Hall2De Simulations with an Anomalous Transport Model Based on the Electron Cyclotron Drift Instability

IEPC-2015-402／ISTS-2015-b-402

*Presented at Joint Conference of 30th International Symposium on Space Technology and Science
34th International Electric Propulsion Conference and 6th Nano-satellite Symposium,
Hyogo-Kobe, Japan
July 4 – 10, 2015*

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Abstract: **In 2004 Adam, Heron, and Laval, based on the results of a 2-D, z-θ particle simulation, proposed that turbulence generated by electron cyclotron drift instability is the cause of the enhanced cross-field electron transport in Hall thrusters. The arguments were supported by both analysis and measurement. A decade later Coche and Garrigues found similar results using a different 2-D, z-θ particle simulation code. Hall2De is a 2-D r-z computational solver of the fluid conservation equations that govern the evolution of the partially ionized gas in Hall thrusters. As compared with z-θ geometry, r-z geometry better describes the features of a Hall thruster such as the finite channel width, magnetic field curvature, and electron flow in the near plume. Because Hall2De includes the near field plume region, the code is able to self-consistently model centrally mounted cathode plasmas. Hall2De has demonstrated sufficient fidelity that it has been successfully used to design a new class of long life magnetically shielded Hall thrusters. However, the code uses an ad hoc empirically based model of "anomalous" electron scattering that in turn makes it dependent on plasma measurements. In this paper, we show that the collision frequency profile is spatially consistent with the rapid growth of electron cyclotron drift waves in the Hall thruster acceleration zone and that the resultant ion waves are transported into the near plume by the main thruster beam ions. Results from the first implementation in Hall2De of a self-consistent electron scattering model based on electron cyclotron drift physics are presented. These results both successfully reproduce Hall thruster plasma features and identify where additional research is needed.**

Nomenclature

ε_0	=	permittivity of free space
$\overline{\overline{\varepsilon}}$	=	dielectric tensor
γ_k	=	growth rate of mode *k*
λ	=	wave length
ν_{AN}	=	anomalous electron scattering frequency
ν_e	=	electron scattering frequency
θ	=	azimuthal dimension

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ω	=	wave frequency
ω_{ce}	=	electron cyclotron frequency
ω_e	=	electron plasma frequency
Ω	=	Hall parameter
σ	=	electrical conductivity
B	=	magnetic field strength
C_A	=	ion acoustic speed
E	=	electric field
e	=	electron charge
$\mathbf{g}$	=	electron guiding center location
I_e	=	electron current
I_{beam}	=	thrust beam ion current
j_e	=	electron current density
k	=	wave number
L	=	channel length
m_e	=	electron mass
m_i	=	ion mass
N_k	=	wave action associated with wave number k
n	=	electron density
n_{fast}	=	fastest growing harmonic of the cyclotron frequency
r	=	radial dimension
u_e	=	electron thermal speed
u_{ion}	=	ion velocity
T_e	=	electron temperature
T_e	=	ion temperature
$\vec{\mathrm{v}}_g$	=	wave group velocity
z	=	axial dimension

I. Introduction

USING 2-D, z-θ particle simulations Adam, et al[1] proposed that the turbulence generated by electron cyclotron drift instability[2] is the cause of the enhanced cross-field electron transport in a Hall thruster. The arguments were based both on analysis and measurement[3-5]. Similar results were found using a different 2-D, z-θ simulation[6]. Hall2De[7] is a 2-D r-z computational solver of the conservation equations that govern the evolution of the partially ionized gas in Hall thrusters. In Hall2De electrons are modelled as a Maxwellian fluid. Excessive numerical diffusion due to the large disparity of the transport coefficients parallel and perpendicular to the magnetic field is evaded in the code by discretizing the equations on a computational mesh that is aligned with the applied magnetic field. Typically, the magnetic-field-aligned mesh spans a computational domain that extends several times the thruster channel length in the axial direction, and encompasses the cathode boundary and the thruster centerline. Because Hall2De includes the near field plume region, the code is able to self-consistently model centrally mounted cathode plasmas. Hall2De has demonstrated sufficient fidelity that it has been successfully used to design a new class of long life magnetically shielded Hall thrusters[8,9]. However, the code uses an ad hoc empirically based model of "anomalous" electron scattering that in turn makes it dependent on plasma measurements. A focused investigation has been undertaken in the last few years that aimed at identifying the precise profile of the required transport coefficient, both in the channel interior and in the near plume of the thruster[10-12]. The motivation behind this focused study was that by identifying the profile we would then be in a more advantageous position to identify the physics that can produce it. In this paper, we show that this newly identified collision frequency profile is spatially consistent with the electron cyclotron drift instability and turbulence[13,14] shown to be the source of anomalous cross field electron transport in plasma devices in the 1970's[15,16] and observed during the past decade in both Hall thruster simulations and measurements[1-5]. This analysis shows how the conditions for the exponential growth of electron cyclotron drift waves are satisfied in the Hall thruster acceleration zone and how the resultant ion waves are transported into the near plume by the main thruster beam ions. Results from the first implementation in Hall2De of a self-consistent electron scattering model based on electron cyclotron drift physics are presented. These results both successfully reproduce Hall thruster plasma features and identify where additional research is needed.

II. Observations from Plasma Measurements and Previous Hall2De Calculations

Over the past decade, the Hall2De code[7] has been used to model the plasmas of several Hall thrusters. Hall2De solves in 2-D, r-z geometry, time dependent fluid continuity and momentum equations for ions and, and steady state continuity, momentum (Ohm's law) and energy equations for electrons. Electron transport includes classical electron scattering with Xenon ions and neutral atoms, channel wall sheath effects, and an ad hoc formulation for "anomalous" scattering. This electron "anomalous" scattering is necessary for the calculation results to replicate the laboratory measurements of Hall thruster plasmas. From the outset, we've recognized the fundamental weakness of using ad hoc, adjustable, "anomalous" scattering. The hope was that examination of the "anomalous" scattering necessary to match the experiments would lead to an understanding of the physical basis of "anomalous" scattering in Hall thrusters and subsequently lead to a first principles implementation for "anomalous" scattering. This paper is the first step in that process.

In the discussion below, we will use the results of a Hall2De simulation of the H6 thruster (without magnetic shielding, called H6US in our previous articles) performed using an ad hoc "anomalous" collision frequency that was chosen to best fit the experimental data. The operating condition chosen are 300V discharge voltage and 20A discharge current. The beam ion current is about 15A. Figure 1 shows a plot of values along the channel centerline from the 2-D Hall2De simulation of the calculated electric potential and the classical (f_ei and f_en in the plot) and "anomalous" scattering frequencies (f_Anom). Also plotted is the sum of all the scattering frequencies (f_e).

