The electron diffusion into the channel of stationary plasma thruster

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Abstract: **In the report presents the results of the analysis of the simulation of the dynamics of the electron in the azimuthal wave, which is observed in the channel stationary plasma thruster SPT. The frequency characteristics of this wave, modulation of ion current density and temperature fluctuations enables to conclude, that these oscillations are obviously connected with the ionization instability. It has been shown, that this wave leads to drift of electrons along the channel thruster wherein the displacement of electrons may reach 2 cm. The main kind of electron scattering, which defines to their diffusion along the cannel, is scattering of electrons on atoms of the working gas. These collisions occur throughout the length of the oscillation amplitude. The ratio of the azimuthal drift velocity to the Larimore frequency, which determines the amount of diffusion of electrons, is increased significantly to the anode. It has been shown that increasing the diffusion of electrons in the azimuthal wave occurs due to those electrons which collision occurs closer to the anode. Presented by the expression for the conductivity of the plasma, when in the channel SPD are observed azimuthal waves. The calculating of Hall parameter, the discharge current of electrons through a channel and the heating power of electrons in the channel had been made on the basis of the above model. It is shown that they are in good agreement with the experimental results for two models of SPT, working on different gases Kr and Xe.**

Nomenclature

τ_0	=	*time of scattering electron on neutral atom,*
τ_i	=	*ionization time,*
L	=	*wavelength of azimuthally wave,*
v_e	=	*azimuthally electron velocity,*
f	=	*wave frequency,*
z	=	*axial coordinate direction,*
v_θ	=	*phase velocity of azimuth wave,*
U	=	*potential of electrical field,*
E	=	*electric field strength,*
B	=	*radial component of magnetic field,*
J_i	=	*ion current,*
J_e	=	*electron current,*

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T_e	=	*electron temperature,*
V_d	=	*drift electron velocity,*
v_{dx}	=	*longitudinal drift velocity of electron*
v_e	=	*electron transport velocity, waves are absent,*
v_{ex}	=	*electron transport velocity in wave field,*
k_l	=	*azimuth wave vector,*
ω_H	=	*electron cyclotron frequency,*
v_x	=	*electron velocity along axes x,*
v_y	=	*electron velocity along axes y,*
β_0	=	*relative amplitude azimuthally wave,*
c	=	*light velocity,*
m_e	=	*electron mass,*
e	=	*electron charge,*
T_{tr}	=	*time of electron transport from exit to anode area in wave field,*
T_{tr}^0	=	*time of electron transport from exit to anode area, waves are absent,*
χ	=	*Hall parameter,*
σ_f	=	*conduction plasma in azimuth wave,*
σ_e	=	*conduction no magnetic field,*
n_e	=	*plasma density,*
N_0	=	*atom density,*
T_o	=	*period basic harmonic modulation of electron along channel*
x	=	*coordinate along channel,*
y	=	*coordinate in azimuth channel,*
x_a	=	*amplitude of oscillation of electrons along the axis,*
α	=	*parameter of relative amplitude azimuthally wave,*
X_c	=	*average displacement of electron,*
Δx	=	*amount of displacement in one collision,*
Q	=	*flow gas,*
T_a	=	*temperature atom,*
m_a	=	*atom mass,*
v_a	=	*atom temperature velocity,*
S	=	*area of channel section,*
p_e	=	*electron pressure,*
W	=	*power of electrons per unit length of channel,*
U_d	=	*the discharge voltage,*
J_d	=	*the discharge current*

I. Introduction

Investigations of stationary plasma thruster were performed long time, however the mechanisms of diffusion and heating electrons is not fully understood[2-5].

The study of oscillations in the channel of the thruster[6-8] showed that near the anode oscillations of the plasma potential synchronous. Starting with $x > (1 \div 2)$ cm, oscillations in the frequency range $f = (60 \div 1000)$ kHz behave as waves, the wave vector is directed both in azimuth and along the channel. The largest relative amplitude of oscillations is observed in the exit, where it reaches (30 ÷ 40) %. The spectral composition of the oscillations varies along the channel, increasing the high-frequency harmonics in the direction to the exit of the thruster. The largest amplitude of the azimuthal waves is observed at frequencies of 60 Hz to 120 kHz. Similar harmonics are also present in the oscillations of the electron temperature and ion current.

