging magnetic field is considered. At the MN throat, a plasma source, Helicon plasma source for instance, feeds the MN with a hot plasma, which indeed presents sonic conditions there, as in a “*de Laval*” nozzle throat. The injected plasma is considered to be in radial equilibrium, an issue further discussed later, and related to the magnetization regime. Thermal electron energy is converted into ion kinetic energy through the plasma ambipolar electric field. The quasi-neutral condition is invoked everywhere, $n_i \sim n_e = n$.

The hybrid model is divided into two different parts, the particle-in-cell model for ions and neutrals, and the electron fluid model. The first one provides the plasma density n and the ion particle flux $\mathbf{g}_i = n\mathbf{u}_i$ or the ion current density $\mathbf{j}_i = en\mathbf{u}_i$. On the other side, the electron sub-model returns the perpendicular electric current $I_\perp$ through each magnetic surface and the thermalized electric potential ϕ_0 , which will be defined later and it is necessary to compute the electric potential ϕ . These two variables, ϕ_0 and $I_\perp$, are one-dimensional and constant along each magnetic surface and inputs for the PIC sub-model. Furthermore, plasma is considered isothermal, so T_e is uniform everywhere and constant in time. Each sub-model will be discussed separately.

Regarding the PIC model¹⁴ approach, ion and neutral particles are moved according to electromagnetic and inertial forces on a radial-axial structured mesh, which is indeed the lengthwise section of a cylindrical prism, and it is presented in Figure 1. An auxiliary rectangular mesh is used for weighting PIC algorithms to evaluate the different moments of the ion distribution function: density, particle fluxes, ion temperatures, etc.. Weighting algorithms use a first-order method (bilinear interpolation method) for the calculation of the mentioned macroscopic properties. A *leap-frog* scheme is implemented to integrate particle trajectories. It results on a first-order model for particle velocities, and second-order for particle positions. Spatially, the model is 2D because it only retains the information on the radial-axial $r - z$ plane. But it is 3D in the velocity space, and retains radial, azimuthal and axial velocities.

Ion particles are injected throughout the panels that form the MN throat section. The algorithm used to inject ions through these panels ensure that the obtained density profile fits the one defined by the radial equilibrium at the HPS, which is discussed later in this paper. This is achieved thanks to include radial

effects in the probabilistic method used for setting the initial position of each injected particle. The setting of the ion initial velocity is split into two components: the axial velocity is chosen considering that they behave as a drift-Maxwellian flux of temperature $T_{i\infty}$ (only $v_z > 0$, with $v_{dr} = c_s = \sqrt{kT_e/m_i}$ the ion sound speed). The transversal component, on the radial-azimuthal plane ($\mathbf{v}_r + \mathbf{v}_{theta}$) considers a full Maxwellian distribution of the same ion temperature $T_{i\infty}$.

A. The electron fluid model

The magnetic field $\mathbf{B}$ that defines the MN, is generated by a coil system, and its circuit/topology is solved during the pre-process using a commercial FEM code. The zero-vorticity and zero-divergence of the magnetic field, $\nabla \times \mathbf{B} = 0$ and $\nabla \cdot \mathbf{B} = 0$ respectively, ensures that a stream function λ exists and satisfies,

$$\frac{\partial \lambda}{\partial z} = -rB_r \quad \frac{\partial \lambda}{\partial r} = rB_z \quad (1)$$

where B_z and B_r are the magnetic field components expressed on a cylindrical reference frame $\{\mathbf{1}_z, \mathbf{1}_r, \mathbf{1}_\theta\}$. Hereafter, the magnetic reference frame will be $\{\mathbf{1}_\perp, \mathbf{1}_\parallel, \mathbf{1}_\theta\}$ in which $\mathbf{1}_\parallel = \mathbf{B}/B = \cos \alpha \mathbf{1}_z + \sin \alpha \mathbf{1}_r$ is the parallel direction, $\mathbf{1}_\perp = \sin \alpha \mathbf{1}_z - \cos \alpha \mathbf{1}_r$ is the perpendicular direction (pointing inward to the axis), $\alpha(r, z)$ is the local magnetic angle, and the azimuthal direction is determined by $\mathbf{1}_\theta = \mathbf{1}_\perp \times \mathbf{1}_\parallel$. The streamline revolution around the axis of symmetry, defines a magnetic surface. The space between successive magnetic surfaces define a differential volume. Each different λ value, belongs to a different magnetic streamline/surface. The perpendicular electric current through each surface $I_\perp(\lambda)$, and the thermalized potential $\phi_0(\lambda)$ are constants to be determined or set as conditions. The magnetic mesh is sketched in Figure 1.

The electron fluid dynamics equations to consider are the mass conservation and momentum equation and are listed below. In this case, energy equation is omitted because T_e is assumed constant.

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{u}_e) = \dot{n}_e, \quad (2)$$

$$n_e m_e \left(\frac{\partial \mathbf{u}_e}{\partial t} + \mathbf{u}_e \cdot \nabla \mathbf{u}_e \right) = -\nabla p_e - en_e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) + \mathbf{M}. \quad (3)$$

$\mathbf{u}_e$ is the electron flow velocity, m_e the electron mass, $p_e = nT_e$ the electron pressure which is considered isotropic. $\dot{n}_e$ is the particle production/loss term due to ionization/recombination. Nevertheless, none of these effects are considered here, assuming that the mean-free path for these collisions is much more larger than other collisional process $\lambda_{rec}, \lambda_{ion} \gg \lambda_{col}$. $\mathbf{M}$ include all collisional effects that modifies electron momentum. It can be written as $\mathbf{M} = -n_e m_e \nu_e \mathbf{u}_e$, where ν_e is the effective electron collision frequency and takes into account *electron-ion* and *electron-neutral* inelastic collisions. This frequency could also include other plasma physical processes related to the plasma wall interaction, virtual cathode considerations and anomalous diffusion as done for the investigation of HETs.¹⁰ But in this work, these contributions are not included anymore.

Equation 2 can be combined with the ion continuity equation, which has the same form. Thanks to the quasineutral plasma hypothesis and the lack of ionization processes, adding together electron and ion continuity equations it yields the charge conservation law,

$$\nabla \cdot \mathbf{j} = 0, \quad (4)$$

where $\mathbf{j} = en(\mathbf{u}_i - \mathbf{u}_e)$ is the electric current density. Turning back to the momentum equation. The inertial term on the left side drops due to the significant difference between the electron mass, and the ion mass $m_e/m_i \ll 1$. Moreover, this convective term is usually much more smaller than the electromagnetic force, on the right side of the equation, $\|n_e m_e \mathbf{u}_e \cdot \nabla \mathbf{u}_e\| / \|en_e \mathbf{u} \times \mathbf{B}\| \sim r_L u_e / R v_{te} \ll 1$. Here, r_L is the Larmor electron gyroradius, R the magnetic nozzle throat radius and v_{te} the electron thermal velocity.