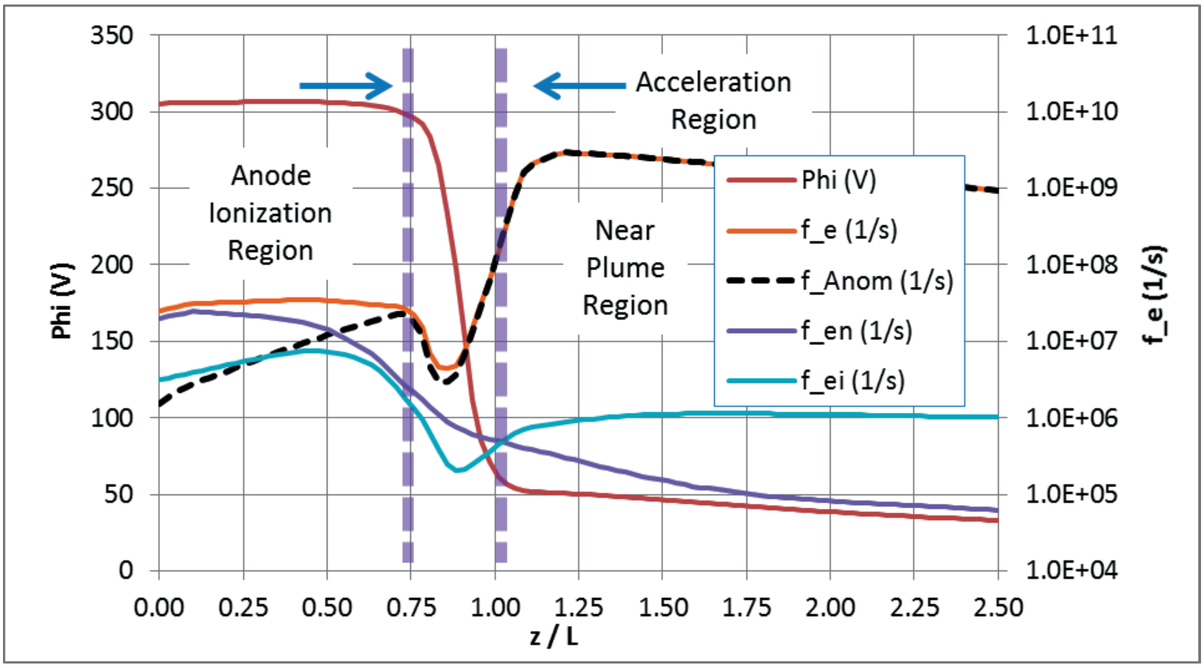

Figure 1. The electric potential electron scattering frequencies along the channel centerline of the Hall2De simulation of the H6 Hall thruster

We have divided the computational space into three regions: Anode Ionization, Acceleration and Near Plume. Examination of the electron transport in each of the these regions leads to the conclusion that the exponential growth of ion sound waves driven by the electron cyclotron drift instability in the Acceleration Region is the source of the "anomalous" scattering.

A. Electron transport in the Near Plume Region

While appearing to be the most benign region, the role of anomalous scattering is more important in the Near Plume region than anywhere else. Before Hall2De, Hall thruster models typically modeled the electron transport in 1-D where the electron current was initiated on a "cathode" field line. Hall2De models the electron current flow in two dimensions, in this case 20A from a centrally mounted cathode, 15A leaving the computational grid to neutralize the beam ions, and 5A going upstream through the Near Plume into the channel and on to the anode.

In the Near Plume, the magnetic field strength within a channel length of the exit plane is generally within a factor of two of the channel centerline peak field. Without "anomalous" scattering, the electric fields required to pull 5A into the channel would be substantial. However, both laboratory measurements and Hall2De show potential drops of only a few tens of volts and electric fields the order of a few hundred volts per meter in the Near Plume. In order to match the experimental results, in the Near Plume Hall2De uses "anomalous" scattering that is everywhere more than 3 orders of magnitude larger than classical scattering.

Even without the Hall2De simulation, a simple analysis shows the magnitude of the scattering necessary to obtain the experimental measurements. Using the definition of the Hall parameter

$$\Omega \equiv \frac{\omega_{ce}}{\nu_e} \tag{1}$$

we can rewrite the plasma conductivity in terms of the density and the Hall parameter

$$\sigma = \frac{n e^2}{m_e \nu_e (1+\Omega^2)} \tag{2}$$

$$\sigma = \frac{n e^2 \Omega}{m_e \omega_{ce} (1+\Omega^2)}. \tag{3}$$

This formulation is very convenient in Hall thrusters, since we know the magnitude of the magnetic field, and thus the electron cyclotron frequency everywhere. Notice that the conductivity is maximum for Ω=1. For higher Hall parameters the magnetic field limits the current; for low Hall parameters, classical resistivity limits the current. Neglecting pressure terms, the electron current density from Ohm's law is

$$j_e = \sigma E = \frac{n e^2 \Omega}{m_e \omega_{ce} (1+\Omega^2)} E \tag{4}$$

Integrating over a cross section of the main ion beam

$$I_e = \int j_e \, dS = \int \frac{n e^2 \Omega}{m_e \omega_{ce} (1+\Omega^2)} E \, dS \tag{5}$$

Using that the electron current in the near plume is about 1/3 of the ion beam current.

$$I_e \approx \frac{1}{3} I_{beam} \tag{6}$$

and relating the ion beam current to the ion density times the ion velocity

$$I_{beam} \approx e u_{ion} \int n \, dS \tag{7}$$

we can estimate the average Hall parameter in the Near Plume from the measured electric field and the ion velocity

$$\left\langle \frac{\Omega}{1+\Omega^2} \right\rangle = \frac{m_e u_{ion} \omega_{ce}}{3 e \langle E \rangle} \tag{8}$$

For parameters like those measured in the H6US near plume

$$u_{ion} \approx 2\times10^{4}\,m\,s^{-1}$$
$$B \approx 100\,G$$
$$E \approx 400\,V\,m^{-1}$$

the average Hall parameter in the near plume is about 6.

$$\langle\Omega\rangle \approx 6$$

Therefore, just based on laboratory electric field measurements in the Near Plume, the effective electron scattering frequency is around

$$\nu_e \approx \frac{\omega_{ce}}{\langle\Omega\rangle} \approx 3\times10^{8}\,s^{-1} \quad . \tag{9}$$

This effective scattering frequency is more than two orders of magnitude larger than the classical electron scattering frequencies in the Near Plume. In fact, it is an order of magnitude larger than the classical scattering frequency anywhere in the thruster, including near the anode. For a typical plume electron temperature, $T_e \approx 4eV$, the effective electron mean free path in the Near Plume is only about 5 mm

The conclusion from this analysis is that in the Near Plume, transport is consistent with low electron Hall parameter, less than 10, in the main beam, as it has also been shown by the numerical simulation results in this region. The effective scattering lengths are shorter than typical distances to the thruster body so wall effects can only play a minor role. This magnitude of scattering is consistent with the fully saturated electron cyclotron drift instability. Computational results presented in Ref [6] show clearly ion density variations consistent with azimuthal ion sound waves convected axially by the supersonic ion beam.

B. Ion Wave generation in the Acceleration Region

Where does the very large effective scattering frequency observed in the Near Plume region originate? An important clue comes from the direction of electric fields necessary to move the electron guiding center towards the anode.

In crossed electric and magnetic fields, without loss of generality, we can transform to electron drift frame, where the electron motion in the plane perpendicular to **B** is a circle around the guiding center, **g**.