The circular frequency of these oscillations coincides with the frequency of inelastic collisions of electrons with neutrals, leading them to the ionization $2\pi f = 1/\tau_i$. Estimates show that for $f = 60$ kHz the wavelength approximately equal to the circumference channel, i.e. oscillation mode m = 1 for the wave vector $k_i \sim 0.35$ 1/cm.

The phase velocity of the azimuthal wave v_0 coincides with the direction of the drift velocity of electrons. In the range voltage (170 ÷ 350) V velocity v_θ increases linearly from $\sim 1.3\times10^6$ cm/s to $\sim 2.4\times10^6$ cm/s. This is more than an order of magnitude less the drift velocity of electrons. Not has been found of a clear dependence of the phase velocity on the magnetic field.

This gave reason to assume that these oscillations may contribute to the increase in the rate of electron transfer along the channel SPT.

In Fig.1 is a view of stationary plasma thruster SPT-70 characteristics which were used for numerical simulation of the dynamics of motion of electrons in channel, calculating changes of the Hall parameter, the electronic current and heating power of electrons along the channel.

The main performances: the distribution of the magnetic field and plasma potential, electron and ion component of current in the channel of the thruster represented in Fig. 2.

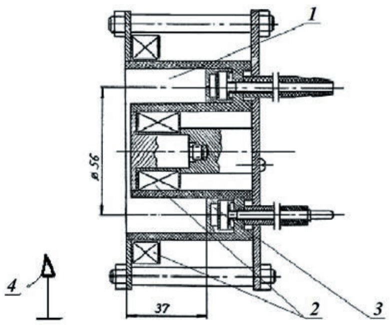

Figure 1. Scheme SPD-70. *(1- channel, 2- magnetic coil, 3- anode, 4-catode)*

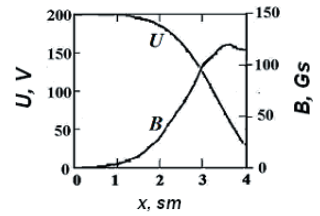

Figure 2. Magnetic field and potential plasma along channel.

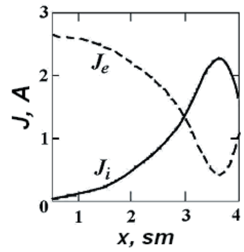

Figure 3. Electron and ion currents along channel.

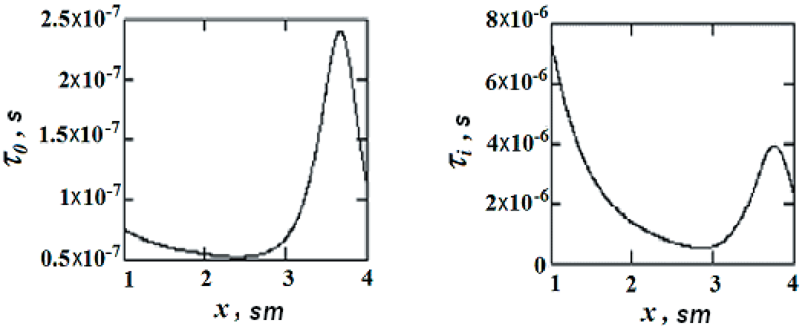

Figure 4. The time dependences a collisions of electrons with neutral along channel $\tau_0(x)$- scattering, $\tau_i(x)$- ionization.

These performances were used to estimate the rate of diffusion of electrons in the channel. Dependence the time of pair collisions of electrons with neutrals, which lead them to scattering $\tau_0(x)$ and ionization $\tau_i(x)$ is represented on Fig. 4. These were made under the assumption that the distribution of electrons $n_e(\varepsilon)$ is the Maxwell function for changing T_e in channel in limit (4-14) eV. These characteristics were obtained when working on Krypton, $Q = 0.55$ cm^3/s, $U_d = 200$ V, $J_d = 2.7$ A. System of equations for numerical simulation of the dynamics of motion of electrons in channel in the field of azimuthal wave has been used next.

$$m\frac{dv_x}{dt} = -eE(x) + \frac{e}{c}v_y B(x) - \frac{mv_x}{\tau_0}$$

$$m\frac{dv_y}{dt} = -eE(x)\beta_i \sin(\omega_i t + k_i y) - \frac{e}{c}v_x B(x) - \frac{mv_y}{\tau_0}$$

Fig. 5 shows the dynamics of electron transport from the exit till the anode of $x(t)$ in dependency on time, when in the channel has an azimuthal wave ($\beta_1 = 0.35$, $k_i = 0.35$ cm^{-1}, $v_\theta = 1.5\times10^6$ cm/c), and when its is not, for $\tau_0 = 3\times10^{-7}$ s. It is seen that the time of electron transport through the channel has decreased with more than 20 µs up to 2.5 µs.

