

Time-Synchronized Laser Induced Fluorescence Measurement of Xenon Ion and Neutral Dynamics in a 350 W Hall Thruster

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The paper presents spatially and time-resolved laser induced fluorescence measurements of the ion and neutral velocity distribution functions in a 350 W laboratory Hall thruster during natural breathing mode oscillations at 23 kHz. Fluctuations in measured axial ion velocity throughout the discharge current cycle are observed at five spatial locations and a transient low velocity ion population arises 15 mm downstream into the plume during the period of minimum discharge current. Time-resolved laser induced fluorescence measurements of excited xenon neutrals reveal that, at the exit plane, the probed state populates and de-populates out of phase with the discharge current oscillation.

Nomenclature

LIF	= Laser induced fluorescence
IVDF	= Ion velocity distribution function
PMT	= Photomultiplier tube

I. Introduction

HALL thrusters¹ exhibit a multitude of coherent and incoherent plasma structures and instabilities when operated under certain conditions.² Perhaps the most ubiquitous is the breathing mode, an ionization instability that manifests itself as strong 10–30 kHz oscillations in the thruster discharge current.³ In order to resolve changing plasma properties and fundamental physical processes occurring at these time scales, time-resolved experimental methods are required. Non-intrusive laser induced fluorescence (LIF) measurements offer a powerful means for obtaining detailed, spatially-dependent information about ion and neutral dynamics in plasma systems.⁴ The velocity of the probed species can be determined from the Doppler shift of the excitation fluorescence spectrum in comparison with a stationary reference.

Different techniques have been developed in recent years to accomplish time-resolved LIF measurements in forced or naturally oscillating plasmas. Biloiu et al.⁵ developed a method to interrogate pulsed plasma sources in argon; Vaudolon et al.⁶ demonstrated a photon counting technique using a Hall thruster with externally stabilized oscillations; and MacDonald et al.⁷ demonstrated a sample-and-hold technique to investigate quasi-periodic plasmas, presenting measurements of 3 kHz natural oscillations in a Diverging Cusped Field Thruster. The same sample-and-hold technique was applied by Young et al.⁸ to capture ion dynamics in an unforced Hall thruster synchronized with 20 kHz natural breathing mode oscillations. Durot et al.⁹ demonstrated a transfer function averaging technique applied to a hollow cathode discharge, forcing periodic current

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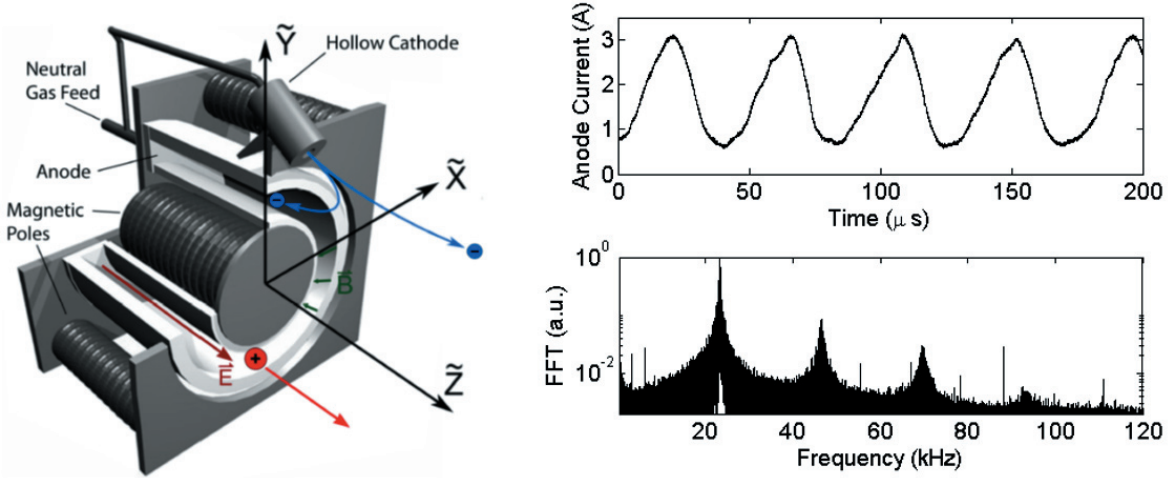


Figure 1. Schematic of the Hall thruster used in this study. The channel centerline is located at $x = 28.5$ mm from the thruster axis. The panel shows the anode current during thruster operation in the breathing mode regime. An FFT of the current reveals a dominant frequency of 23 kHz.

oscillations at 10 kHz. Finally, Diallo et al.¹⁰ proposed a heterodyne technique based on Fourier decomposition of the periodic LIF signal, applying the method to a Cylindrical Hall Thruster with forced oscillations at 12 kHz.