The electric field $\mathbf{E}$ derives from a scalar potential $\mathbf{E} = -\nabla \phi$ as well. This is because both, the magnetic field fluctuations in time are negligible $\nabla \times \mathbf{E} = 0$ and plasma is quasineutral $\nabla \cdot \mathbf{E} \simeq 0$. Accounting for all these hypotheses, the momentum equation is reduced to,

$$\mathbf{0} = -T_e \nabla n_e + en_e \nabla \phi - en_e \mathbf{u}_e \times \mathbf{B} - n_e m_e \nu_e \mathbf{u}_e, \quad (5)$$

which can be projected on the parallel direction $\mathbf{1}_\parallel$,

$$0 = -T_e \frac{\partial n_e}{\partial \mathbf{1}_{\parallel}} + en_e \frac{\partial \phi}{\partial \mathbf{1}_{\parallel}}. \quad (6)$$

The collisional term in Eq. 6 has been dropped off because it is expected to be small in comparison to pressure gradients,

$$\frac{\|n_e m_e \nu_e \mathbf{u}_{e\parallel} / B\|}{\|T_e \partial n_e / \partial \mathbf{1}_{\parallel}\|} \sim \frac{u_{e\parallel} R}{v_{te} \lambda_{col}} \ll 1,$$

in which λ_{col} is the electron collision mean-free-path with ions/neutrals ($\lambda_{ei}/\lambda_{en}$). The ordering of the different lengths that characterize the helicon plasma at the MN stage is assumed the following: $\lambda_D \ll r_L \ll R \ll \lambda_{col}$; λ_D is the Debye length. Integrating Eq. 6 it yields the well-known Boltzmann relation

$$\phi(r, z) = \phi_0(\lambda) + \frac{T_e(\lambda)}{e} \ln \left(\frac{n_e(r, z)}{n_{ref}} \right); \quad (7)$$

$\phi_0(\lambda)$ is the thermalized potential and n_{ref} is a reference density, say $n(0, 0)$. The Boltzmann relation determines the electric potential in terms of one-dimensional variables ($T_e(\lambda)$ and $\phi_0(\lambda)$) and the plasma density distribution $n_e(r, z)$ provided by the PIC sub-model.

Analogously, the momentum equation is also projected on $\mathbf{1}_{\perp}$ and $\mathbf{1}_{\theta}$ respectively,

$$-T_e \frac{\partial n_e}{\partial \mathbf{1}_{\perp}} + en_e \frac{\partial \phi}{\partial \mathbf{1}_{\perp}} + en_e u_{e\theta} B - \nu_e m_e n_e u_{e\perp} = 0, \quad (8)$$

$$-en_e u_{e\perp} B - \nu_e m_e n_e u_{e\theta} = 0. \quad (9)$$

After some straightforward algebra, expressions for $\mathbf{u}_{e\perp}$ and $\mathbf{u}_{e\theta}$ are easily determined,

$$u_{e\perp} = \frac{r}{\chi_e} \frac{\partial \phi_0(\lambda)}{\partial \lambda}, \quad (10)$$

$$u_{e\theta} = -\chi_e u_{e\perp}. \quad (11)$$

$\chi_e = \Omega_{ce}/\nu_e$ is the electron Hall parameter, and Ω_{ce} is the electron gyrofrequency. The Hall parameter χ_e compares electron magnetization against electron collisions. Cross-field diffusion increases with χ_e decreasing, so in this work the parametric analysis is done in terms of χ_e . Note that Eq. 10 is equivalent to the reduced Ohm's law.

B. The plasma equilibrium at the MN throat

In our previous work¹⁵ the fully-ionized and free-current plasma jet was injected through the MN throat assuming it was fully magnetized. This considered the radial equilibrium of the plasma pressure against the magnetic pressure, and therefore the null radial electric field (i.e. $\phi(r) = \text{cnt} = 0, r \in [0, R]$). Combining that equilibrium with the Boltzmann relation, it yielded the thermalized potential condition for all lines crossing the MN throat, $\phi_0(\lambda), \forall \lambda \in (\lambda_0, \lambda_{ed})$,

$$\phi_0(\lambda) = -(T_e/e) \ln (n_e(r)/n_0)|_{z=0},$$

n_0 is the plasma density at the axis just at the MN throat section ($(z, r) = (0, 0)$). The density profile shape was taken parabolic because its resemblance to the 0th Bessel function. Actually, this is the trend fulfilled by the radial density profile in the asymptotic limit analysis of an infinite magnetized plasma column,¹⁶ $n(r)/n_0 \propto J_0(r/R)$.

The present work moves an step forward, and the radial equilibrium at the MN throat is solved for each regime during the pre-process time, and it is set as a boundary condition in the hybrid model. The model implemented here requires solving the 2D fluid-dynamics of a magnetized plasma column as we described in Ref. 17. Specifically the radial equilibrium at any radial section is solved through the following set of

derivative equations,

$$\begin{aligned}\left(\frac{1}{\hat{u}_r} - \hat{u}_r\right) \frac{d\hat{u}_r}{d\hat{r}} &= \hat{\omega}_{lh} \hat{u}_{\theta e} + \hat{\nu}_i \hat{u}_r + \frac{\hat{\nu}_w}{\hat{u}_r} - \frac{1}{\hat{r}} - \frac{\hat{u}_{\theta e}^2}{\hat{r}}, \\ \hat{u}_r \frac{d\hat{u}_{\theta e}}{d\hat{r}} &= \hat{\omega}_{lh} \hat{u}_r - \hat{\nu}_e \hat{u}_{\theta e} - \frac{\hat{u}_{\theta e} \hat{u}_r}{\hat{r}}, \\ 0 &= -\frac{d \ln \hat{n}}{d\hat{r}} + \frac{d\hat{\phi}}{d\hat{r}} - \hat{\omega}_{lh} \hat{u}_{\theta e} + \frac{\hat{u}_{\theta e}^2}{\hat{r}}, \\ \frac{d\hat{\phi}}{d\hat{r}} &= -\hat{u}_r \frac{d\hat{u}_r}{d\hat{r}} - \hat{\nu}_i \hat{u}_r.\end{aligned}$$

From the previous set of equation, the radial profile of all dimensionless variables are solved: radial plasma velocity $\hat{u}_r$, density $\hat{n}$, electric potential $\hat{\phi}$ and electron azimuthal velocity $\hat{u}_{\theta e}$. The results depend on the dimensionless parameters $\hat{\nu}_e, \hat{\nu}_i, \hat{\omega}_{lh}$, electron and ion frequency of collisions and the lower hybrid frequency $\omega_{lh} = m_i m_e / \sqrt{eB}$. $\hat{\nu}_w$ is the eigenvalue for the previous set of equations, determined through the fulfilment of boundary conditions at the Debye sheath transition, say the Bohm condition $\hat{u}_r(\hat{r} = 1) = 1$. Reference values for dimensionless variables are: the tube/throat radius R , for lengths, the ion sound speed $c_s = \sqrt{kT_e/m_i}$, for velocities, the electron temperature T_e , for energies, and the radial transit frequency c_s/R , for all frequencies. Once $\phi(r)$ and $n(r)$ are known, the condition $\phi_0(\lambda)$ is set for all streamlines that cross the MN throat. In the Results section, the radial profiles will be presented for some cases of interest.

Note that this model¹⁷ considers local current ambipolarity everywhere, which is appropriate to characterize the collisional regime of the plasma fluid-dynamics within the Helicon source. Consequently, the radial *plasma* confinement, or magnetization regime within the source, is measured in terms of the ratio $\chi_H = \omega_{lh}/\nu_e$, which is a parameter that appears naturally in the previous set of equations. This differs from the electron Hall parameter χ_e in a factor of $\sqrt{m_i/m_e}$, for instance $\sqrt{m_i/m_e} = 490$ considering the Xenon mass. The electron Hall parameter controls the *electron* magnetization regime at the nozzle, in a similar fashion as it controls electron cross-field diffusion in a Hall effect thruster.