Since the electron drift velocity in azimuth direction more than azimuthal phase velocity of the wave $V_d >> v_\theta$, the motion of electrons in the wave field k_1 can consider in the quasi stationary approximation, as in the field of the standing wave.

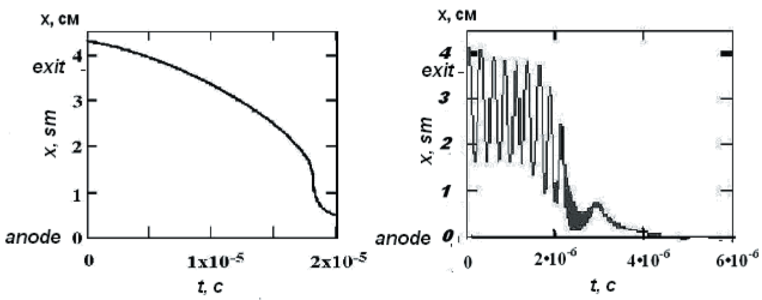

Figure 5. a,b the dynamics of electron transport from the exit till the anode of *x*(*t*) in dependency on time *a) in channel wave not, b) in the channel is azimuthal wave $\beta_1 = 0.35$, $k_i = 0.35$ cm^{-1}.*

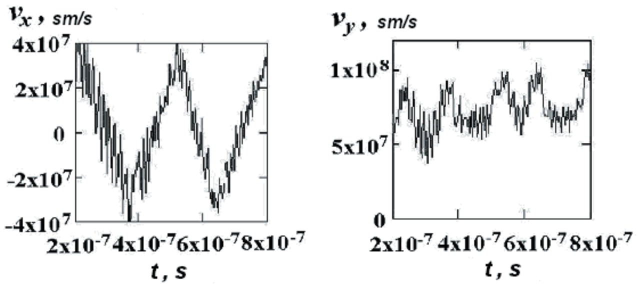

Figure 6. Oscillation of longitudinal v_x and azimuthal v_y velocity electrons

In Fig. 6 has been shown oscillation of longitudinal v_x and azimuthal v_y velocity electrons.

The period T_0 of the basic harmonic modulation of the longitudinal drift velocity $v_{dx}(t)$ can be estimated as $T_0 \approx \frac{2\pi}{V_d k_i}$, therefore, $v_{dx}(t) = c\beta_1 E(x)\sin(2\pi t/T_0)/B(x)$. The amplitude of oscillation of electrons along the axis $x_a = \int_0^{T_0/4} v_{dx}(t)dt$ is equal to β_i / k_i, = 1 cm, which coincides with the results of the calculation.

As seen from figure 6, period basic harmonic of oscillation v_x is T_0 and specter oscillation v_y has harmonics with T_0 and $T_0/2$.

Calculations of the rate of diffusion of electrons from the wave parameters and the frequency of elastic collisions of electrons with neutrals for a range of changes τ_0, k_i and amplitude of the oscillations, for a range of changes τ_0, k_i, β, which are realized in the channel of the thruster, showed.