In this work, a continuous wave laser induced fluorescence sample-hold technique is synchronized with the 23 kHz breathing mode of a Hall thruster operating on xenon in the 350–450 W power range, providing time-varying ion and neutral velocity data throughout the discharge current oscillation. This method employs a scanning external cavity diode laser modulated at a characteristic frequency such that a lock-in amplifier can discriminate the induced fluorescence signal from the bright background emission using homodyne detection. Time-resolved ion velocity measurements on the same thruster were reported in a previous work.⁸ This paper presents plasma dynamics recorded at a new thruster operating condition with enhanced time resolution throughout the breathing mode cycle. Measurements are extended to new spatial locations and neutral dynamics are investigated along with ions to provide additional insight into the overall time-varying characteristics of the thruster plume.

II. Experimental Setup

Experiments are conducted in the Stanford Plasma Physics Laboratory large vacuum facility, which has been described in detail elsewhere.¹¹ The 4 m long, 1.25 m diameter stainless steel vacuum chamber is equipped with a two-stage cryogenic pumping system: two shrouds cooled with a Polycold Fast Cycle Water Vapor Cryopump (PFC, Model 1100) followed by two cryopanel cooled to 16 K with cryogenic helium. The system provides a base pressure of 5×10^{-7} torr and maintains 5×10^{-5} torr (uncorrected for xenon) while operating the thruster.

Figure 1 shows a schematic of the Hall thruster used in this work. The annular channel outer diameter spans 72 mm, with an inner diameter of 42 mm and a depth of 23 mm. The thruster is operated at an average power of 408 W (240 V anode voltage, 1.7 A anode current) with a xenon mass flow rate of 1.97 mg/s. The external barium oxide IonTech cathode is located 2 cm downstream and 7 cm radially outwards from the thruster main axis, pointing at about 45° , and operates on $150 \mu\text{g/s}$ of argon. The radial magnetic field strength at the exit plane along the channel centerline is ~ 160 G. Natural breathing mode oscillations at 23 kHz are observed at these conditions, as shown in the right panel of Fig. 1. The thruster is mounted on a two-axis horizontal motorized stage for obtaining spatially resolved measurements.

The optical setup for performing time-synchronized laser induced fluorescence velocimetry is identical to that used previously.^{8,11} For measuring ion velocities, the $5d[4]_{7/2} - 6p[3]_{5/2}$ Xe II transition (834.953 nm vacuum) is pumped with ≈ 40 mW of diode laser power and the resulting fluorescence from the $6s[2]_{3/2}$

– $6p[3]_{5/2}$ transition at 542.1 nm (vacuum) is collected through a Hamamatsu 1P21 photomultiplier tube (PMT) for determining the Doppler shift of the absorbed radiation. The exciting laser beam is generated by a New Focus TA-7600 semiconductor tapered amplifier seeded by a 20 mW tunable external-cavity diode laser (New Focus, Model TLB-6817-P) through a polarization-maintaining optical fiber. For xenon neutrals we apply a resonant LIF scheme using the $\text{Xe I } 6s[3/2]_{2}^{\circ} - 6p[3/2]_{2}$ transition (823.390 nm vacuum) optically pumped by a 13 mW external cavity diode laser (New Focus, Model 6017). The induced fluorescence photons originating from the same transition are collected through a Hamamatsu R3896 PMT.

In both cases, beam diagnostics accurately reconstruct the laser wavelength scan as a function of time. A Fabry-Perot interferometer provides fixed frequency markers ($\text{FSR} = 1.5 \text{ GHz}$) throughout the scan and a stationary optogalvanic reference signal from the $6p'[3/2]_{1} - 8s'[3/2]_{1}$ Xe I transition (834.973 nm vacuum) is obtained (for ions) from a hollow cathode lamp. This line is spaced 9.03 GHz from the target stationary ion transition. For neutral LIF, the 823.390 nm line provides its own stationary optogalvanic reference. An additional Burleigh WA-1500 wavemeter provides a visual wavelength reading while tuning the laser parameters. The collected light (background emission plus induced fluorescence) is passed through optical shortpass and (10 nm) bandpass filters before PMT detection to reject photons originating from other transitions. A 2 mm mechanical iris reduces the light collection volume and defines the $\sim 1 \text{ mm}^3$ spatial resolution of the measurement.