C. Solving $I_{\perp}(\lambda)$ and $\phi_0(\lambda)$

For all lines intercepting the throat, and due to the setting of throat boundary condition, $\phi_0(\lambda)$ is constant in time and uniform along each one of them. Thus, once the PIC sub-model provide the plasma density, the local potential is determined by Eq. 7 and the electric field can be computed, $\mathbf{E} = -\nabla\phi$. Otherwise, the unique one-dimensional variable that has to be evaluated is the perpendicular current through magnetic surfaces $I_{\perp}(\lambda)$. This depends on both electrons and ions, and it is calculated as the integral over the magnetic surface of the local current density,

$$I_{\perp}(\lambda) = \iint_{S(\lambda)} (\mathbf{j} \cdot \mathbf{u}_{\perp}) dS = \int_{\Gamma(\lambda)} 2\pi r n_e (\mathbf{u}_i - \mathbf{u}_e) \cdot \mathbf{u}_{\perp} d\chi = I_{i\perp}(\lambda) + I_{e\perp}(\lambda) \quad (12)$$

The ion current $I_{i\perp}$ is provided by the PIC sub-model, while the electron current $I_{e\perp}$ is computed accounting for $u_{e\perp}$ as defined in Eq. 10, this yields,

$$I_{e\perp}(\lambda) = - \int \frac{2\pi e n_e r^2}{\chi_e} ds \left(\frac{\partial \phi_0}{\partial \lambda} \right). \quad (13)$$

ds is the magnetic line differential length.

Lets consider the charge conservation law, Eq. 4, which is also projected on the magnetic mesh, obtaining the following law after applying the Divergence theorem,

$$\frac{\partial I_{\perp}(\lambda)}{\partial \lambda} = - \left. \frac{2\pi j}{(B \cos \gamma)} \right|_0 - \left. \frac{2\pi j}{(B \cos \gamma)} \right|_1. \quad (14)$$

The last expression indicates that the variation of the perpendicular current through each magnetic surface is equivalent to the loss or gain of net charge current at the differential domain boundaries. Subscript 0 and 1 refers to plasma properties just at the magnetic streamlines boundaries, being 0 the boundary located closest to the left bottom corner of the PIC domain (say magnetic throat), and 1, the streamline end located

downstream. Current densities j are positive according to the orthogonal outward vector to the PIC domain boundary. γ is the angle between the mentioned vector and the local vector $\mathbf{1}_\perp$. For all analysed cases, the local current ambipolarity is imposed at the throat, $j_0 = 0$. So, the current downstream j_1 is obtained from Eq. 14,

$$j|_1 = -\frac{\partial I_\perp(\lambda)}{\partial \lambda} \frac{B \cos \gamma}{2\pi}. \quad (15)$$

Considering the absence of any external electrode/cathode, plasma must be globally current-free. Defining λ_{ed} as the magnetic stream line that defines the plasma-vacuum “edge”, which for instance coincides with $r = R$ at the MN throat, and λ_0 the one that corresponds to the axis; the area between them corresponds to the denser plasma jet region. In comparison to fully fluid models,¹⁸ which consider a perfect edge between the plasma and vacuum, the imbalance committed on the current-free condition within the dense plasma domain, $\lambda \in [\lambda_0, \lambda_{ed}]$, would be,

$$\epsilon_c = -I_\perp(\lambda_{ed}) + \int_{\lambda_0}^{\lambda_{ed}} (\mathbf{j}(\lambda) \cdot d\mathbf{S}(\lambda))|_1. \quad (16)$$

Moreover, this hybrid model does not forbid the possibility of ions to diffuse into this highly rarefied area, a result that occurs in the demagnetized regime as it will be shown in the results section.

Outside the region in which magnetic lines do not cross the MN throat, a closure of the boundary conditions of the electron fluid equations must be postulated. There are still some uncertainties about which is the most appropriate solution. This depends on several aspects: the dynamics of electrons trapped on the magnetic field, wall properties (dielectric, conducting), the geometry of the magnetic field itself, whether lines intersect the wall or intersect the vacuum boundary.

In the current scenario the authors propose that the lateral wall is made of a dielectric material. In other words, local and global zero net current is imposed there. The problem now remains on the transition to this zero net current from the usually non-zero perpendicular current determined at the edge of the dense plasma $I_\perp(\lambda_{ed})$ (Eq. 12). A potential drop of this magnitude in the λ coordinate, with δ a parameter to control it, is suggested

$$I_\perp(\lambda) = I_\perp(\lambda_{ed}) \left(\frac{\lambda - \lambda_w}{\lambda_{ed} - \lambda_w} \right)^\delta \quad (17)$$

λ_w is the longest magnetic line of the magnetic mesh that intersects the dielectric wall on its both ends, its top end is approximately located at the left-upper corner of the PIC mesh. The main idea is that the net current through λ_w , and the rest of lines that also intersect the wall in the same manner must be 0. Since in the current domain, there are magnetic streamlines that intersect the wall in one end but not in the other, because it intersects the “free” boundary, a zero net current along these fragment of line is not a mandatory condition. According to this argument it is reasonable to impose the mentioned law for $I_\perp$. Also note that in this region, the plasma density is expected to be very low, so electron collisionality becomes smaller, and consequently electron cross field diffusion should be reduced as well. As a result the perpendicular current should also decrease. The physical interpretation of δ is unclear and it is just imposed to force the fulfilment of plasma-wall boundary conditions. At this low density/outer region, $\phi_0(\lambda)$ is determined recombining Eq. 12 - 14, with $j_0 = 0$, and also $j_1 = 0$ if magnetic lines intercept the dielectric wall at its both ends.

III. Simulation results

The Hall parameter χ_{lh} and the plasma electron temperature T_e are considered inputs and identify all the studied cases. χ_{lh} is defined at the center axis line just at the MN section ($r = 0, z = 0$), which is the interface between the helicon tube and the MN plume. At the mentioned location, plasma density is set to $n_0 = 10^{19} \text{ m}^{-3}$ and it is kept constant for all analysed cases. χ_{lh} is written in terms of B_0 , n_0 and the electron-ion rate of collisions, R_{ei} .

$$\chi_{lh} = \frac{eB_0}{n_0 R_{ei} \sqrt{m_e m_i}}. \quad (18)$$

Electron-neutral collisions are included in the hybrid model to compute the equivalent electron frequency of collisions ν_e . However, on the definition of the reference Hall parameter χ_{lh} at the MN throat, plasma is assumed to be injected as a fully-ionized gas, so $\nu_{en} \simeq 0$. The presence of neutrals in the hybrid simulation is only due to ion recombination at the left dielectric wall that closes the domain over the nozzle throat. The

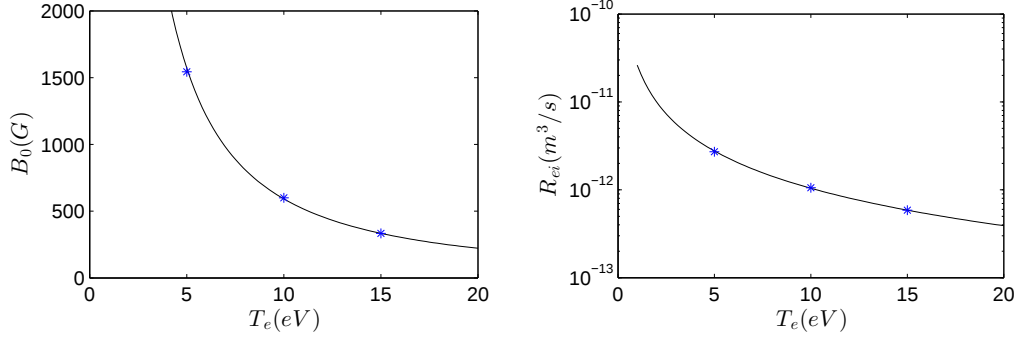


Figure 2. Left: Required B_0 (G) to keep $\chi_e = 1000$ ($\chi_{lh} \simeq 2$) constant when varying the electron temperature. Right: variation of $R_{ei}(T_e) \propto T_e^{-3/2}$. Asterisks corresponds to $T_e = 5, 10, 15$ eV.

dimensionless electron-ion collision rate $R_{ei}(T_e)$, which is proportional to $T_e^{-3/2}$, decreases with increasing the electron temperature, resulting $\hat{\nu}_{ei} = \nu_{ei}R/c_s = 195.72, 50.67, 23.84$ at $T_e = 5, 10, 15$ eV respectively, as shown in Fig. 2. This figure also indicates how the magnetic field strength B_0 is tuned in order to adjust the Hall parameter to the desired value.