1. The transport time of electrons through the channel decreases with increasing amplitude, as $T_{tr}(\beta_i) = 8\times10^{-6}/(1 + C\beta_i^2)$ or $T_{tr}(\beta_i)/T^0_{tr} = 1/(1 + C\beta_i^2)$, where $C = 22$, T^0_{tr}–transport time when $\beta_i = 0$. The ratio of $T_{tr}(\beta_i)/T^0_{tr} = 1/C\alpha$, where $\alpha = (1 + C\beta_i^2)/C$ or $\alpha \approx \beta_i^2$ for $\beta_i^2 > 1/C$. If $\beta_i = 0.35$, the value $\alpha \approx 1/2\pi$.
2. Time $T_{tr}(k_i)$ increases linearly with increasing wave vector azimuthal waves as $T_{tr}(k_i) \approx 1.1\times10^{-5}k_i$. For the considered waves $k_i = 1/v_\theta\tau_i$ in that case the dependence transport time on the time of ionization can be represented as $T_{tr}(\tau_i) \approx 7.3\times10^{-12}/\tau_i$.
3. The transport time in the azimuthal wave for $k_1 = 0.3$ cm^{-1}, $v_\theta = 1.5\times10^6$ cm/s, $\beta_i = 0.35$ increases as $T_{tr}(\tau_0) \sim \tau_0^2$. The absence of collisions $\tau_0=\infty$ does not lead to the transport of electrons along the axis of thruster.

These results allowed concluding, that the transport in the wave field, compared to the classical conductivity, increases in proportion to τ_0. The transport of electrons through the channel in that case can be represented as $T_{tr} / T_{tr}^0 = \tau_0 / \alpha\tau_i$, or $v_{xt} / v_e = \alpha\tau_i / \tau_0$. In the region of exit $\alpha \approx 1/2\pi$, and ratio τ_0/τ_i changes slightly, therefore the velocity of the transport here $v_{xt}(\tau_0, \tau_i) \sim \tau_i/2\pi\tau_0^2$.

Longitudinal wave has significantly smaller amplitude than azimuthal wave and does not influence on transport electron through channel.

II. General Guidelines

A. The mechanism of electron transfer in the azimuthal wave.

Qualitatively, the increase rate of electron transport in the azimuthal wave can be considered as follows. The movement of electrons, when there are no waves[9], in crossed electric and magnetic fields is trochoid a superposition of the Larmor rotation and the electric drift. It is known, that the amount of displacement electron in one collision is $\Delta x = V_d / \omega_H$. In the case where there is the azimuthal wave, the electrons drift azimuth with the velocity V_d, so and along the axis of channel with a velocity v_{dx}. Estimates show that over the length of the channel the period of longitudinal wave is $T_0 > \tau_0$. Magnitude of motion of the electrons (double amplitude of the oscillations $2x_a$) can be up to half the length of the channel. Collisions of electrons may occur at any point along the line of the amplitude.

In particular, if $T_0 = \tau_0$, X_c average displacement can be estimated as

$$X_c(x_0) = \frac{1}{2x_a} \cdot \int_{x_0-2x_a}^{x_0} \frac{V_d(x)}{\omega_H(x)} dx$$

It is obvious that the greatest contribution to the average displacement gives the electrons in which the ratio of the drift velocity to the cyclotron frequency has a maximum value.

Figure 7 shows a dependence the displacement of electron for collisions with neutral along channel SPD-70 $\Delta x(x) = V_d(x) / \omega_H(x)$ and the results of the calculation X_c near the exit (3.7- 2.7)sm when a wave $\beta_i = 0.35$, $k_i = 0.34$, $x_a = 1$cm. It is seen that when the electron moves to the anode, Δx increases by more than an order of

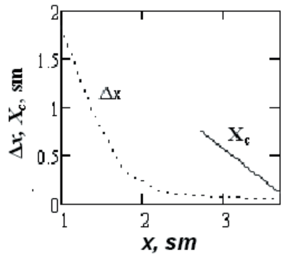

Figure 7. The amount of displacement electron Δx in one collision along channel, the amount of average displacement X_c near the exit (3.7- 2.7) sm for wave $\beta_i = 0.35$, $k_i = 0.34$.

magnitude. The value X_c compared to Δx becomes several times larger due to the fact that the place of electron scattering which gives the greatest contribution to X_c, moves closer to the anode to a value (x_0 - $2x_a$), where V_d / ω_H has the greatest value. Amplitude $x_a = \beta_i / k_i$, therefore maximum value of x_a will have for $k_i = 0.35$.

Estimates X_c, derived from such representations, are consistent with the results of numerical calculations.