We track the time evolution of the ion and neutral velocity distribution functions at several points throughout the thruster plume using a sample-and-hold approach. An acquisition gate is generated at a fixed phase of the oscillating period, in which the PMT signal is sampled, averaged, and held until updating in the following gate one oscillation period later. A lock-in amplifier then extracts only the fluorescence signal originating from the target phase of the current, producing a velocity distribution function representative of the instantaneous plasma conditions during that short time interval. Further details of the method and the hardware used for its implementation are described separately,¹² as is a parallelized high-efficiency version of the system.¹³

III. Results and Discussion

A. Time-Resolved Ion Velocity Measurements

We track the time-varying ion velocity distribution functions downstream of the exit plane ($z = 0$) at axial positions $z = 5, 15$, and 20 mm along the channel centerline ($x = 28.5 \text{ mm}$). At $z = 15 \text{ mm}$ we also move off-center in either direction, radially inwards to $x = 23.5$ and radially outwards to $x = 33.5 \text{ mm}$. We use a 2 μs acquisition gate spaced in 2–2.5 μs increments throughout the $\sim 43 \mu\text{s}$ breathing mode oscillation period.

The measured time evolution of the ion fluorescence excitation spectra throughout the breathing mode oscillation are summarized in Figs. 2 and 3. Figure 2 shows how the time-dependent IVDF changes spatially in the axial direction along the channel centerline, and Fig. 3 illustrates the time evolution of the IVDF in the radial direction at $z = 15 \text{ mm}$. Strong changes in ion velocity are observed over the oscillation period. In the contour plots on the right side of the figures, each LIF trace at a given instant is normalized relative to its maximum intensity (locally in time) to highlight the observed structures in velocity space. The left half of the figures show the most probable ion velocity as a function of time within a breathing cycle (defined by the peak of the acquired fluorescence lineshape).

All of the interrogated locations show a higher ion velocity in the current trough between consecutive discharge current peaks. The velocities attain their lowest values during the ramp up in discharge current, after which ions are accelerated to near their maximum velocities as the current ramps down. At $z = 5 \text{ mm}$ the velocity doubles from 7.5 km/s to 15.4 km/s, indicating strong changes in the local accelerating potential structure over the breathing mode cycle. This behavior is also seen downstream, where velocities at $z = 15 \text{ mm}$ and $z = 20 \text{ mm}$ fluctuate up to 38% and 34%, respectively, about their median values.

As expected, ion acceleration is observed in the axial direction moving downstream of the thruster. The increase in velocity between $z = 5 \text{ mm}$ and $z = 15 \text{ mm}$ demonstrates that the acceleration front extends outside the thruster channel. The acceleration in this region is most pronounced in the first half of the discharge current cycle, in which most probable ion velocities rise from 8 km/s to 12 km/s. After the discharge current peak additional ion acceleration between $z = 5 \text{ mm}$ and $z = 20 \text{ mm}$ is minimal, indicating the bulk of the accelerating potential drop has moved upstream. Ions slow substantially at $z = 5 \text{ mm}$ as the current builds back up from the trough, presumably due to the main potential gradient returning downstream. At $z = 15 \text{ mm}$ the ion acceleration is almost fully developed throughout the entire cycle, and