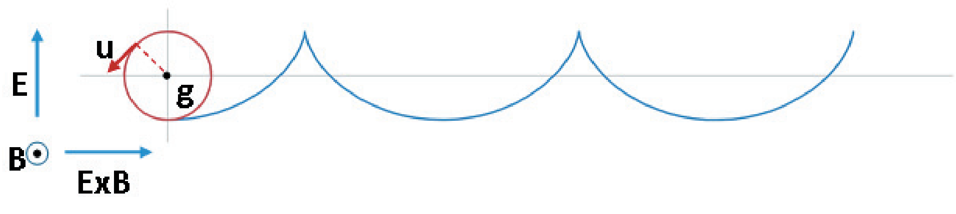

Figure 2. In the drift frame the projection of electron motion in the plane perpendicular to B is a circle around the guiding center

$$\mathbf{u} = (u_{\mathbf{E}\times\mathbf{B}}, u_{\mathbf{E}}, u_{\mathbf{B}})$$
$$\mathbf{r} = (r_{\mathbf{E}\times\mathbf{B}}, r_{\mathbf{E}}, r_{\mathbf{B}})$$
$$\mathbf{g} = (r_{\mathbf{E}\times\mathbf{B}} - u_{\mathbf{E}} / \omega_{ce}, r_{\mathbf{E}} + u_{\mathbf{E}\times\mathbf{B}} / \omega_{ce}, r_{\mathbf{B}})$$

The location of the guiding center, **g**, in the direction of the applied electric field, **E**, can only move by changes in the drift direction velocity, $u_{\mathbf{ExB}}$. This is an import result. It explains why attempts to model Hall thruster transport by full particle simulations[17] or multi-scale modeling in 2-D r-z geometry[18] have still had to rely on additional "anomalous" scattering to enhance the cross field electron transport. Only models that can resolve fields in the **E**x**B** dimension, such as the z-θ codes[1,6], are able to obtain Hall thruster like behavior without the addition of ad hoc "anomalous" scattering. The observed scattering is from plasma waves with electric field components in the ExB direction.

The very high effective scattering frequency necessary to explain the measured low electric fields in the near plume require high amplitude, millimeter length waves. Published 2-D, z-θ simulations, analyses, and measurements, have all shown that the condition exist in the Hall thruster acceleration region for the electron cyclotron drift instability. The criterion for this instability is that the relative ion electron drift speed,

$$u_{drift} = \frac{E}{B}, \tag{10}$$

exceed the ion sound speed,

$$C_A = \sqrt{\frac{eT_e}{m_i}}. \tag{11}$$

In the Acceleration Region, azimuthal electron drift velocities, as shown in Figure 3, are hundreds of times faster than the ion sound speed, much greater than the minimum necessary to generate ExB drift waves, or even conventional ion acoustic waves. This high speed electron drift drives the growth of large amplitude azimuthal ion sound waves. In this region, the steep potential gradient accelerates ions axially from subsonic to speeds several times the ion acoustic velocity. Thus means that ion acoustic waves generated in the acceleration region move downstream with the supersonic beam ions into the near plume.

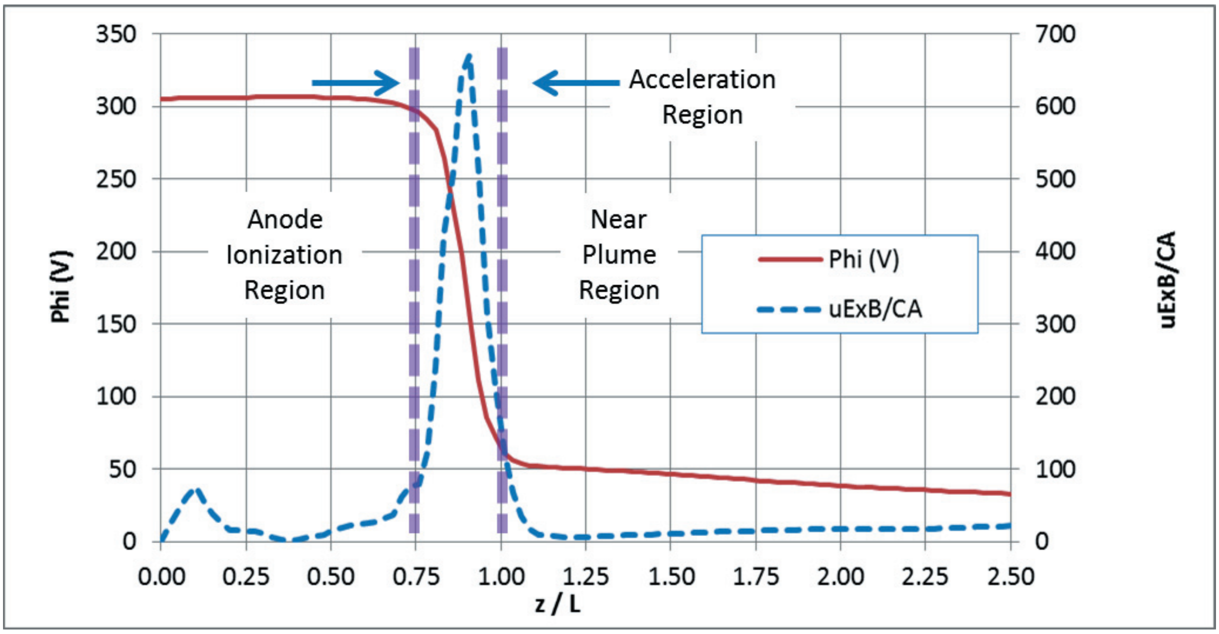

Figure 3. In the Acceleration Region the azimuthal ExB drift velocity is hundreds of times faster than ion sound speed.

The growth of ExB drift waves have been investigated by many researchers over the past half century. The initial wave growth occurs near harmonics of the local electron cyclotron frequency. Unfortunately, these investigations, with the notable exceptions of the two z-θ Hall thruster particle codes[1,6] were for uniform, homogenous plasma with constant applied magnetic and electric fields. However in the Hall thruster, the plasma electron temperature and density and the magnetic and electric fields are all changing rapidly in the ion frame of reference. This makes the direct application of the growth rates for the ion cyclotron drift instability problematical. For a first attempt at an implementation in a 2-D r-z Hall code with fluid electrons, based on the observation by Lampe, et al.[13] that as the amplitude increases the wave transitions to "ordinary ion sound modes that would occur in an unmagnetized plasma", we use the linear growth rate of ordinary ion acoustic waves to calculate the wave energy of the high amplitude waves.

C. The Anode Ionization Region

As seen in Figure 1, electron transport in the Anode Ionization region is dominated by classical electron neutral and electron ion scattering. However, in much of this region the ExB drift velocity exceeds the ion acoustic speed. The greatest difficulty that we have encountered in implementing an anomalous electron collision frequency based on the ion drift wave instability isn't whether the waves will grow, but limiting the growth to the Acceleration Region and the Near Plume. We have not identified a physical mechanism that prevents the ion wave energy from moving upstream toward the anode. As discussed in the results section below, invariably the steady state, self-consistently calculated, anomalous collision profile was extended far upstream compared the ad-hoc collision frequency used in the original Hall2De. This led to very high electron currents (greater than 25 A) and potential profiles much broader than observed in Hall thrusters. To overcome this difficulty and obtain a reasonable Hall2De solution using a self-consistent anomalous collision profile we found it necessary restrict the growth of the wave energy to downstream of the minimum in the anomalous collision frequency from Figure 1, about 0.85 of the channel length.

III. Hall2De Implementation of Ion Acoustic Instability based Anomalous Scattering

A. Plasma Wave Kinetic Equation

Plasma turbulence has been observed in Hall thruster plumes to propagate in the azimuthal direction[4]. Provided this turbulence is primarily the electron cyclotron drift instability, we know that it gains energy at the expense of momentum of the E x B electron drift in the azimuthal direction. The increase in wave energy therefore is balanced by a slowing down of the electrons, which in turn can be modeled as an effective collision frequency. The goals of this section are 1) to demonstrate how the turbulent amplitude can be related to an effective collision frequency and

2) to use wave kinetic theory to motivate a simple, self-consistent model for how the effective collision frequency should evolve in the plasma plume.