B. Comparison results of the modeling of transport electrons in the channel SPD with experimental studies.

Value of electron transport in the azimuthal wave compared to the classical diffusion increases in $\alpha\tau_i$ / τ_0 times. Assuming that the azimuthal velocity in one and the other case is the same, and the Hall parameter in the classical diffusion $\chi_0 = \omega_H\tau_0$, then the Hall parameter for the diffusion of electrons in the azimuthal wave can be represented as $\chi_2 = \omega_H \tau_0^2 / \alpha\tau_i$. Experimental Hall parameter is determined as $\chi_1 = j_d / j_e$, where $j_e = J_e(x) / S$, $j_d(x) = cen_e(x)E(x)/B(x)$, where $n_i(x) = J_i(x)/eSv_i(x)$, $n_i(x)$ - ion density distribution along the channel, $n_i(x) = n_e(x)$, $v_i(x)$ - velocity of the ions. Ion velocity was obtained from the distribution of the plasma potential in the channel of the thruster,
$v_i(x) = [2e (U_d - U(x)) / m_i]^{0.5}$.

Figure 8 shows the changes in the parameters of the Hall along the channel:

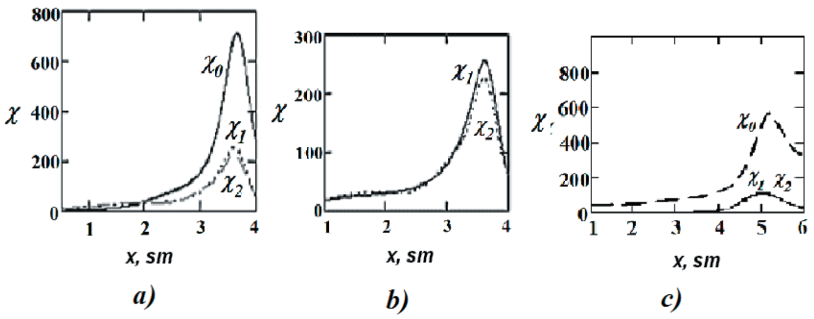

Figure 8. The changes in the parameters of the Hall χ along the channel- a, b) SPT-70, c) SPT-130, (χ_0 - classical diffusion, χ_1 – experiment, χ_2 – result of calculation).

1) The classical diffusion χ_0,
2) The diffusion resulting from experiment χ_1,
3) The diffusion, calculated in the wave field χ_2.

Values of change τ_0 and τ_i along the channel were obtained from experimental measurements of Figure 4. As can be seen, the Hall parameter χ_1 is in good agreement with value of $\chi_2 = \omega_H \tau_0^2 / \alpha\tau_i$, for a value of $\alpha = 1 / 2\pi$, which is obtained in the analysis of the dynamics of electrons in the azimuthal wave. This coincidence is shown when $x > 2.5$ cm, where there is a wave with $\beta_i = 0.35$ and $\tau_0 / \tau_i \approx$ constant.

It should be noted that the relative amplitude of the oscillations $\beta_1 = 0.35$, which was used in the calculations is reduced in direction the anode. This can take into account by fitting variation coefficient α_1 (x) along the channel. If for SPD -70 $\chi_2 = \omega_H \tau_0^2 / \alpha_1\tau_i$, where $\alpha_1(x) = \alpha$ (1 + exp (5.56 - (x / l) 2)), l = 1cm, the calculation results of the Hall parameter $\chi_2(x)$ are in good agreement with $\chi_1(x)$ over the entire length of the channel of the thruster fig.6. Obviously, if relation $\alpha_1 \approx \tau_0 / \tau_i$, which is already being implemented at $\beta_1 < 0.1$, then the transport of electrons will be in good agreement with the classical conductivity. Figure 7 shows the change of the Hall parameter along the channel of model SPD-130 [13, 14], the working gas is Xe, Q = 6.3 mg/s, U_d = 200V, the J_d = 6.7A [11]. Method of calculation has been used is the same as for the SPD-70. Hall parameter χ1 (x) agrees quite well with the results of the calculation of $\chi_2(x) = \omega_H\tau_0^2 / \alpha\tau_i$ for $\alpha = 1/2\pi$ in half of the channel adjacent to the exit, which is consistent with the results obtained for the SPT- 70. Taking into account the decrease in the amplitude of the azimuthal wave for this model as $\alpha_2 = \alpha$ (1 + exp (1 - (x / l) 2)), results of calculation of the Hall parameter $\chi_1(x)$, as shown from Figure 8, are in good agreement with accuracy up to 15% along the entire length of the channel SPT-130.

