Small scale fluctuations and anomalous electron current in Hall plasmas

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A system of nonlinear fluid equations has been developed to describe Hall plasma turbulence relevant to electric propulsion. The model takes into account electron collisions with neutrals, density and magnetic field gradients, and parallel (to the magnetic field) electron motion. The dominant nonlinearity comes from the electron convective flow due to $E \times B$ drift, but the higher order nonlinearities in the electron polarization current are also included taking into account the gyro-viscous cancellation. The linear limit of the model contains the gradient drift and low hybrid type instabilities. The linear initial value simulations have been benchmarked against the liner eigenmode solutions. The nonlinear simulations demonstrate turbulence saturation and anomalous electron current independent of the initial conditions. The anomalous electron current exceeds classical collisional current by a factor of six for the case of the resistive driven fluctuations.

I. Introduction

Plasma systems with closed electron drift in crossed electric and magnetic fields are of interest for a number of applications such as space propulsion and material processing plasma sources. In these devices, the strength of the external magnetic field is such that the electrons are magnetized, $\rho_e \ll L$, but ions are not, $\rho_i \gg L$, where L is the characteristic length scale of the plasma region in the device. These conditions, generically referred to as Hall plasmas, are typical for many technological applications. The ions, due to large Larmor radius, are unmagnetized and accelerated in the externally applied electric field $\mathbf{E}_0$, while the electrons are confined by a strong magnetic field with a nominally small current along the electric field $\mathbf{E}_0$ due to collisions. This principle is used in Hall plasma thrusters, magnetron devices for material processing, magnetic filters and Penning type plasma sources.

Despite many successful applications and the long history of Hall thrusters and other Hall plasma sources, some aspects of their operation are still poorly understood. One notable problem is the anomalous electron mobility,[1–5] which is far above the classical collisional values. Hall plasmas demonstrate numerous oscillations

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in a wide range of frequencies[6–9] and it is generally believed that these fluctuations are likely responsible for the anomalous mobility and generation of low frequency coherent structures such as spokes.[10]

An inhomogeneous plasma immersed in the external electric and magnetic fields is not in a state of thermodynamic equilibrium. The equilibrium $\mathbf{E}_0 \times \mathbf{B}_0$ electron drift, plasma and magnetic field gradients, and ion flow are all sources of plasma instabilities in Hall plasmas.[11] The low-hybrid instability and the modified two-stream instability of Hall plasma with transverse current[12–14] are thought to be particularly important. The effects of plasma and magnetic field gradients on low hybrid instability were studied in kinetic theory in Refs. 15–18. The modified two-stream instability was studied in details (also in kinetic theory) in Refs. 19, 20. The lower hybrid instability is typically a short wavelength mode with $k_\perp \rho_e \simeq 1$, where ρ_e is the electron Larmor radius, while the modified two stream version has the most unstable modes for longer wavelengths $k_\perp \rho_e < 1$,[19] but requires a finite component of the wavevector along the magnetic field. The short wavelength low hybrid modes are also a special case of more general beam cyclotron instabilities,[20–22] in which higher cyclotron harmonics are included. The nonlinear stage of such cyclotron instabilities, driven by the transverse current was analyzed in Refs 20,21,23, where it was concluded that these small scale modes saturate at relatively low amplitude due to ion trapping.

The $\mathbf{E}_0 \times \mathbf{B}_0$ instability driven by the combination of the magnetic field and density gradients was experimentally and theoretically identified as a possible source of fluctuations and anomalous mobility in Hall plasma thrusters.[24,25] This is the long wavelength instability obtained by neglecting the effects of electron inertia, $m_e \to 0$. Therefore it is not directly related to the low hybrid modes, nor it requires a finite value of the wavevector along the equilibrium magnetic field, $k_\parallel$. Theoretical studies have revealed the existence of the collisions and ionization,[26,27] Rayleigh shear flow, and resistive instabilities of low-hybrid and Alfven waves.[28,29] Kinetic studies[22] have also identified the high-frequency instability driven by the resonances between the electron cyclotron harmonics and the $\mathbf{E}_0 \times \mathbf{B}_0$ drift.