Once this parameter is fixed, the radial equilibrium is solved according to the model in Section II B. At the throat, the necessary ion mass flow rate $\dot{m}$, is determined integrating the density radial profile and assuming that axially, plasma flows at its sonic velocity c_s ,

$$\dot{m} = 2\pi R^2 n_0 m_i c_s \int_0^1 \hat{n} \hat{r} d\hat{r}. \quad (19)$$

After determining the plasma flow, all conditions for the hybrid model are totally set, and the numerical simulations can be performed. All the results that will be shown in this section, have been obtained from time-averaged simulations that cover a total elapsed real time $\Delta t_T = 4\Delta z/c_s$, being Δz the axial length of the simulation domain, for instance $\Delta z = 0.3$ m ($\Delta z = 30R$).

Two different studies have been carried out. The first one consists on a validation of the highly magnetized regime, for the three mentioned temperatures. The second one sets up the temperature at 10 eV and explores the transition from the magnetized to the demagnetized regimes by decreasing χ_{lh} progressively.

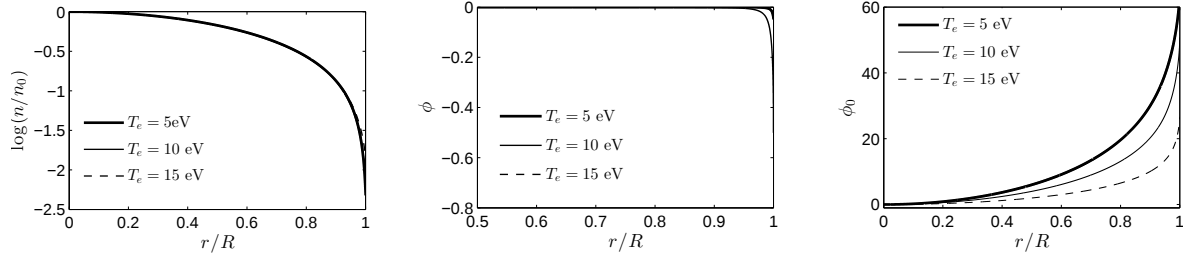


Figure 3. Radial equilibrium at the MN throat according to Sec. II B, at different electron temperatures, $T_e = 5, 10, 15$, thick, thin, and dashed line respectively. All results are evaluated at constant magnetization $\chi_{lh} = 2$ ($\chi_e \simeq 1000$). Left: density profile; centre: electric potential profile; right: thermalized potential profile for setting the MN electron model conditions.

A. The highly magnetized regime

The magnetic field B_0 has been tuned according to Figure 2 to keep $\chi_{lh} = 2$ constant for the three different electron temperature scenarios, $T_e = 5, 10, 15$ eV. The variation of B_0 is proportional to $R_{ei}(T_e) \propto T_e^{-3/2}$. Figure 3 depicts the radial profiles of the plasma density n , the electric potential ϕ and the thermalized potential ϕ_0 at the throat. The flat potential of ϕ , except at the vicinity of the Debye sheath, and the fact

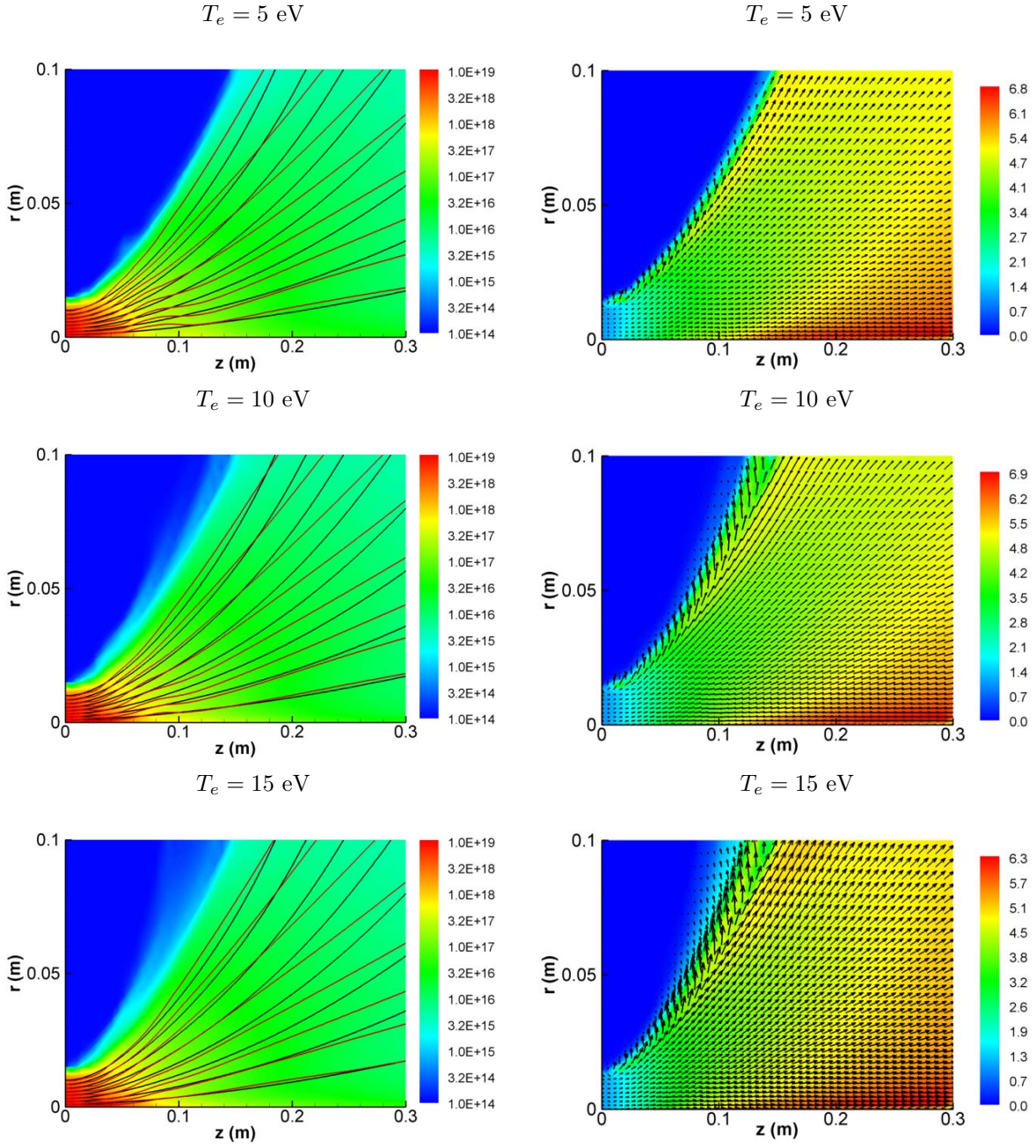


Figure 4. Left side: plasma density contourmaps (m^{-3}), ion streamlines in red, and magnetic streamlines in black. Each graph is obtained at different electron temperature: 5 eV, 10 eV and 15 eV from the top to the bottom respectively. Right side: contourmaps of the ion Mach number $\|u_i\|/c_s$. Black arrows indicate the ion flow direction. Arrow lengths scale with the velocity. All results are at constant magnetization $\chi_{lh} = 2$.

that the density drops at least two orders of magnitude when approaching the lateral wall, indicate that although $\chi_{lh} \sim 1$, plasma within the helicon source can be still considered *well* confined.