With this in mind, we begin with a local, kinetic description for the electrons in the plasma, which is governed by the Boltzmann equation:

$$\frac{\partial f_e}{\partial t} + \vec{\mathrm{v}} \cdot \frac{\partial f_e}{\partial \vec{r}} + \frac{e}{m_e}\left(\vec{E} + \vec{\mathrm{v}} \times \vec{B}\right) \cdot \frac{\partial f_e}{\partial \vec{\mathrm{v}}} = \left.\left(\frac{\partial f_e}{\partial t}\right)\right|_c \tag{12}$$

where f_e denotes the electron distribution function and $\left.\left(\frac{\partial f_e}{\partial t}\right)\right|_c$ is the classical collision operator. In order to elucidate the role of electrostatic waves in this equation, we expand the parameters to first order

$$\begin{aligned} f_e &= f_{e0} + f_{e1} \\ \vec{E} &= \vec{E}_0 + \vec{E}_1 \end{aligned}.$$

Here the zeroth order component encapsulates the slowly varying parameters in the plasma, i.e. bulk fluid velocity and density, and the first order component represents the effects of the quickly varying waves. In this formulation, we have neglected first order perturbations in $\vec{B}$ since the electron cyclotron drift instability is assumed to be electrostatic, and we assume the first order quantities are summations over all possible propagating modes that satisfy the electron cyclotron drift instability dispersion relation:

$$\begin{aligned} f_{e1} &= \sum_{\vec{k}} f_{\vec{k}1} e^{i(\vec{k}\cdot\vec{r}-\omega t)} + \text{c.c.} \\ \vec{E}_1 &= \sum_{\vec{k}} \vec{E}_{\vec{k}1} e^{i(\vec{k}\cdot\vec{r}-\omega t)} + \text{c.c.} \end{aligned},$$

where we have employed the eikonal approximation, $f_{\vec{k}1}, \vec{E}_{\vec{k}1}$ denote the Fourier amplitudes of the $\vec{k}$, and c.c. is the complex conjugate.

The quantities of interest to us are macroscopic in nature, i.e. bulk fluid velocity and density, which are governed by f_{e0}. We therefore seek a form of Eq. 12 that self-consistently describes the temporal and spatial variation of this parameter. To this end, we substitute in the first order expansion of the quantities and take a phase average over the quickly varying components:

$$\frac{\partial f_{e0}}{\partial t} + \vec{\mathrm{v}} \cdot \frac{\partial f_{e0}}{\partial \vec{r}} + \frac{e}{m_e}\left(\vec{E}_0 + \vec{\mathrm{v}} \times \vec{B}\right) \cdot \frac{\partial f_{e0}}{\partial \vec{\mathrm{v}}} = \left.\left(\frac{\partial f_{e0}}{\partial t}\right)\right|_c - \left\langle \frac{e}{m_e} \vec{E}_1 \cdot \frac{\partial f_{e1}}{\partial \vec{\mathrm{v}}} \right\rangle \tag{13}.$$

Here $\langle .. \rangle$ denotes the phase average, and "phase" refers to the argument of the exponential quantities in the eikonal approximation. This type of averaging is consistent with the random phase approximation and the physical interpretation that the quickly varying components of the Fourier modes in the turbulence interfere destructively. The only surviving components thus are the terms independent of time. With this in mind, we take the first moment to arrive at an equation for the electron fluid velocity:

$$m_e n_{e0} \frac{D}{Dt}\left[\vec{\mathrm{V}}_e\right] + \nabla \mathrm{P}_e - e n_{e0}\left(\vec{E}_0 + \vec{\mathrm{V}}_e \times \vec{B}\right) = -\nu_{ce} m_e \vec{\mathrm{V}}_e + e n_e \vec{S}_e, \tag{14}$$

where n_{e0} denotes the background electron density, $\vec{\mathrm{V}}_e = \frac{1}{n_{e0}} \int \vec{\mathrm{v}}\, f_{e0} d\vec{\mathrm{v}}^3$ is the electron fluid velocity, P_e is the electron pressure, $\frac{D}{Dt}$ is the convective derivative, $\nu_{ce} = \frac{1}{n_{e0}\vec{\mathrm{V}}_e} \int \vec{\mathrm{v}} \left(\frac{\partial f_{e0}}{\partial t} \right)\Big|_c d\vec{\mathrm{v}}^3$ is the classical electron collision frequency and we have defined $\vec{S}_e = \frac{1}{n_{e0}} \int \left\langle \vec{E}_1 f_{e1} \right\rangle d\vec{\mathrm{v}}^3$. This last term has units of force density per unit charge and represents an effective drag the plasma turbulence has on the bulk plasma parameters.

To examine the role this drag plays in cross-field transport, we derive an Ohm's Law for the electron current from Eq. 14. In particular, we are interested in finding an expression for the electron current density in the direction across magnetic field lines, i.e. $j_{ez} = en_{e0}V_{ez}$. In keeping with the standard formulation for Ohm's law in Hall thrusters where we neglect the convective derivative for the electron time, we can solve Eq. 14 to find

$$j_{ez} = \frac{e^2 n_{e0}}{m_e \nu_{ce} \left(\Omega_e^2 + 1\right)} \left[E_{0z} + S_{ez} - \Omega_e S_{e\theta} \right], \tag{15}$$

where we have folded the electron pressure into the electric field. The first term in the bracket represents classical cross-field transport driven by the axial electric field. Transport results in this case because classical collisions prevents pure E x B drift and therefore allow a fraction of electrons to cross field lines. The second term represents the driving influence of turbulence propagating in the cross-field direction. Its force is also directed across field lines, and just as with the steady-state electric field, it is capable of driving some cross-field electron transport through classical collisions. It is significant to note that in a strongly magnetized plasma such as a Hall thruster ($\Omega_e >> 1$), the cross-field current resulting from these two terms goes to 0. On the other hand, the last term represents the driving force of the turbulence in the azimuthal direction, which is re-directed into cross-field transport by the Lorentz force. In this case, the effect is independent of classical collisions in the limit of strong magnetic field, suggesting that the turbulence can be dominant driver of cross-field transport. This interpretation is consistent with the single –particle description outlined in Sec. II.

With this effect in mind and recalling that the electron cyclotron drift instability is largely azimuthal in nature, we choose to neglect the S_{ez} component for the remainder of this investigation. Similarly, assuming large Hall parameter, we can simplify Eq. 15 to

$$j_{ez} = en_{e0} \left[\frac{E_{0z}}{B} \right] \frac{\nu_{eff}}{\omega_{ce}}, \tag{16}$$

where we have defined an effective collision frequency as

$$\nu_{eff} = \nu_{ce} - \left[\frac{e}{m_e} \right] \left[\frac{B}{E_{0z}} \right] S_{e\theta}. \tag{17}$$

This expression in turn allows us to identify the wave turbulence with an anomalous collision frequency:

$$\nu_{AN} = -\left[\frac{e}{m_e} \right] \left[\frac{B}{E_{0z}} \right] S_{e\theta}. \tag{18}$$

This form for the turbulence-driven collision frequency lends itself to a physically intuitive description. If we recognize that $V_{e\theta} = \frac{E_{0z}}{B}$, i.e. the electron velocity in the azimuthal direction is given by the E x B drift, then we can see that $m_e V_{e\theta} \nu_{AN} = -eS_{e\theta}$. In other words, the effect of the wave turbulence is to act as a drag term on the electron drift in the azimuthal direction. This is consistent with our physical interpretation of the electron cyclotron drift instability: it grows at the expense of drag on the electrons.