We had recently revisited the problem of the long wavelength $\mathbf{E}_0 \times \mathbf{B}_0$ instability in plasmas with inhomogeneous magnetic field and plasma density gradients which was originally studied in Ref. 25 and later in Ref. 30. We had shown that quantitative corrections (to the previous theory) are required for the accurate determination of the conditions for the instability and its characteristics (real part of the frequency and growth rate).[31,32] We had shown that in inhomogeneous magnetic field the modes have finite perturbations of the electron temperature and developed a three-field fluid model that includes electron temperature perturbations. Applications of general stability criteria to stability of some realistic profiles in Hall thrusters have been reported.[32]

The full picture of instabilities in Hall plasmas is complex and, for typical experimental conditions, likely involves a number of interacting modes which require numerical simulations. Particle-in-cell kinetic simulations offer a first principles description that has provided valuable results on plasma dynamics.[33–36] Fully kinetic simulations could be the most realistic approach to study the experimental conditions, however they could also be difficult to interpret. This becomes even more difficult when practical limitations of the modern computer capabilities and available resources are taken into account. Hybrid simulations where electrons are treated as a fluid and ions are treated as particles have also been used to study the plasma behavior in a Hall thruster but they suffer from similar issues as the particle approach.[5,37] On the other hand, fluid simulations are faster and cheaper numerical tools for simulations of nonlinear plasma dynamics, easier to interpret and provide much greater flexibility in separating various physics elements. Fluid models have been important contributors and have been indispensable for the development of tokamak physics.[38] The fluid models are especially valuable in complex magnetic geometries where the direct kinetic simulations may become prohibitively expensive.[39] We have developed a set of nonlinear fluid codes to investigate the turbulent fluctuations and transport in Hall plasmas relevant to electric propulsion.[40] These fluid simulations capabilities are being developed in conjunction with kinetic simulations being performed at the Princeton Plasma Physics Laboratory (PPPL).[41] Synergy of kinetic and fluid simulations with similar experimental conditions and addressing the same set of physics problems allows to achieve a better understanding and, via mutual complementarity, in part overcome natural limitations of each method.

Our methodology is based on advanced multi-fluid models including the collisional and ionization processes. Our fluid simulations are based on the BOUT++ code framework.[42] BOUT++ is a platform for fluid and plasma simulations in curvilinear magnetic field geometry using finite-difference discretization and a variety of numerical methods and time-integration solvers. It was designed and tested with reduced plasma fluid models applications in mind by LLNL, University of York (UK) and others and has been widely used for studies of edge tokamak phenomena, 3D plasma structures and plasma turbulence and structures in Large

Plasma Device (LAPD).[43–47] BOUT++ has been recently adapted for simulations of Hall plasmas relevant to electric propulsion.[40]

We have set up simulations of density gradient driven modes in a rectangular geometry model (uniform magnetic field). This fluid model includes electron inertia and electron neutral collisions; therefore the lower hybrid modes destabilized by $\mathbf{E}\times\mathbf{B}$ drift and collisions are also included in the model. Our nonlinear model includes the electron gyroviscosity and the so called gyroviscous cancellation effects.[48] The gyroviscosity effects are of the same order as the electron inertia and have to be retained in a finite electron temperature plasma to properly account for the correct behavior at high values of the perpendicular wave-vector as well as for the proper energy balance among nonlinear terms. Note that both growth rate and real part of the density gradient driven modes increase linearly with the wave-vector $\mathbf{k}$. The inertia and gyroviscosity effects introduce physics based cutoff at high $\mathbf{k}$, which are important for simulations in finite size regions.

II. Basic equations

Here we formulate the set of basic fluid equations appropriate for plasma dynamics in a Hall thruster. In general, we consider the cylindrical region inside a Hall thruster channel. In this region, there is a background magnetic field that is mainly directed in the radial direction, $\mathbf{B}_0 = B_0\hat{\mathbf{r}}$. The magnitude of the magnetic field is chosen in such a way that it does not affect the motion of the ions, such that the ion Larmor radius is much larger than the characteristic length of the channel. The background electric field is fixed in the axial direction, $\mathbf{E}_0 = E_0\hat{\mathbf{x}}$ resulting in an electron drift in the azimuthal direction. The background plasma density and magnetic field gradients are present in the axial direction.