Plasma density contourmaps are drawn in Figure 4, from the throat section ($z = 0$) and until 30 cm downstream. Concerning the effect of the temperature, qualitatively, results can be considered exactly the same. The denser plasma is located close to the MN throat and along the axis. Magnetic streamlines, which define the nozzle geometry, are depicted in black, overlapping the density contourmap (note that nozzle divergence is rather small). For each magnetic streamtube, an ion streamtube with the same origin at the MN throat is depicted in red. The figure shows that ions effectively detach from magnetic streamlines, reproducing this way the results in Refs. 19,20.

Contourmaps of the ion Mach number, i.e. the module of the ion velocity over the ion sound speed $\|\mathbf{u}_i\|/c_s$, are depicted for the mentioned scenarios in Fig. 4. All them present a very similar response and differences can be attributed to the numerical noise intrinsic to the PIC model and its weighting algorithms. Black arrows indicate the velocity vector direction. These maps stress the fact that ions are also radially accelerated, and this causes thrust degradation or beam divergence inefficiencies. Larger axial ion acceleration is located close to the axis (red coloured area).

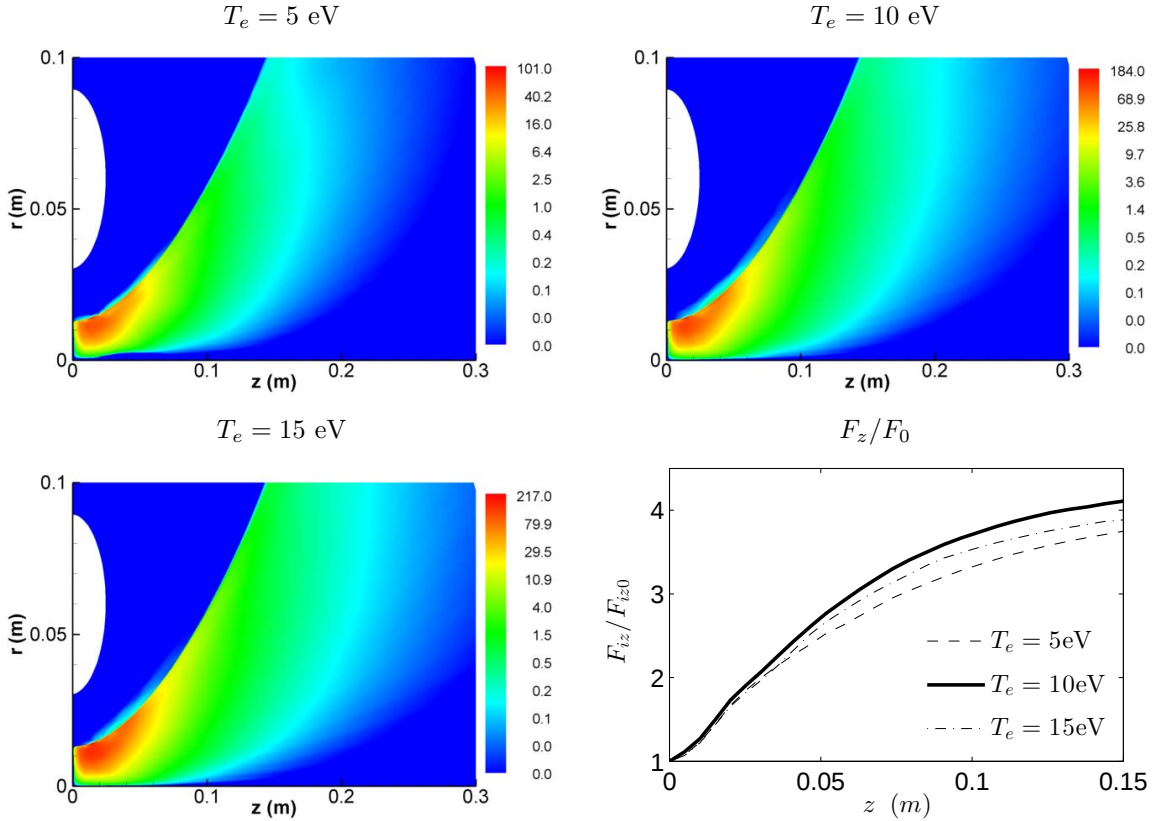


Figure 5. Countormaps of the magnetic force density (N/m^3), for the same cases of Fig. 4. The last plot depicts the thrust gain along the diverging MN, normalized with the ion momentum at the MN throat for the same cases as well.

The axial component of the magnetic force density, $F_m = -j_\theta B_r$, is drawn in Figure 5. At higher temperatures the magnetic force density is more intense, basically because F_m scales with the electron temperature, $F_m \propto j_\theta \equiv j_{e\theta} \propto u_{e\theta} \propto \phi_0 \propto T_e$, according to Eqs. 7, 10 and 11. However, this force should be normalized with the injected plasma momentum at the throat, which indeed it is also proportional to T_e , resulting the dimensionless thrust almost invariant with the temperature.

Finally, the thrust/ion momentum gain F_{iz} (normalized with the ion momentum at the throat F_{iz0}) should be equivalent to the volume integral of the magnetic force density. This is depicted in Figure 5. Thrust gain throughout the nozzle increases monotonically. However, in the current simulations there is an evident limitation on the computational domain. The plasma beam intercepts the upper boundary,

approximately at $z = 0.15$ m causing a loss of particles and as a consequence an artificial drop of the total ion momentum gain. Results beyond that location do not reproduce the expected thrust trend and are not shown.

Apart from this issue, there is another matter in question regarding the non negligible difference between these F_{iz} curves obtained at each temperature, while these were expected to follow approximately the same trend. The problem here is due to how the current model behaves at the plasma-vacuum edge or its surrounding area. Note that at this highly magnetized regime, plasma density drops abruptly at the “edge” and consequently the electric potential and its subsequently electric field are quite steep as well. Macro-ions at this region feels this electric force and the current leap-frog scheme with a constant time step is not able to provide a feasible integration of neither its trajectory, nor its velocity. The error committed cannot be accepted and these particles are also suppressed as they were being lost at the upper boundary. The ion mass flow rate lost at the edge due to the fail of particle trajectory integration is 10%, 2.5% and 1.2% for $T_e = 5, 10$ and 15 eV respectively. This also explains the “perturbation” of the local ion velocity vectors at the edge (see Fig. 4). If these particles were kept in the simulation, they would introduce larger errors on the computation of plasma properties within the dense plasma beam.

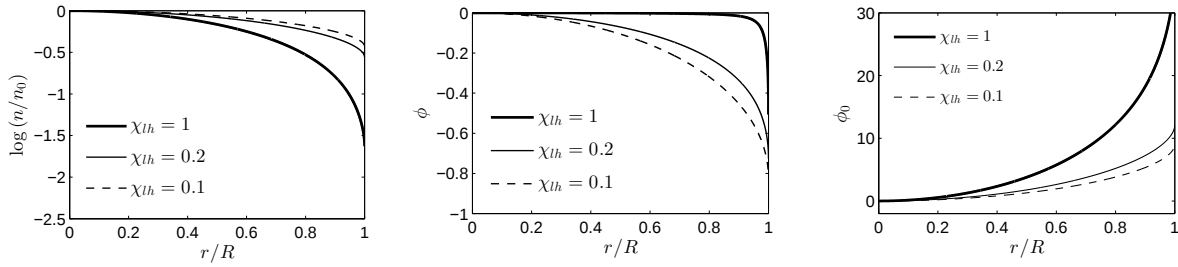


Figure 6. Radial equilibrium at the MN throat according to Sec. II-B, at different levels of magnetization, $\chi_{lh} = 1, 0.2, 0.1$, thick, thin, and dashed lines respectively. All results evaluated are at constant electron temperature $T_e = 10$ eV. Left: density profile; centre: electric potential profile; right: thermalized potential profile for setting the MN electron model conditions.