The implication of Eq. 18 is that we have framed the discussion of anomalous transport in terms of turbulent amplitude. In order to find a self-consistent formulation for the anomalous collision frequency, it is necessary to find an expression for how the wave turbulence evolves in the plasma. With this end goal in mind, we begin by explicitly performing the phase average in Eq. 13 to find

$$\begin{aligned} \vec{S}_e &= \frac{1}{n_{e0}} \int \left\langle \vec{E}_1 f_{e1} \right\rangle d\vec{v}^3 = \frac{1}{n_{e0}} \sum_{\vec{k}} \vec{E}_{\vec{k}1} n_{(e)\vec{k}1} = \frac{1}{n_{e0}} \sum_{\vec{k}} i\vec{k} \varphi_{\vec{k}1} n_{(e)\vec{k}1} \\ \vec{S}_e &= \frac{1}{n_{e0}} \sum_{\vec{k}} \vec{k} \varphi_{\vec{k}1} \operatorname{Im}[n_{(e)\vec{k}1}] \end{aligned} , \tag{19}$$

where we have used the definition $n_{e1\vec{k}} = \int f_{(e)\vec{k}1} d\vec{v}^3$ and the electrostatic approximation $\vec{E}_{\vec{k}1} = i\vec{k}\varphi_{\vec{k}1}$. By solving the collisionless Boltzmann equation (Eq. 14) and employing the electrostatic dispersion relation,

$$D(\vec{k},\omega) = \vec{E}_{\vec{k}1} \cdot \overline{\overline{\varepsilon}} \cdot \vec{E}_{\vec{k}1} = k^2 \varphi_{1\vec{k}}^2 + \frac{1}{\varepsilon_0} \sum_s e \varphi_{1\vec{k}} n_{(s)\vec{k}1} = 0, \tag{20}$$

where $\overline{\overline{\varepsilon}}$ is the dielectric response of the plasma, we can show that

$$\vec{S}_e = -\frac{1}{n_{e0} e} \sum_k \vec{k} \gamma_k N_k . \tag{21}$$

Here we have introduced N_k, the wave action density[19] and the growth rate γ_k, for the kth mode. Substituting this relation back into Eq. 18, we find

$$\nu_{AN} = \left[\frac{e}{n_{e0} m_e} \right] \left[\frac{B}{E_{0z}} \right] \sum_k \gamma_k k N_k \tag{22}$$

where we have made the approximation that k is primarily in the azimuthal direction.

In this first examination of the problem, we are only interested in high amplitude waves. Therefore, following Lampe et al.[13], we make the assumption that the growth rate is the linear growth rate for unmagnetized ion acoustic waves:

$$\gamma_k = -k C_A \sqrt{\frac{\pi}{8}} \left(\sqrt{\frac{m_e}{m_i}} \left(1 - \frac{u_{drifit}}{C_A} \right) + \left(\frac{T_e}{T_i} \right)^{3/2} \exp\left(-\frac{T_e}{2T_i} \right) \right) \tag{23}$$

By invoking this definition of the growth rate, the expression for the anomalous collision frequency (Eq. 22) can be written in the limit of large electron drift as

$$\nu_{AN} = \left[\frac{e}{n_{e0} m_e}\right]\left[\frac{B}{E_{0z}}\right]\left(\frac{\pi}{8}\right)^{1/2}\left(\frac{m_e}{m_i}\right)^{1/2} u_{rdrift} \sum_k k^2 N_k \tag{24}$$

Furthermore, since the relative electron drift is in the azimuthal direction and is given by the E x B velocity, we can simplify this expression further to

$$\nu_{AN} = \left[\frac{e}{n_{e0}}\right]\left(\frac{\pi}{8}\right)^{1/2}\left(\frac{1}{m_i m_e}\right)^{1/2} \int k^2 N_k dk\ , \tag{25}$$

where we have expressed the summation as an integral over k.

For the rest of this investigation, we assume that there is a dominant wave vector, primarily oriented in the azimuthal direction but with sufficient spread in k space that phase averaging is still applicable. In this case then, we can write the wave action density as

$$N_{k'} = N\delta(k' - k) \tag{26}$$

such that

$$\begin{aligned} &\int k'^2 N_{k'} dk' = k^2 N \\ &\int N_{k'} dk' = N \end{aligned} \quad . \tag{27}$$

The equation for the anomalous collision frequency thus becomes

$$\nu_{AN} = \left[\frac{e}{n_{e0}}\right]\left(\frac{\pi}{8}\right)^{1/2}\left(\frac{1}{m_i m_e}\right)^{1/2} k^2 N\ . \tag{28}$$

This equation relates the effective collision frequency to the wave action. The advantage of writing the collision frequency in this way is that there is an extant formulation for how the wave action density should evolve in an inhomogeneous plasma, the plasma wave kinetic equation[19]:

$$\frac{\partial N_k}{\partial t} + \frac{\partial}{\partial \vec{r}} \cdot \left[\frac{\partial \omega}{\partial \vec{k}} N_k\right] - \frac{\partial}{\partial \vec{k}} \cdot \left[\frac{\partial \omega}{\partial \vec{r}} N_k\right] = 2\gamma_k N_k \quad , \tag{29}$$

where $\omega(\vec{k})$ is a function of the group velocity through the dispersion relation: $\omega = c_s k + \vec{k} \cdot \vec{V}_{di}$. Using the acoustic dispersion relation in the laboratory frame, we find

$$\frac{\partial N_k}{\partial t} + \frac{\partial}{\partial \vec{r}} \cdot \left[\left(c_s \frac{\vec{k}}{|\vec{k}|} + \vec{V}_{di}\right) N_k\right] - \frac{\partial}{\partial \vec{k}} \cdot \left[\frac{\partial \omega}{\partial \vec{r}} N_k\right] = 2\gamma_k N_k \quad . \tag{30}$$

Now, again invoking the assumption that k is primarily in the azimuthal direction and also noting uniformity (by symmetry) in this direction, we find that in the r-z geometry,

$$\frac{\partial N_k}{\partial t} + \frac{\partial}{\partial \vec{r}} \cdot \left[\vec{V}_{di} N_k\right] - \frac{\partial}{\partial \vec{k}} \cdot \left[\frac{\partial \omega}{\partial \vec{r}} N_k\right] = 2\gamma_k N_k \quad . \tag{31}$$

Finally, we substitute in our delta-expression for N_k and integrate over k-space to find

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial \vec{r}} \cdot \left[\vec{V}_{di} N\right] = 2\gamma_k N \quad . \tag{32}$$

We then can employ our expression for anomalous collision frequency with single-valued, unchanging k to find the evolution equation

$$\frac{\partial\left[n_{e0}\nu_{AN}\right]}{\partial t} + \frac{\partial}{\partial \vec{r}} \cdot \left[\vec{V}_{di} n_{e0}\nu_{AN}\right] = 2\gamma_k n_{e0}\nu_{AN} \quad , \tag{33}$$

which, subject to the assumption above of single-valued k, propagating primarily in the azimuthal direction, yields an estimate for the evolution of the anomalous collision frequency in the Hall thruster.

B. Self- Consistent Hall2De Calculation

The ground breaking numerical simulations[1,6] showing the importance of the electron cyclotron drift instability on electron transport in Hall thrusters, were generated using 2-D z-θ codes that treated the electrons as particles. The physical design features of Hall thrusters are better represented in the r-z geometry with the assumption of azimuthal symmetry. The Hall2De code models the thruster in 2-D axis-symmetric r-z geometry, along with an additional assumption that the electrons act as a Maxwellian fluid. This choice of r-z geometry and fluid electrons enables calculations that directly address thruster design, performance and life issues and provide engineering insight during Hall thruster development. The caveat is that an ad-hoc "anomalous" collision profile is needed. Typically the parameters of the ad-hoc collision frequency are guided by measured potentials and plasma parameters. In this paper we present the first results of computing self-consistently an enhanced electron scattering frequency for use in the electron fluid equations based on the growth of ion acoustic waves through interaction of ExB drifting electrons in the Acceleration Region.