Our model includes electron collisions with neutrals that couple gradient drift and resistive instabilities due to electron-neutral collisions.[28] Plasma is assumed to be quasineutral. Ion are collisionless and unmagnetized, and for simplicity we assume they are cold, $T_i = 0$. With these assumptions, the ion continuity and momentum equations are

$$\begin{aligned} \frac{\partial n_i}{\partial t} + \nabla\cdot(n_i\mathbf{v}_i) &= 0, && (1)\\ \frac{\partial \mathbf{v}_i}{\partial t} + (\mathbf{v}_i\cdot\nabla)\mathbf{v}_i &= -\frac{e}{m_i}\nabla\phi. && (2) \end{aligned}$$

The electron continuity equation can be written as

$$\frac{\partial n_e}{\partial t} + n_e\nabla_\perp\cdot\mathbf{v}_{e\perp} + \mathbf{v}_{e\perp}\cdot\nabla_\perp n_e - \frac{1}{e}\nabla_{||}\cdot\mathbf{J}_{e||} = 0, \tag{3}$$

where the electron velocity is separated into perpendicular and parallel (with respect to the magnetic field) components.

The electron equation of motion in the direction perpendicular to the magnetic field can be written as

$$m_e n_e\left(\frac{\partial}{\partial t} + \mathbf{v}_{e\perp}\cdot\nabla\right)\mathbf{v}_{e\perp} = e n_e(-\nabla_\perp\phi + \mathbf{v}_{e\perp}\times\mathbf{B}) - T_e\nabla_\perp n_e - \nabla\cdot\pi_e - m_e n_e\nu\mathbf{v}_{e\perp}, \tag{4}$$

from which the electron perpendicular velocity can be written as

$$\begin{aligned} \mathbf{v}_{e\perp} &= \mathbf{v}^{(0)} + \mathbf{v}^{(1)}, && (5)\\ \mathbf{v}^{(0)} &= \frac{\mathbf{b}}{B_0}\times\nabla_\perp\phi - \frac{T_e}{e n_e}\frac{\mathbf{b}}{B_0}\times\nabla_\perp n_e && (6)\\ \mathbf{v}^{(1)} &= \frac{e}{m_e}\frac{1}{\Omega_e^2}\left(\frac{\partial}{\partial t} + \mathbf{v}_{\mathbf{E}}\cdot\nabla + \nu\right)\nabla_\perp\left(\phi - \frac{T_e}{e}\frac{n_e}{n_0}\right). && (7) \end{aligned}$$

In calculating the first order electron velocity $\mathbf{v}^{(1)}$, gyroviscous cancellation has been taken into account. The plasma is fully described by the system of equations (1) - (4) and quasineutrality.

III. Linear benchmarking and nonlinear simulations

The basic equations above allow some reduction. Thus, the ion dynamics is conveniently described by the potential function $\mathbf{v}_i = -\nabla\chi$. Then, the ion continuity equation can be written in the following form

$$\frac{\partial n}{\partial t} = -\mathbf{v}_0\cdot\nabla_\perp n + n_0\theta, \tag{8}$$

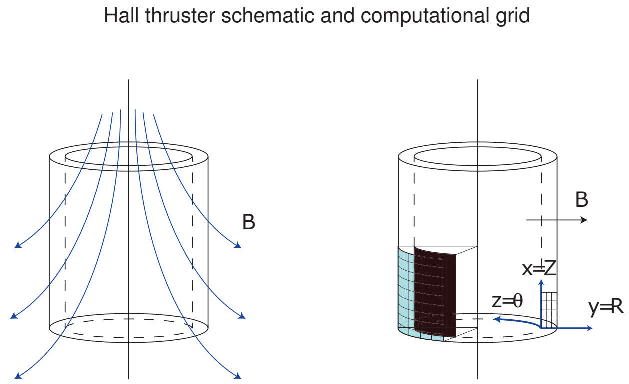


Figure 1. Schematics of the geometry of a Hall thruster and the computational grid for Eqs. (12) - (14). The magnetic field is mainly in the radial direction, the electric field and the density gradients are in the axial direction, while the electron $E_0 \times B_0$ drift is in the azimuthal direction. In the grid used by BOUT++, the axial, radial and azimuthal directions correspond to the $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ coordinates respectively.

$$\theta = \nabla_\perp^2 \chi. \tag{9}$$

And, the ion momentum equation reduces to

$$\frac{\partial \theta}{\partial t} = -\mathbf{v}_0 \cdot \nabla_\perp \theta + \frac{e}{m_i} \nabla_\perp^2 \phi. \tag{10}$$