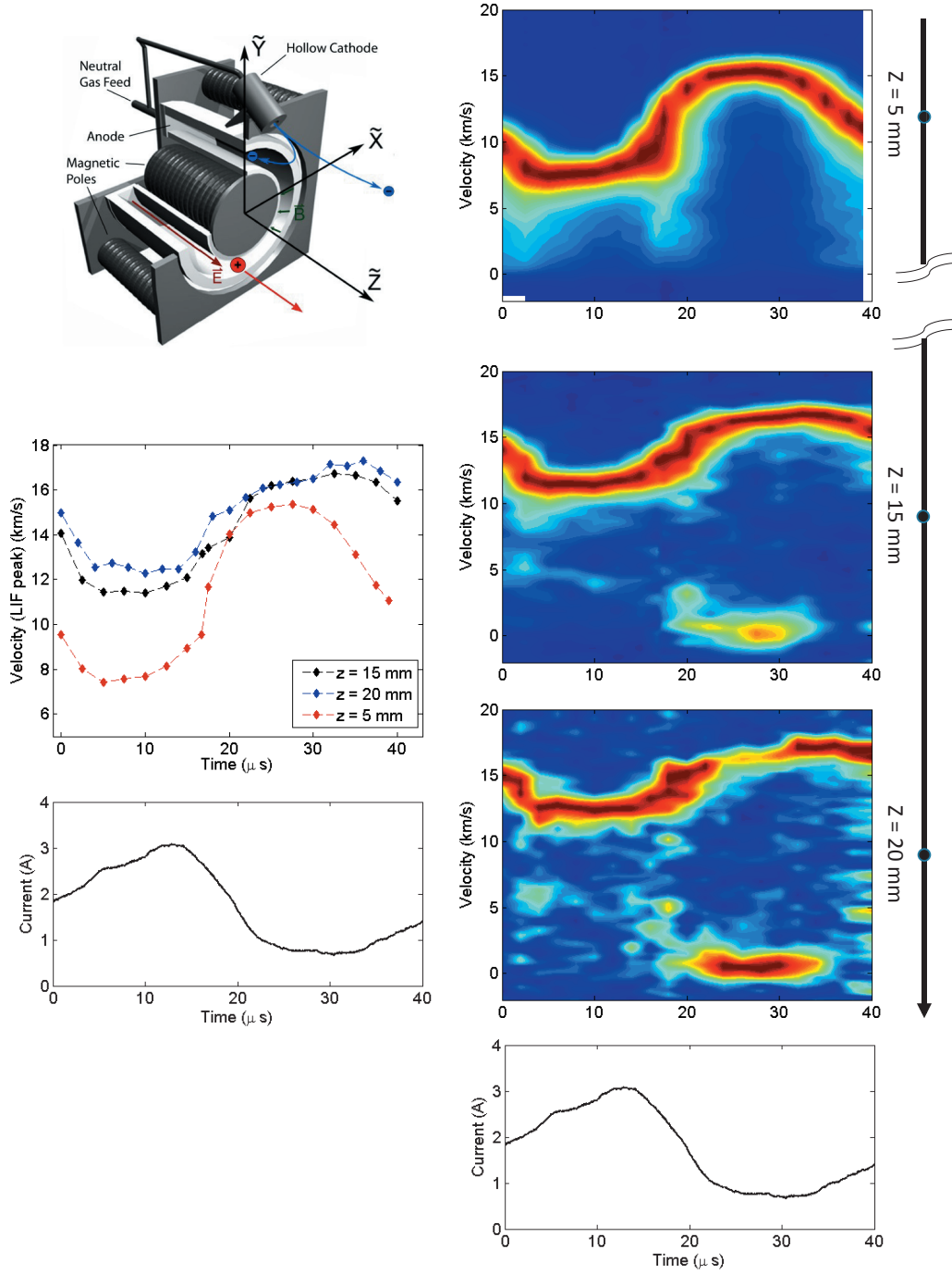


Figure 2. (Left) Time histories of the most probable ion velocities (defined as the peak in the measured xenon ion LIF lineshape) over a breathing mode cycle for three different locations along the centerline. (Right) Contour maps of the acquired lineshapes; a single laser scan constitutes a vertical strip at a given time delay. Each trace is normalized by its maximum to show structure despite variations in signal intensity (see Fig. 4a).