The physical picture that we are trying to model is of sound waves propagating periodically around the thruster channel. Published photos of wear tested Hall thrusters[20] show regular period striations on both the inner and outer channel ceramics. We hypothesize that these striations are generated by ion density fluctuations associated with high amplitude sound waves driven by the electron cyclotron drift instability. The regular nature of the observed striations suggests that a single mode, m, dominates the ion waves. This pattern is consistent with the physical picture that in a periodic system, the fastest growing mode with a wavelength commensurate to the period length dominates. Since the wave group velocity is much less that the ion axial velocity at the exit plane, the structure appears like a standing wave with a very slight azimuthal slope. In order to maintain phase coherence across the channel, the wave number must vary inversely with the radius.

$$\begin{aligned} \lambda_m(r) &= \frac{2\pi r}{m} \\ k(r) &= \frac{2\pi}{\lambda(r)} = \frac{m}{r} \end{aligned} \tag{34}$$

For electron cyclotron drift waves, the wavelengths need to be less than or equal to the electron cycloidal distance in the laboratory frame.

$$\lambda \le \frac{2\pi u_{drift}}{\omega_{ce}}$$

$$k \ge \frac{\omega_{ce}}{u_{drift}} \tag{34}$$

The range of cycloidal distances based on the Hall2De calculation is shown in Figure 4. Wavelengths less than a few millimeters are consistent with published erosion patterns[21].

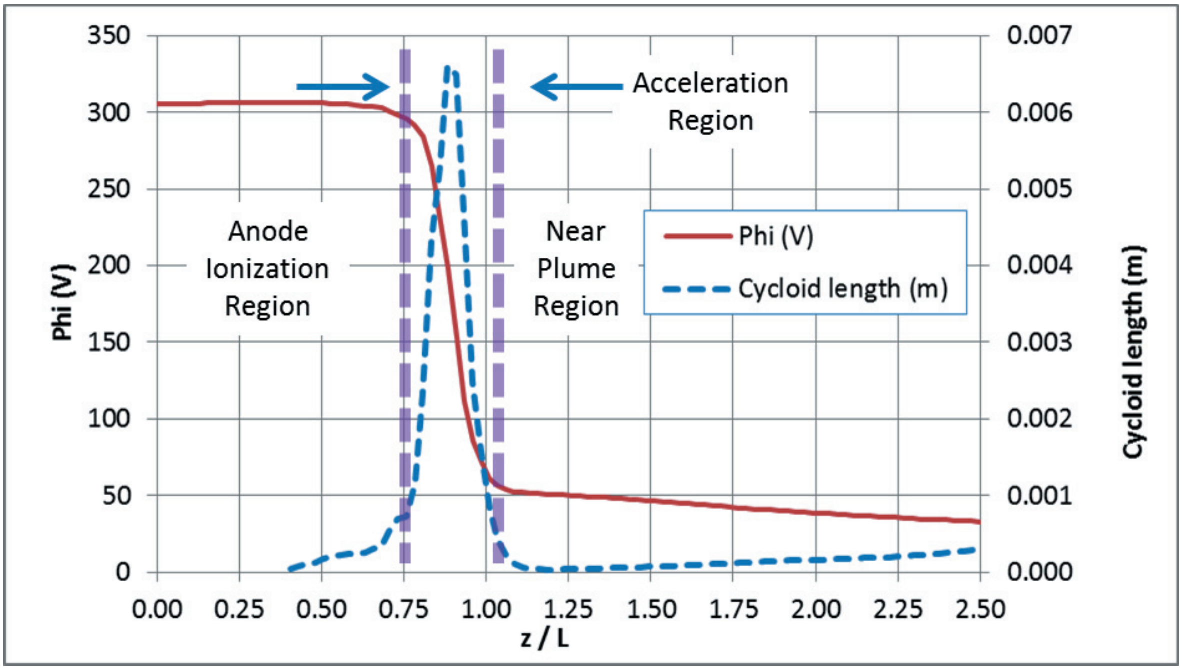

Figure .4 The cycloidal length at the channel center as a function of axial location

Following Lampe[13], the fastest growing instability is the harmonic of the cyclotron frequency falling closest to

$$n_{fast} = \frac{1}{\sqrt{2}}\left(\frac{u_{drift}}{u_e}\right)\left(\frac{\omega_e}{\omega_{ce}}\right) \tag{35}$$

where

$$u_e \equiv \sqrt{\frac{eT_e}{m_e}} \tag{36}$$

Based on this criterion, the wavelength of the fastest growing modes are shown in Figure 5. Since the largest growth rates are in the Acceleration region, we would expect the wavelength of the dominant mode to be lie somewhere between 0.1 mm and 0.4 mm.

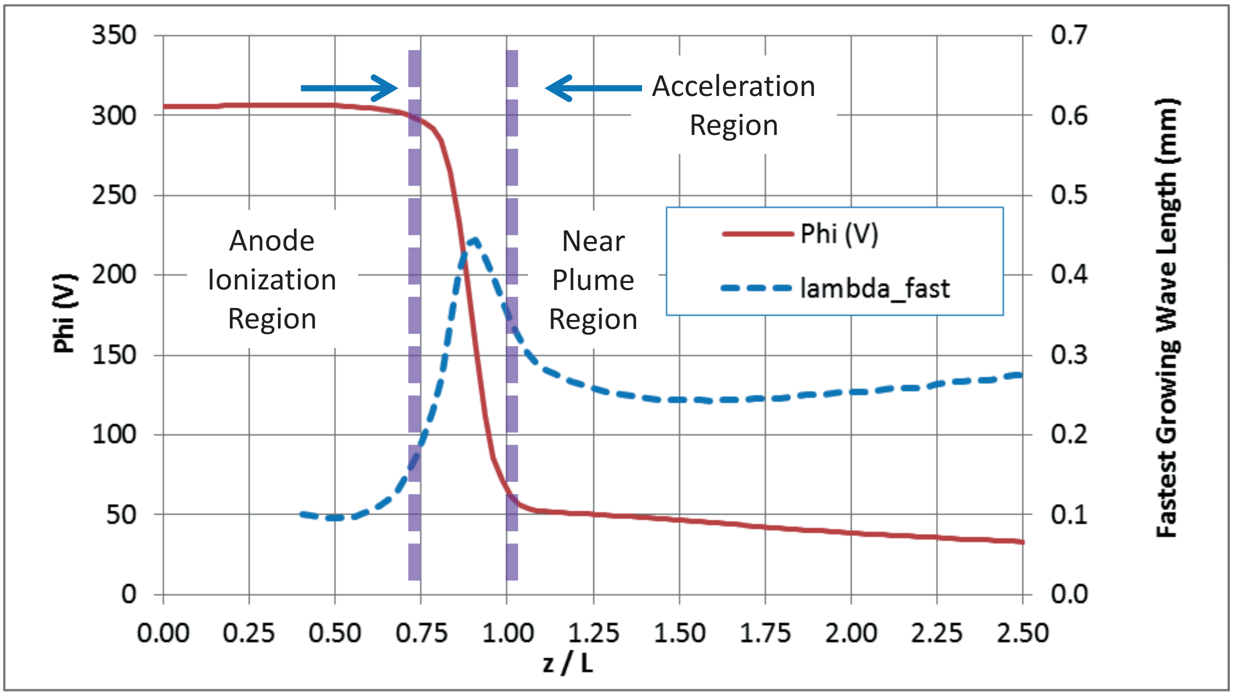

Figure . 5. Fastest growing wavelength for the electron cyclotron drift instability.