Using the quasineutrality condition in the electron continuity equation, along with the expression for the electron velocity, and after some algebraic manipulation, the following equation is obtained

$$\begin{aligned} \frac{\partial \varpi}{\partial t} + u_0 \frac{\partial \varpi}{\partial z} + \nu_e \varpi - \nu_e \left(\frac{n}{n_0}\right) + v_n \frac{\partial}{\partial z}\left(\frac{e\phi}{T_e}\right) + \frac{T_e}{eB_0}\left\{\varpi, \frac{e\phi}{T_e}\right\} & \\ + \frac{T_e}{eB_0}\rho_e^2 \left\{\nabla_\perp\left(\frac{e\phi}{T_e}\right), \nabla_\perp\left(\frac{n}{n_0}\right)\right\} &= 0, \end{aligned} \tag{11}$$

$$\begin{aligned} \varpi &= \rho_e^2 \nabla_\perp^2 \left(\frac{e\phi}{T_e} - \frac{n}{n_0}\right) + \frac{n}{n_0}, \\ v_n &= \frac{T_e}{eB_0}\left(\frac{1}{n_0}\frac{\partial n_0}{\partial x}\right), \end{aligned}$$

where ρ_e is the electron Larmor radius, ϖ is the modified vorticity and v_n is the density gradient drift velocity

These equations describe the gradient drift, the lower hybrid and the resistive mode rendered unstable by electron collisions.[29] BOUT++ is an initial value code, so it picks up the most unstable mode from the initial background noise. For numerical simulations, the equations are written in dimensionless form using the following variables: the lower hybrid frequency scale, $\omega_{lh} = \sqrt{\omega_{ci}\omega_{ce}}$, $t' \rightarrow \omega_{lh} t$, $\nabla \rightarrow (1/\rho_l)\nabla'$, $v \rightarrow c_l v'$, $\phi \rightarrow (T_e/e)\phi'$, $n \rightarrow n_0 n'$, $c_l^2 = T_e/\sqrt{m_i m_e}$, $\rho_l = c_l/\omega_{lh}$:

$$\frac{\partial n}{\partial t} = -v_0 \frac{\partial n}{\partial x} + \theta, \tag{12}$$

$$\frac{\partial \theta}{\partial t} = -v_0 \frac{\partial \theta}{\partial x} + \mu_e \nabla_\perp^2 \phi, \tag{13}$$

$$\frac{\partial \varpi}{\partial t} = -u_0 \frac{\partial \varpi}{\partial z} - \nu_e \varpi + \nu_e n - v_n \frac{\partial \phi}{\partial z} + \{\phi, \varpi\} + \mu_e \{\nabla_\perp n, \nabla_\perp \phi\} - D_\varpi \nabla^4 \varpi, \tag{14}$$

$$\varpi = \mu_e \nabla_\perp^2 (\phi - n) + n,$$

where $\mu_e = \sqrt{m_e/m_i}$. Note that the equations (12) - (14) are written in coordinates usually assumed in BOUT++, namely the axial direction is along the $\hat{\mathbf{x}}$ axis, the radial direction is along the $\hat{\mathbf{y}}$ axis and the $E_0 \times B_0$ azimuthal direction is along the $\hat{\mathbf{z}}$ axis. A schematic of the computational grid used to solve the system in Eqs. (12) - (14) is shown in Fig. 1

To benchmark the code we compare the linear initial value BOUT++ results with the solution of the 1D (azimuthal) linear eigenvalue problem. This comparison is shown in Fig. 2. The remaining discrepancy is due to the finite value of the (axial) k_x wave vector which is present in BOUT++ runs.

In the nonlinear simulations presented here, we assume the case with absence of the gradient drift modes by taking uniform plasma density and magnetic field. This leaves only the resistive low hybrid mode instability as a source of plasma turbulence.[29] The nonlinear equations were solved using a rectangular mesh with 128×256 points in the axial and azimuthal directions for three different initial profiles of the perturbed plasma parameters, namely the product of a sinusoidal function and a gaussian, a purely gaussian perturbation and a purely sinusoidal perturbation. The saturation of the instability in the system was studied by checking the energy type integral

$$E_n = \int |n|^2 \, dx dz. \tag{15}$$

The energy calculated from Eq. (15) is shown in Fig 3 for the three different initial states.

From Fig. 3, it can be seen that the energy saturates after around $t = 400$. Though, the time to reach saturation could be different, the final turbulent state is generally independent of the initial conditions.