B. The transition to the demagnetized regime

In this subsection, the demagnetization transition at the MN is explored by reducing the Hall parameter at constant electron temperature $T_e = 10$ eV. This is achieved by weakening the magnetic field strength. The radial equilibrium within the tube is solved in order to set the boundary conditions at the MN throat. The same hypothesis as in the previous analysis are assumed: $\nu_e \sim \nu_{ei}$, $n_0 = 10^{19} \text{m}^{-3}$ constant at the axis line, which implies that the mass flow rate has to be readjusted. Three cases, $\chi_{lh} = 1, 0.2, 0.1$, are considered here in addition to the previous one $\chi_{lh} = 2$. The required magnetic field, considering the aforementioned conditions is $B_0 \simeq 305, 61, 30.6$ Gauss at $\chi_{lh} = 1, 0.2, 0.1$ respectively; and considering the use of Xenon, the mass flow rate is $\dot{m} = 1.42, 2.1, 2.29$ mg/s for each case.

Figure 6 depicts the radial profiles of the plasma density n , the electric potential ϕ and the thermalized potential ϕ_0 at the throat. Here, in comparison to the highly magnetized regime, the radial profile of the ambipolar potential ϕ drops gently, except at $\chi_{lh} = 1$ that is similar to the one obtained at $\chi_{lh} = 2$. This indicates that within the source, the role of the radial electric field becomes important, and compensates the lack of magnetic pressure. So, at $\chi_{lh} = 0.2, 0.1$ the plasma confinement within the source is already very poor. This issue is also stressed by their corresponding density profiles. At the sheath transition, plasma density values are much more larger than in other cases, pointing out that plasma losses to the wall will reduce source efficiency, but this is not the object of this work (see Ref. 17 for further details on this topic). The shape of these profiles allows us to conclude that plasma within the source is already demagnetized.

Plasma density contourmaps are drawn in Figure 7, from the throat and until 30 cm downstream. The denser plasma is still located close to the MN throat and at the axis line. However, the effect of reducing the hall parameter has a huge impact on the ions dynamics. Below $\chi_{lh} = 1$, their streamlines, coloured in red, are diverging more than magnetic field streamlines. This highlights that plasma demagnetizes and opens outward when χ_{lh} is reduced, causing a huge deterioration of MN performances. This cannot be predicted by fully fluid models.¹² Next, several plasma properties are presented to support these results.

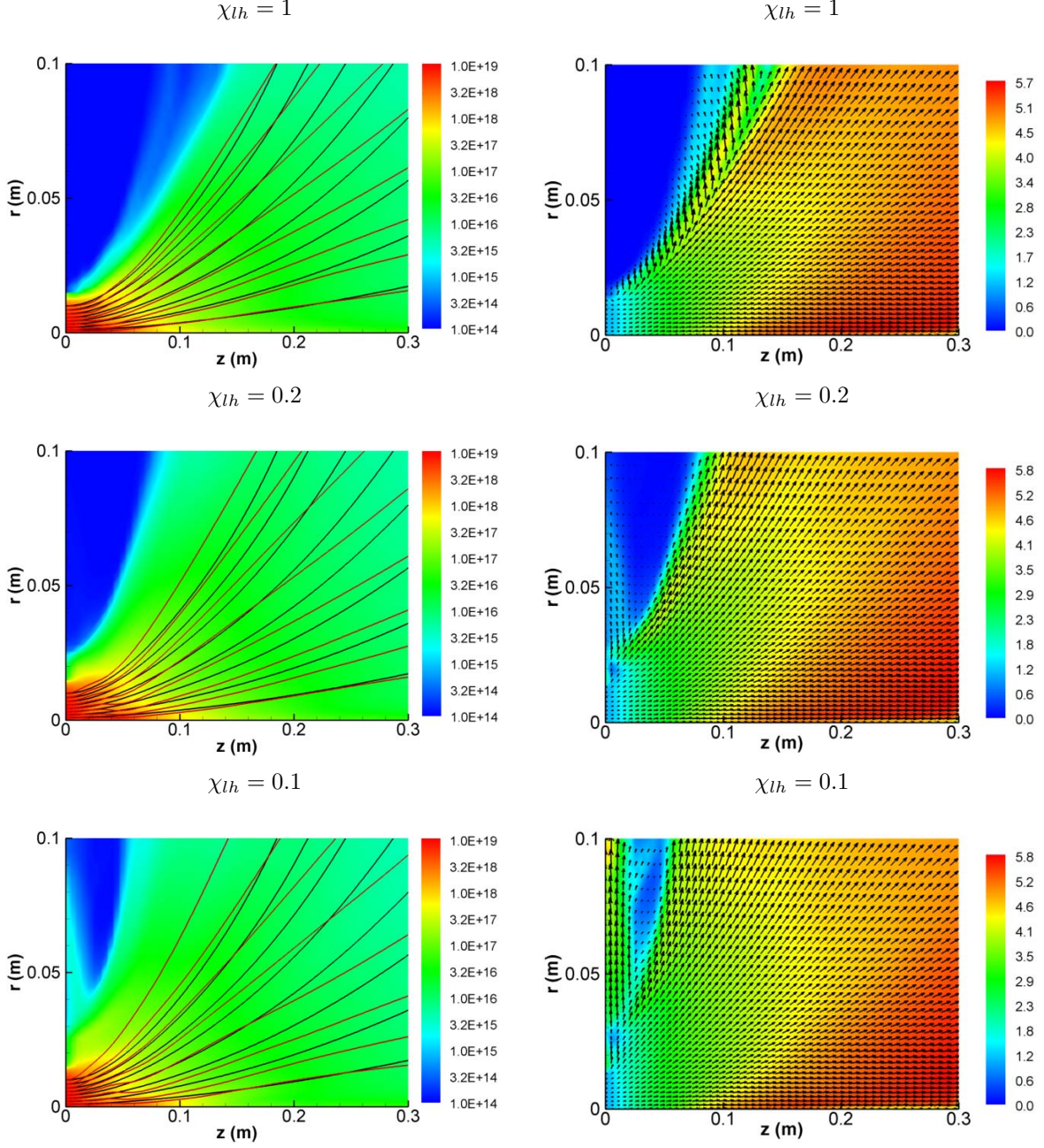


Figure 7. Left side: plasma density contourmaps (m^{-3}), ion streamlines in red, and magnetic streamlines in black. Each graph is obtained at a different value of the Hall parameter, $\chi_{lh} = 1, 0.2$ and 0.1 , from the top to the bottom respectively. Right side: contourmaps of the ion Mach number. Black arrows indicate the ion flow direction. Arrow lengths scale with the velocity.

The ion Mach number is depicted for the same scenarios in Fig. 7. This shows a larger area occupied by ions with higher radial velocities when the magnetic field strength has been reduced (χ_{lh} decreasing). Concluding that the undesired radial acceleration emerges due to the loss of confinement.

A direct consequence of demagnetization is the increasing of plasma-wall interaction. This can be shown in Figure 8, in which neutral density countormaps are painted. The rising of this quantity in the current scenario in which a fully ionized gas is injected, indicates that more ions are colliding to the lateral dielectric wall, so new neutrals born due to recombination and are introduced to the PIC domain diffusively. Another effect is that neutrals induce “electron-neutral” collisions, $\nu_{en} = n_n R_{en}$. This modifies the equivalent ν_e , decreasing the local Hall number, and thus increasing the electron cross-field diffusion. Then, the solution for Φ_0 outside of λ_{ed} will be modified, interfering strongly on the ambipolar potential ϕ (see case $\chi_{lh} = 0.1$ in Fig. 8). At this weakly magnetized regime, it seems that the increase of electron perpendicular mobility (or inversely, the drop of its resistivity), produces a smoother potential outwardly, although the perpendicular current is larger. At the opposite regime (the highly magnetized, see $\chi_{lh} = 1$ in Fig. 8), the resistivity is much more higher and it seems to produce a sharp rising of the potential at the vacuum side of the plasma edge. This behaviour might be reasonably justified by the Ohm’s law.