The zeroth order wave action equation, Eq. 23, was implemented in Hall2De with the same numerical algorithms used to solve the ion mass continuity equations. The velocity used in this equation was for main beam singly charged Xe ions only. The calculation started from the converged solution obtained using an ad hoc "anomalous" collision frequency profile, as shown by the dashed black profile in Figure 1. The fraction of the collision frequency from the ExB drift generated ion sound wave calculated using the wave action equation was slowly increased until eventually all the "anomalous" collision frequency was from ion sound waves. In the simulations a minimum value of ν_{AN} $=2.7\times10^{6}$ s^{-1} was specified.

Ideally, the implementation of the zeroth order wave action equation in Hall2De would require two parameters, the mode number of the wave and its starting amplitude. However, the zeroth order equation implemented has no way for wave energy to radiate away from a fluid element. As a result, since the ion fluid velocities are very low in the Anode Ionization Region, small positive growth rates in this region led to high amplitude waves. The result was the Acceleration Region broadening upstream toward the anode, allowing an order of magnitude more electron current than in the actual thruster. We think that inclusion of a first order wave action equation that includes a term equivalent to sound pressure should alleviate this unphysical result

Because of this difficulty, in order to obtain self-consistent results with electron current close to that of the laboratory thruster and a potential profile similar to those using an ad hoc collision frequency, we set the wave growth rate to zero from the anode to the location of the scattering minimum as shown in Figure 1.

$$\gamma_k(z) = 0, \qquad {}^{z}\!/_{L} < 0.85 \tag{35}$$

The wave amplitude was assumed to be saturated when the electron scattering frequency equaled the electron cyclotron frequency. With these conditions we were able to find a stable self-consistent Hall2De solution where all of the non-classical collision frequency came from the wave action equation for current driven ion sound waves. The results are shown in Figures 6 & 7 are for a wavelength of λ=0.25 mm. Figure 6 compares the computed classical frequencies for electron-ion (designated as f_ei in the plot) and electron-neutral (f_en) collisions, with the newly computed anomalous collision based on the first-principles arguments embodied in Eqs. 31 above (f_Anon ExB $=\nu_{AN}$ here). Also shown for comparison by the light dashed line is the ad hoc anomalous collision frequency used in previous simulations.

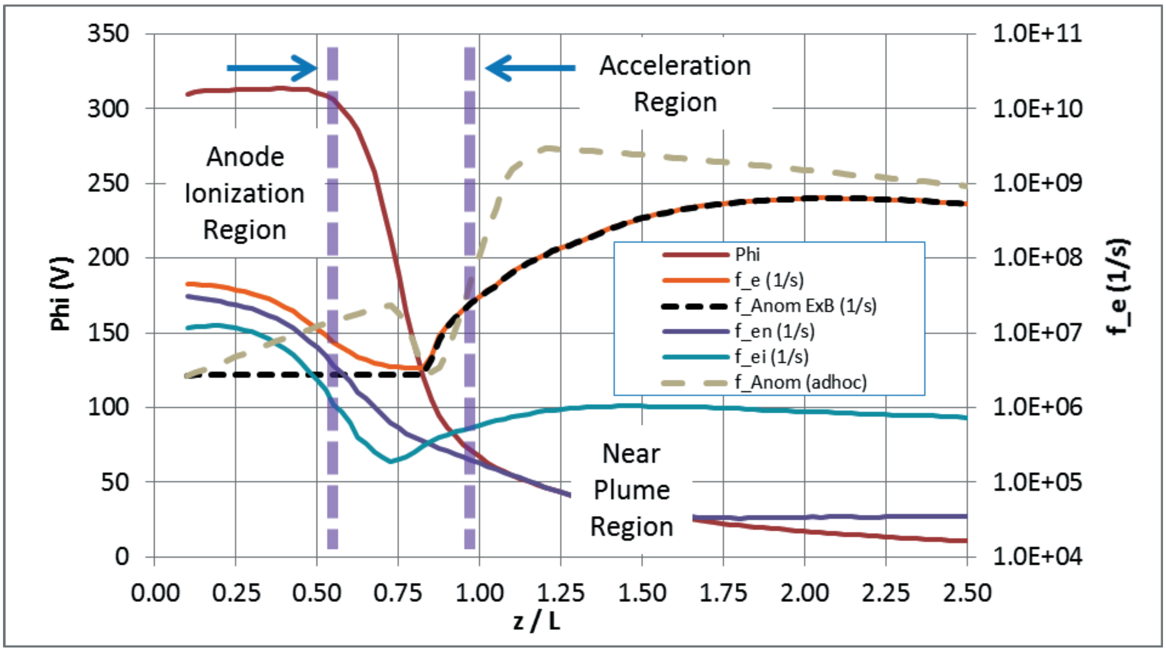

Figure 6. Collision frequencies for the self-consistent steady-state solution along the channel centerline from Hall2De simulations of the H6US thruster

Figure 7 compares the computed plasma potential and electron temperature with experimental measurements obtained using injected probes[14]. Due to now well-established evidence of probe perturbations that have shown to yield shifts in the location of the acceleration zone of several mm, the precise location of the maximum electric field and temperature are unknown. Thus, the experimental profiles in Figure 7 have been shifted axially to match the computed maxima. The Hall2De solution at the chosen wavelength (λ=0.25 mm) yields a discharge current of 20.3 A, which is 1.5% higher than the measured value.

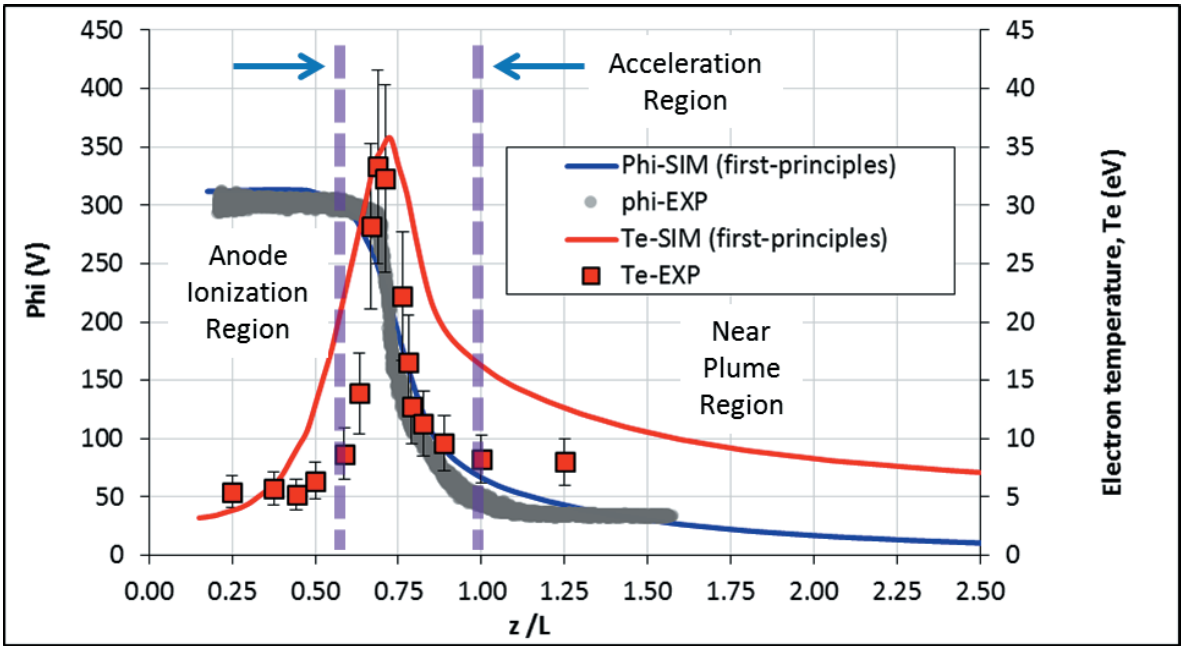

Figure 7. Potentials and electron temperatures from the self-consistent steady-state solution along the channel centerline compared with laboratory measurements on the H6US thruster. (SIM=simulation, EXP=experiment)