IV. Anomalous electron current

The perturbed nonlinear electron density can be expressed by the product of the perturbed electron density and the perturbed electron velocity. This current density is then

$$\mathbf{J}_e = -e \, \tilde{n} \, \tilde{\mathbf{u}}_e. \tag{16}$$

In this equation, the perturbed electron velocity, $\tilde{\mathbf{u}}_e$ is the perturbed electron $\mathbf{E} \times \mathbf{B}$ drift velocity produced by the perturbed electric field and the radial magnetic field

$$\tilde{\mathbf{u}}_e = \frac{\hat{\mathbf{b}}}{B_0} \times \nabla \tilde{\phi} = \frac{1}{B_0} \frac{\partial \tilde{\phi}}{\partial z} \hat{\mathbf{x}} - \frac{1}{B_0} \frac{\partial \tilde{\phi}}{\partial x} \hat{\mathbf{z}}. \tag{17}$$

Using Eqs. (17) and (16), the axial current density becomes

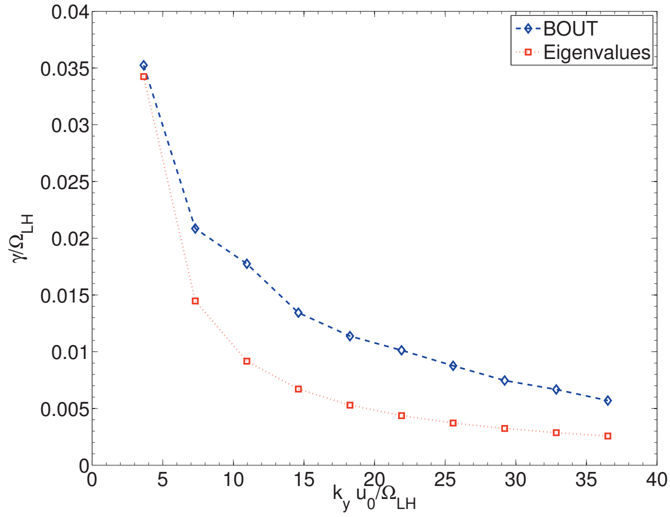
$$J_{ex} = -\frac{e \, \tilde{n}}{B_0} \frac{\partial \tilde{\phi}}{\partial z}. \tag{18}$$

The local value of the oscillating electron current given by the equation (18) is space averaged at every time step

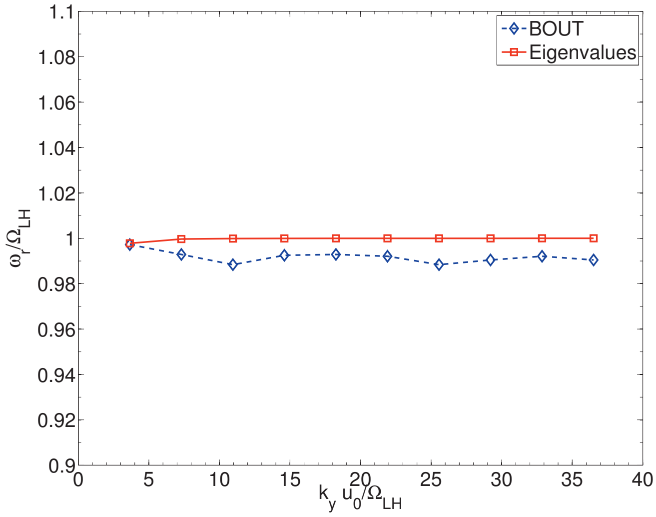
$$\bar{J}_{ex}(t) = \frac{1}{L_z \, L_x} \int \int \tilde{n}(t,x,z) \frac{\partial \tilde{\phi}(t,x,z)}{\partial \tilde{z}} \, dx \, dz. \tag{19}$$

The perturbed axial current density calculated from Eq. (19), along with the mean average value over 5 lower hybrid periods and the classical current density given by Eq. (20) are shown in Fig. (4). The classical collisional current density is given by

$$J_{ex}(classical) = \frac{e n_0}{B_0} \frac{\nu_e}{\Omega_{ce}} E_0. \tag{20}$$



a)



b)

Figure 2. a) Growth rate and b) real part of the frequency as obtained from BOUT++ simulations and the eigenvalue problem for Eqs. (8), (10) and (11). The electric field E_0 = 1.5 × 10^4 V/m, the collision frequency ν_e =10^6 rad/s, mean thruster radius R_{t0} = 5 cm.