The axial component of magnetic force density is drawn in Figure 9. These contourmaps show that the magnetic force strength seems larger in the cases with higher electron magnetization. However, there are several mixed effects. First, the force is proportional to B_0 , so larger magnetic field yields larger forces. Second, the force is also proportional to the azimuthal current, which at the same time has been defined as $j_\theta \propto nu_{e\theta}$. For the cases simulated here, and due to the radial profiles imposed at the throat according the magnetization regime: the lower the χ_{lh} is, the weaker the azimuthal currents become. The strength of the interaction of this currents with the applied field,¹⁸ or the resulting magnetic thrust drops when χ_{lh} decreases.

Once again the thrust/ion momentum gain along the MN, normalized by the ion momentum at the throat accompanies the previous F_m figures. Thrust gain must be a monotonically increasing function in isothermal models. The fact of these are peaked is because the particle loss through the upper “vacuum” boundary. The slope of F_{iz}/F_{iz0} is larger on the high magnetized regime concluding that it produces thrust more efficiently.

Regarding the particle integration issue at the surroundings of the plasma edge, which has been discussed at the high magnetized regime, the performances of the current model are better at the medium-low magnetized regimes. This is mainly because the electric potential presents a flatter transition to the vacuum area when plasma opens gently due to its demagnetization (see again Fig. 8).

IV. Discussion

According to the results presented in previous section, the hybrid model reproduce the results of other models as we compared in Ref. 21 at the high magnetized regime. Furthermore, this adds some reality to the physics, for instance, it includes electron collisions, which are relatively important in the near plume close to the MN throat.

Regarding the electron thermodynamics, the present model is still suffering of the same uncertainties of other fluid models. It takes them isothermal everywhere, an assumption that is not fully consistent. Considering thermalized electrons along magnetic streamlines yields some unrealistic limits of the potential drop. For this reason and referring to the conclusions in Ref. 13, the region where isothermality is guaranteed is reduced to a small area close to the throat. Beyond that point, Boltzmann condition is broken, as well as the formulation of the thermalized potential ϕ_0 . For example, the mentioned reference concluded that electron temperature drops a 20% when the magnetic field is reduced 5 times. This would means that in the current topology, isothermality could not be assumed beyond $z = 0.065$ m.

Concerning the closure of the conservation charge equation, Figure 10 depicts the shape of $I_\perp(\lambda)$. It evolves different depending on the magnetization regime. Considering first the results at $\chi_{lh} = 1$ and $T_e = 10$ eV, $I_\perp(\lambda)$ receives contributions from ions that detach inward and the electron perpendicular diffusion, both contributing positive to the current. The match at λ_{ed} is smooth enough to accept the validity of the proposed law in Eq. 17. On the other hand, profiles at $\chi_{lh} = 0.2$ present some peculiarities: First, the perpendicular current is higher than in the previous case, and this behaviour is driven by the stronger cross field diffusion of electrons (lower χ_{lh}). Second, after the peak of $I_\perp$ and $I_{e\perp}$, the decreasing slop is interrupted at B (See Fig. 10), and increases abruptly. This response is caused by the $\phi_0(\lambda)$ profile

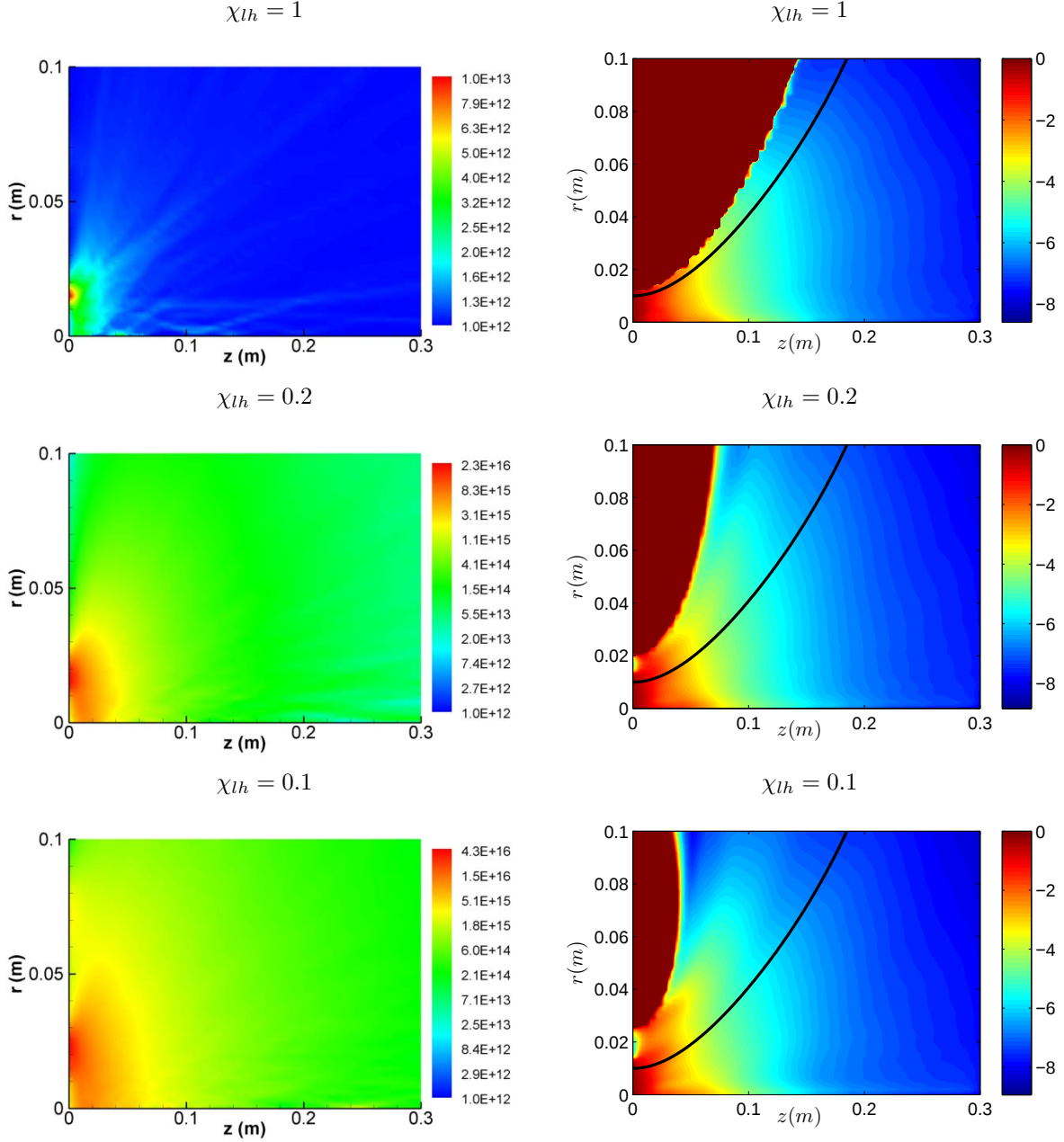


Figure 8. Left side: neutrals density contourmaps (m^{-3}), showing that ions impact the lateral wall and produce new neutrals due to wall-recombination. Each graph is for a different value of the Hall parameter, $\chi_{lh} = 1, 0.2$ and 0.1 , from the top to the bottom respectively. Right side: normalized electric potential $e\phi/kT_e$. The origin of the potential $\phi = 0$, is set at $(z, r) = (0, 0)$, and also is the upper cut-off, so in the outer region the potential increases largely and it is not shown (dark red area). All results are at constant temperature $T_e = 10$ eV. The black line is the λ_{ed} streamline.