Two-dimensional contours comparing solutions with the ad hoc and the first-principles anomalous collision frequency models are shown in Figure 8. Though some of the finer details of the ad hoc solution are not captured, the comparison shows that the overall expected behavior of the plasma is re-produced by the first-principles model. Self-consistent simulations were also run using two other wavelengths, (λ=0.35 mm and λ=0.05 mm). All three cases produced similar results with discharge currents varying by about ten percent. The acceleration region potential was steeper and the electron temperature peaked higher and further upstream for the shorter wavelengths.

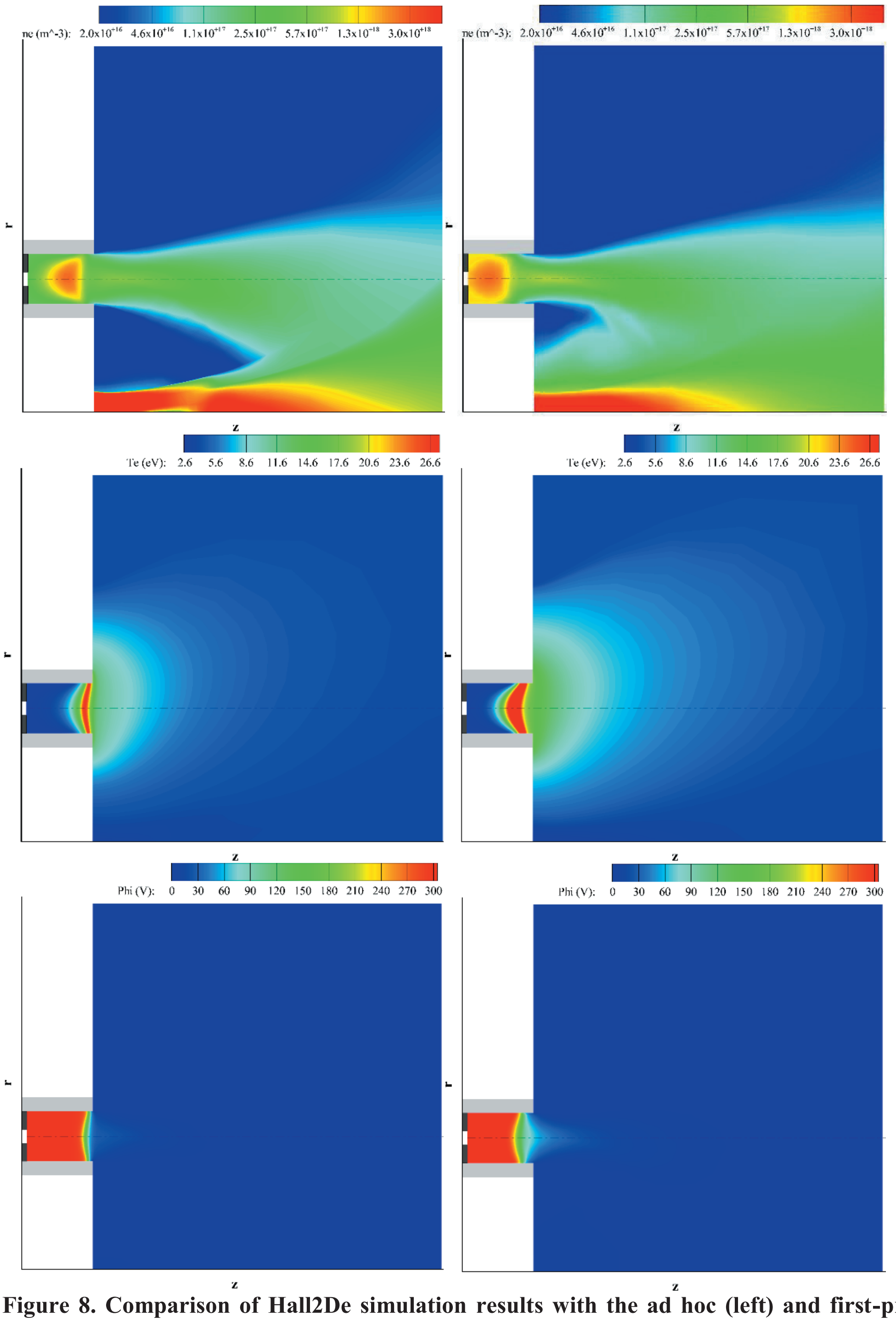

Figure 8. Comparison of Hall2De simulation results with the ad hoc (left) and first-principles (right) anomalous collision frequency models. Top: Electron number density. Middle: Electron temperature. Bottom: Plasma potential.

IV. Conclusion

The results presented here are from a first attempt at incorporating in the 2-D, r-z, Hall2De code a first principles self-consistent formulation for “anomalous” scattering based on ion acoustic waves generated in the Acceleration Region by the electron cyclotron drift instability. We presented arguments that show why this instability is the source of the “anomalous” scattering used in our Hall2De simulations. First, the growth rate for the electron cyclotron drift instability peaks in the Acceleration Region, as does the growth rate of the ad hoc “anomalous” scattering used in Hall2De. Second, the ion sound waves generated by the instability have their dominant wave vector in the azimuthal direction, the direction necessary for scattering that moves the electron guiding across the magnetic field. Third, ion sound waves generated in the Acceleration Region into the near plume grow and convect downstream into the Near Plume, the same direction of ad hoc “anomalous” scattering frequency growth. Finally, the scattering frequency of the saturated waves in the Near Plume region is high enough to explain the observed small electric fields in that region.

In order to implement a self-consistent scattering model based on ion acoustic waves generated by the electron cyclotron drift instability. in the 2-D r -z Hall2De code a large number of simplifications were made. While we demonstrated that the conditions for the electron cyclotron drift instability are satisfied in the Acceleration Region, in this first implementation we actually used the growth rate for ordinary, non-magnetized, ion acoustic waves, justified by published observations[13]. The code only solved the continuity equation for the wave action, assuming the wave action was convected by the fluid ion motion, neglecting the propagation of ion sound. This approximation is good when the ion velocity is much greater than the ion acoustic speed, e.g. in the Near Plume region, but it fails throughout the Anode Ionization region and in the upstream portion of the Acceleration region. We think that the failure of this approximation is why we found it necessary to spatially fix the location where waves start to grow. As a result, this first attempt, while producing extremely promising results, failed to accomplish one of our major goals, having the code predict the location of the acceleration region. Additionally, all of the basic theory is for a uniform, homogeneous plasma, conditions far from those found in a Hall thruster Acceleration region. We plan to address these weaknesses in future work.

Acknowledgments

The research described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. The support of NASA’s Space Technology Mission Directorate through the Solar Electric Propulsion Technology Demonstration Mission (SEP TDM) project is gratefully acknowledged.

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