V. Conclusions

The reduced nonlinear fluid model has been developed to simulate plasma turbulence and anomalous transport in Hall plasmas. The nonlinear model has been implemented in the high performance BOUT++ framework. We have performed the linear bench-marking of the BOUT++ initial value simulations against the results obtained by the eigen-value solvers. In our linear simulations we observe clear transition from gradient driven modes[31,32] to the low-hybrid modes destabilized by collisions[29] (and density gradients) and subsequent decay of the growth rate at high $\mathbf{k}$. We have also investigated the saturation of the turbulence and formation of the stationary turbulent spectra in nonlinear simulations. It is shown that the nonlinear simulations reach saturation at the level which is independent of the initial state. The related anomalous electron transport has been calculated. It can be seen from Figs. 4 that the perturbed current is strongly oscillating and is generally smaller than the classical collisional current during the first stage of the simulation corresponding to the linear phase. Once the nonlinearities start to dominate the system and it approaches saturation, the perturbed current grows and becomes larger than the classical collisional current density. Once the system reaches saturation, the current density also stabilizes. The value of the perturbed current density in saturation state is larger than the classical current density from collisions by a factor of around 6 for the values used in the simulations indicating that the nonlinear effects are responsible for the anomalous transport observed in Hall plasma experiments. The parameters used in the simulations correspond to the region where the electrons are strongly magnetized and the strongest deviation from the classical current are expected.[4,5]

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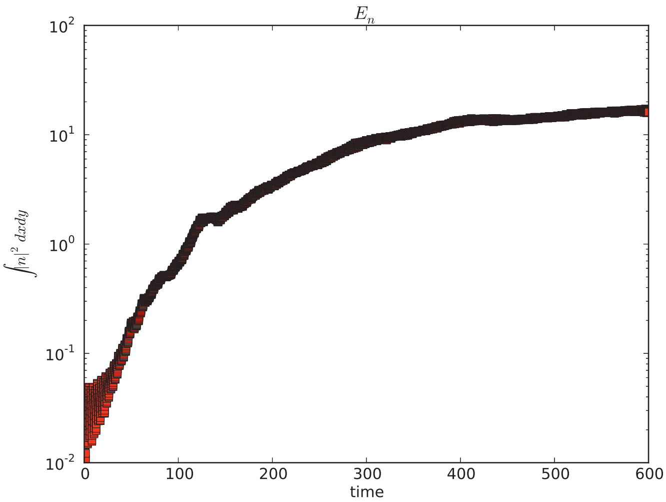
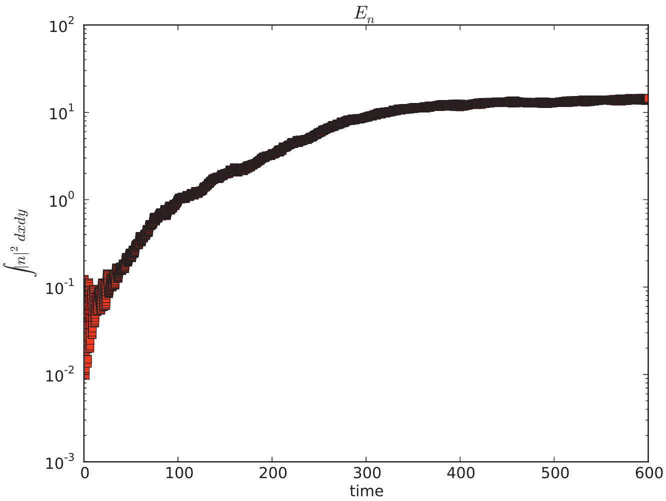


a)