Conductivity of the plasma in the absence of a wave along the channel[10].

$$\sigma_x = \sigma_e \cdot \frac{1}{1+\omega_H^2 \cdot \tau_0^2}$$

, where $\sigma_e = e^2 n_e \tau_0 / m$.

As shown, time the transport of electrons in the presence of azimuthal wave and Hall parameter, compared to classical diffusion, decreases in τ_0 / $\alpha_1\tau_i$ times. Consequently, the conductivity of the plasma in this case can be represented as $\sigma_f = \sigma_e \cdot \frac{\alpha_\beta \tau_i / \tau_0}{1+\omega_H{}^2 \cdot \tau_0{}^2}$ if $\omega_H\,\tau_0 >> 1$ $\sigma_f = \frac{e^2 \bullet n_e \bullet \tau_i \bullet \alpha_\beta}{m \bullet \omega_H{}^2 \cdot \tau_0{}^2}$, where $\alpha_\beta = \alpha_1$ for SPT-70 and $\alpha_\beta = \alpha_2$ for SPT-130.

Knowing the change in conductivity of the plasma along the canal σ(x), we can calculate the electron current $J_e(x) = \sigma(x)\ S\ (-E(x) - (\text{grad}(p_e(x)))/en_e)$ and the heating power of electrons per unit length of channel $W(x) = \sigma(x)\ S\ (-E(x) - (\text{grad}(p_e(x)))/en_e)^2$.

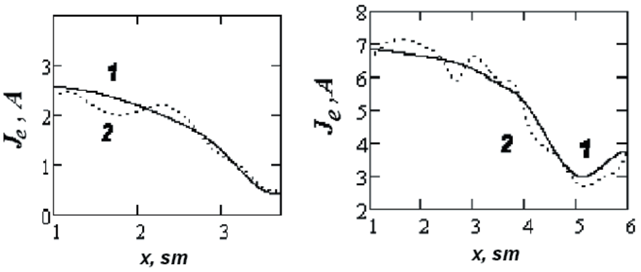

Figure 9. Variation the electron current along channel *a) SPT-70, b) SPT-130, 1-experiment, 2-result of calculation.*

Figure 9 illustrates the value $J_e(x)$ obtained numerically, and the electron current obtained from the experiment for two models of SPT, without taking into account the electron pressure gradient. As can be seen from the figures, the calculation results are in good agreement with the experimental values.

Figure 10 shows the variation of the heating power of the electrons along the channel SPT. Numerical calculations were performed also excluding the electron pressure gradient. The experimental value of $W(x)$ was evaluated as $W(x) = J_e(x)\ E(x)$.

Heating power, which get electrons when an azimuthal wave is in the channel SPT, agrees quite well with the values of $W(x)$, obtained from experimental measurements.

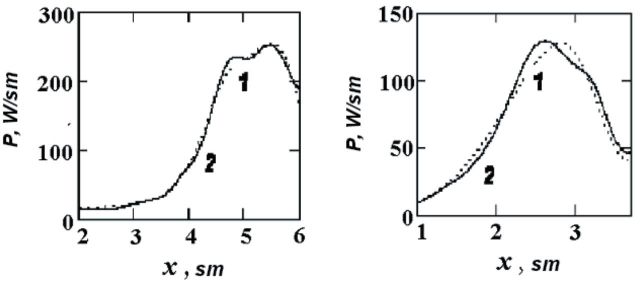

Figure 10. Variation of the heating power of the electrons along the channel SPT *a) SPT-130, b) SPT-70, 1- experiment, 2- result of calculation.*

C. Discussion of the results

The presented results show that the mechanism of electron diffusion in the azimuthal wave makes it possible to describe the transport and heating of the electrons in the channel of the SPT.

Azimuthal oscillations give rise to electron drift along the channel of thruster. The amplitude of the drift motion of x_a electrons depends on parameters of the azimuthal wave (amplitude and magnitude of the wave vector). Collisions of electrons may occur at any point along the line of the amplitude x_a. The largest contribution to the average displacement X_c will give electrons, in which the ratio of the drift velocity to the cyclotron frequency V_d / ω_H has a maximum value. Since in direction to anode ω_H decreases, and the drift velocity V_d increases, the place of electron scattering, which gives the largest contribution to X_c, moves closer to the anode.