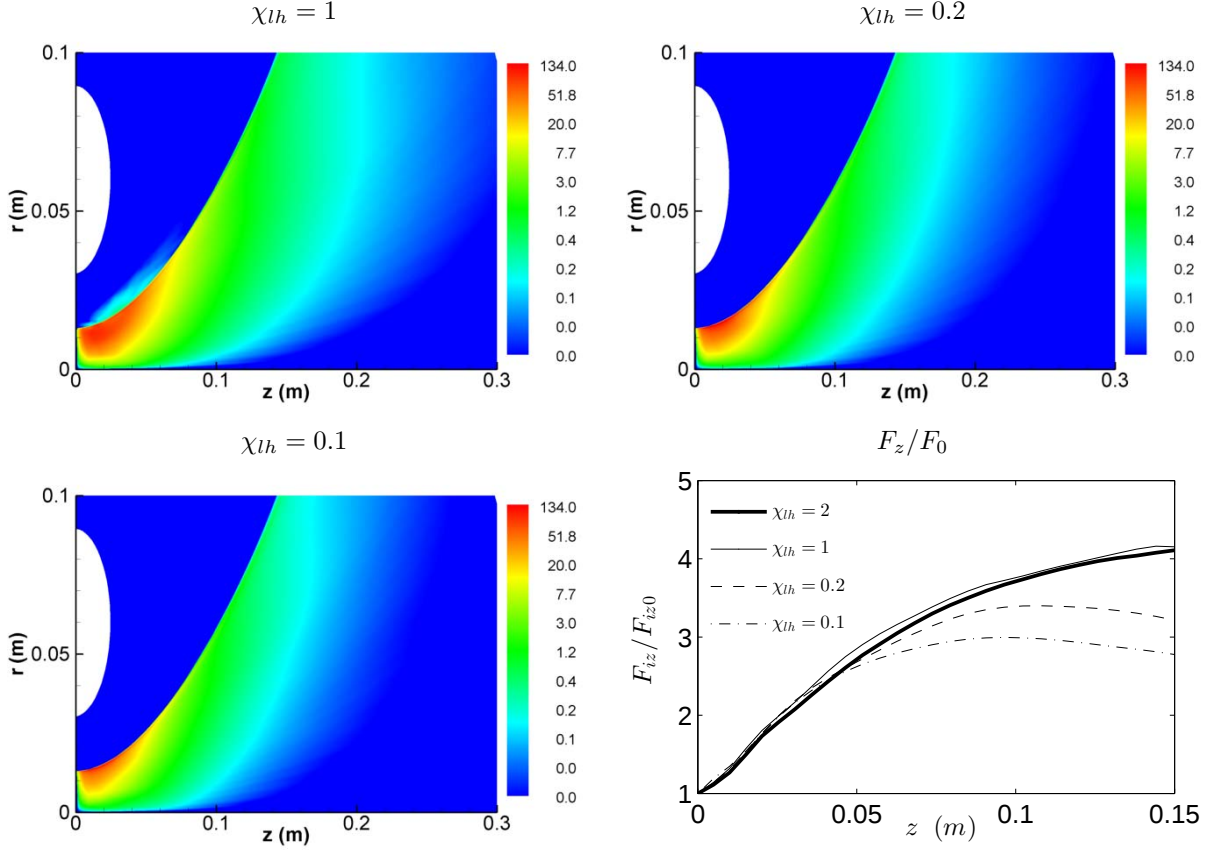


Figure 9. Contourmaps of the magnetic force density (N/m^3), for the same magnetization regimes of Fig. 7. It is only depicted on the region occupied by magnetic streamlines that pass through the MN throat. The last plot depicts the ion momentum gain along the diverging MN, normalized with the ion momentum at the MN throat for the same cases as well.

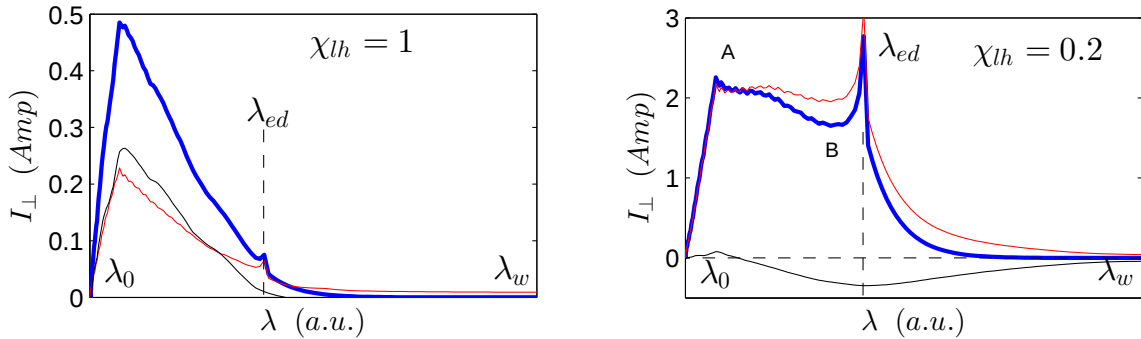


Figure 10. Perpendicular current $I_{\perp}(\lambda)$ in Amperes (blue line), the solid black is the perpendicular ion current $I_{i\perp}$ (Amp), and the red line is the electron perpendicular current $I_{e\perp}$ (Amp). Left: case $\chi_{lh} = 1$, $T_e = 10$ eV. Right: case $\chi_{lh} = 0.2$, $T_e = 10$ eV. Label A corresponds to the magnetic streamline that intercepts the upper-right corner of the PIC domain. The strong gradient of $\phi_0(\lambda)$ close to the plasma edge (see Figure 3) induces the increasing trend of $I_{\perp}$ and $I_{e\perp}$ close to λ_{ed} , it is labelled by B.

imposed in the range $\lambda \in (\lambda_0, \lambda_{ed})$ according to the MN throat conditions, and also by the fact that the local Hall parameter decreases downstream along the magnetic streamlines (electron demagnetization), thus increasing the electron cross field diffusion ($I_{e\perp}$ raises). Third, the ion current becomes negative meaning that ions detach outwardly. The first two issues are related to the electron perpendicular velocity. The law of the closure of $I_{\perp}$ influences the shape of the potential at the vicinity of λ_{ed} . How δ modifies the results predicted by the current model is an issue not explored yet but it may cause an important impact on the results. The aforementioned failure on the particle trajectory integration, which occurs mainly on the plasma-vacuum edge, is also influenced by the resolution of the electric potential at the plasma-vacuum edge, and consequently by the $I_{\perp}(\lambda)$ law. Related to this, it is important to note that the assumption of quasi-neutrality is another limitation of the hybrid model. Note that at the high magnetized regime, the drop of the density at the edge is of several orders, thus the premise of quasi-neutrality (zero-Debye length limit) it is not necessarily preserved there. However, observing again the ion Mach figures (4 and 7), this is approximately sonic and opens outwardly. So, the edge could be understood as an electrostatic sheath that separates the high density area from the near vacuum area.

V. Conclusions

A hybrid model, fluid for electrons and PIC for ions has been developed to reproduce the main phenomena of a magnetized/demagnetized plasma flow through a diverging magnetic nozzle.

In the high magnetized regime, or nearly collisionless regime, the model matches the results of other models, for instance DIMAGNO.¹⁹ This is because the similarity of hypotheses concerning electron thermodynamics. Consequently, the same uncertainties of these models also applies to the results presented here.

This model illustrates the transition between the highly magnetized and demagnetized regimes. Ions detach inward in the first regime, while in the second regime, their streamlines become much more divergent than magnetic streamlines. Special attention has been invested on the study of boundary conditions at the throat, which are determined by the radial equilibrium of the plasma within the cylindrical source.

A closure of the charge conservation equation has been proposed in the area of low density plasma. However, further investigation is required on the determination of a better closure of these boundary conditions or how this modifies the dense plasma jet area.

Acknowledgments

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