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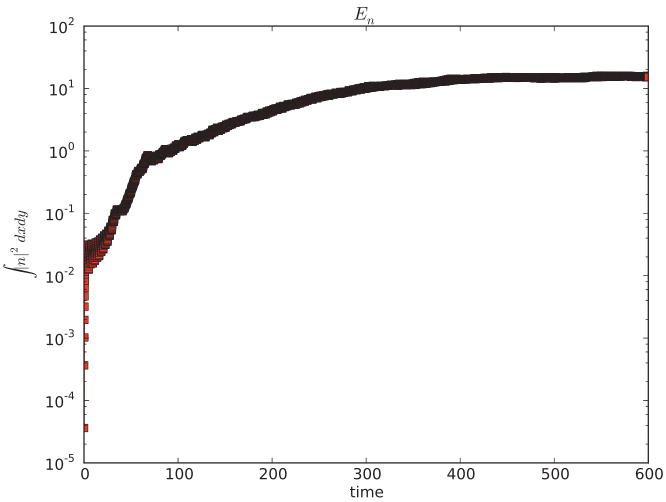
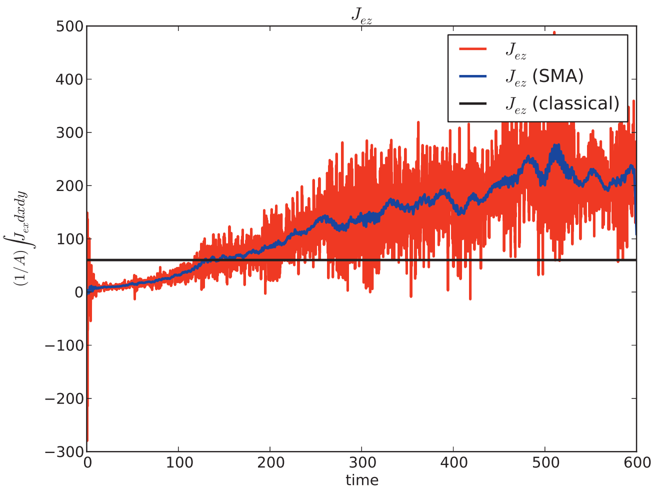
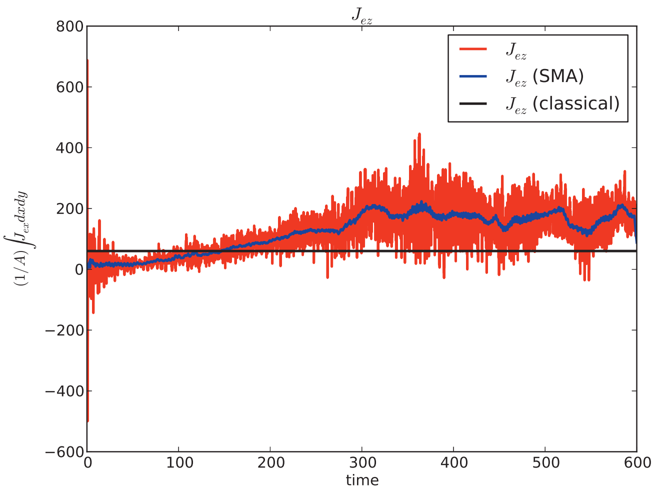


c)

Figure 3. Saturation of the energy integral as a function of time ($\omega_{ln} t$) for three different initial states: a) Product of sinusoidal and gaussian, b) Gaussian, c) Sinusoidal.



a)



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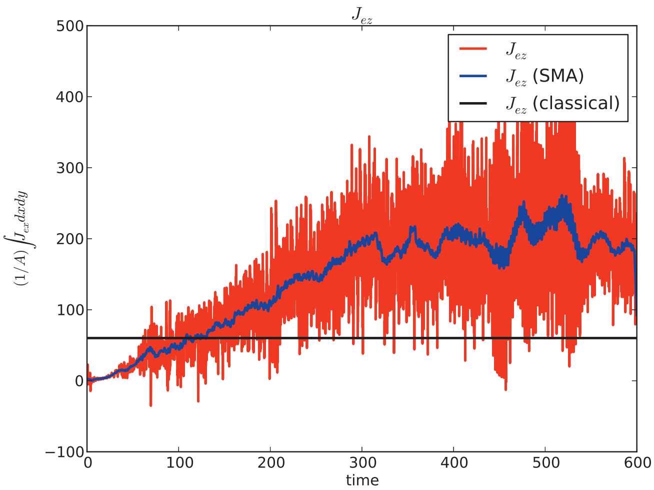

c)

Figure 4. Axial current density as a function of normalized time ($\omega_{ln}t$) for cold electrons model for three different initial profiles for plasma parameters. a) Product of sinusoidal and gaussian, b) Gaussian, c) Sinusoidal. The SMA (Simple Mean Average) is performed over 5 lower hybrid periods. The parameters used are $n_0 = 10^{18}$ m^{-3}, $E_0 = 10^4$ V/m, $B_0 = 10^{-2}$ T, $\nu_e = 6.6 \times 10^5$ Hz. The ions are Xe^+. The value of the classical current density is $J_{ex}(classical) = 60$ A/m^2.