The results which have been obtained by numerical simulation of the dynamics of electrons in the azimuthal waves show that the diffusion of electrons as compared to the classical diffusion is increased in $\alpha\ \tau_i$ / τ_0 times.

Diffusion of electrons is determined only by the scattering of electrons by atoms. Azimuthal wave increases the rate of transport due to the appearance of electron drift along the channel and redistribution of place electron scattering.

The results, which were obtained by numerical simulation of the dynamics of electrons in the azimuthal wave, analysis of the Hall parameter, the electron current, electron power heating show a good agreement between the results of calculation and experimental research.

The absence of model instability processes, which leads to the appearance of azimuthal ionization waves in the channel, SPT does not allow more detail to determine the amplitude and spectral composition oscillation and their dependence on the parameters of the thruster. Azimuthal wave characteristics are important for real calculations of diffusion and the heating of electrons in the channel of the SPT and especially when need optimize of parameters thruster.

From this point of view, there are two processes that should be taken into account the analysis work of SPT in a wide range of its parameters - it Coulomb scattering of electrons and the interaction of electrons with the channel walls.

III. Conclusion

In the report presents the results of the analysis of the simulation of the dynamics of the electron in the azimuthal wave which is observed in the channel stationary plasma thruster SPT.

Numerical calculations show that the azimuthal wave lead to an increase in the rate of electron transport along the axis of the thrusters compared to the classical conductivity.

This occurs due to the fact, that the azimuthal wave creates drift oscillation electron along channel SPT and the displacement it in direction the anode can reach more than 2 cm.

The greatest contribution to the average displacement in moment collision gives the electrons in which the ratio of the drift velocity to the cyclotron frequency has a maximum value. This value increases in direction anode. The increase velocity of transport compared to classical diffusion occurs due to the fact that the place of the electron scattering moves closer to the anode.

Has been obtained the conductivity of the plasma in presence the azimuthal wave in channel SPT. Values of the Hall diffusion, electrons current through the channel and the heating power of the electrons in the channel, which were obtained from the present model, are in good agreement with the experimental results for two models of SPD, working on different gases Kr and Xe.

References

Periodicals

[1]*A.I. Morozov, K. Kozubsky et al // Sov. Phys. Plasma, Vol. 29, No. 3, 2003, pp. 261.*

[2]*Janes G., Lowder R., Anomalous electron diffusion and ion acceleration, Physics of Fluids. 1966. Vol. 9, No. 6. pp. 1115-1123.*

[3]*A.I. Morozov, Yu.V.Esipchuk, A.M.Kapulkin, V.A.Nevrovskiy, V.A.Smirnov, Azimuthely asymmetric oscillation and the electron anomalous conduction in acceleration closed drift electron, Sov. Phys. Tech. Yournal, Vol. XLIII, No.5, 1973, pp. 972-982.*

Proceedings

[4]*L. Garriques, J. Perez-Luna, J. Lo, G.J.M. Hagelar, J.P. Boeuf //Paper IEPC-2009-82. Michigan. 2009.*

[5]*J.M. Gallardo, E. Ahedo // Paper IEPC-2005-117, Princeton University, 2005.*

[6]*A.N. Veselovzorov, E.D. Dlougach, A.A. Pogorelov, E.B. Svirsky, V.A. Smirnov Paper IEPC-2013-080 Washington October 6 – 10, 2013*

[7]*A.N. Veselovzorov, E.D. Dlougach, A.A. Pogorelov, E.B. Svirsky, V.A. Smirnov //. Paper IEPC-2011- 060 -Wiesbaden Germany. September 11 – 15, 2011.*

Periodicals

[8]*A.N. Veselovzorov, E.D. Dlougach, A.A. Pogorelov, E.B. Svirsky, V.A. Smirnov Sov. Phys. Tech. Yournal, Vol.58, 2013, pp., 968-972.*

Books

[9]*V.E. Golant, A.P. Chilinskiy, S.A. Sakharov // Fundamentals of Plasma Physics. Atomizdat, 1977, pp., 245.*

[10]*A.I. Morozov, Physical Fundamentals of Cosmic Electro-Jet Thruster, Vol.1, 1978, pp. 256.*