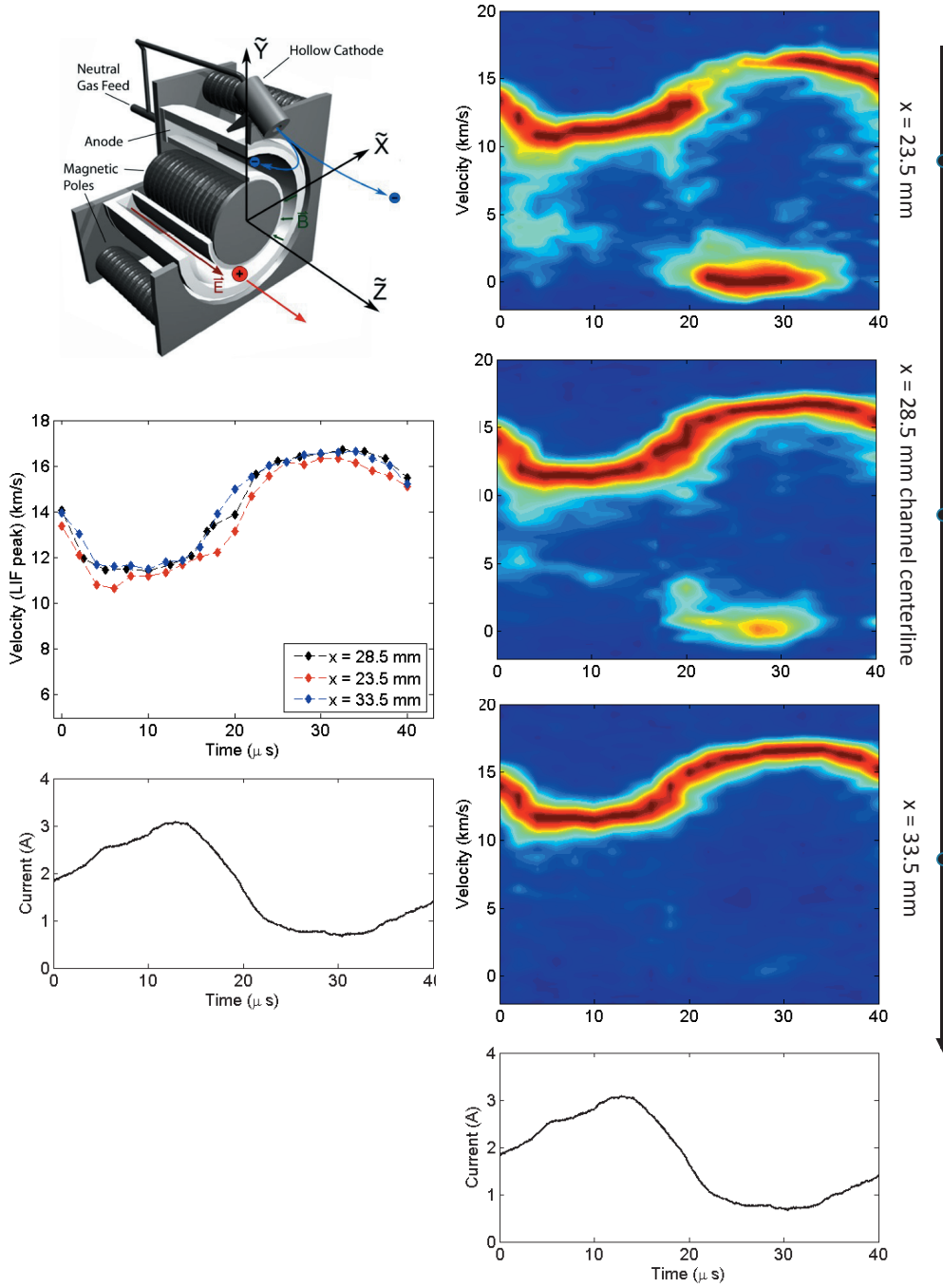


Figure 3. (Left) Time histories of the most probable ion velocities (defined as the peak in the measured xenon ion LIF lineshape) over a breathing mode cycle for three different radial locations 15 mm downstream of the thruster exit plane. (Right) Contour maps of the acquired lineshapes; a single laser scan constitutes a vertical strip at a given time delay. Each trace is normalized by its maximum to show structure despite variations in signal intensity (see Fig. 4a).

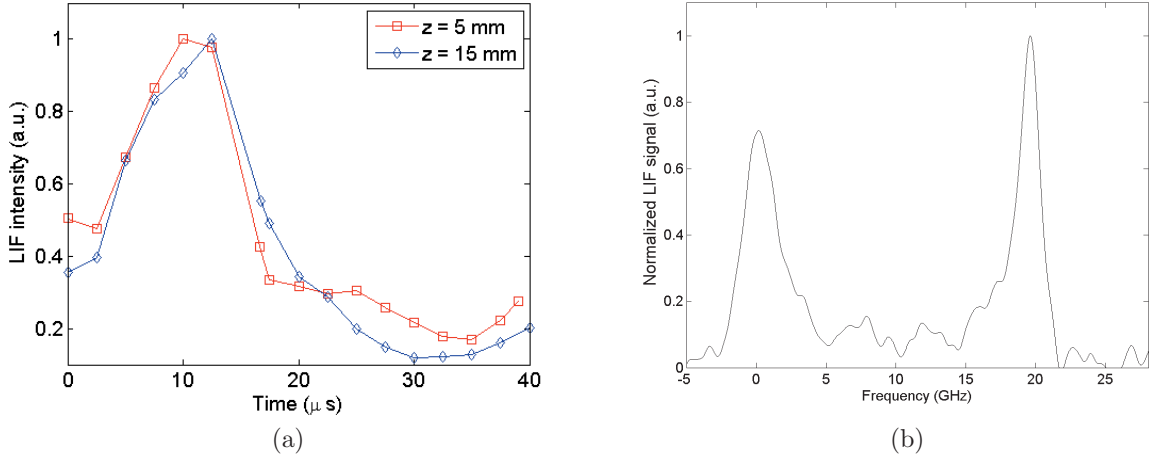


Figure 4. (a) Time history of the fluorescence signal intensity normalized to its maximum value at two axial locations on the channel centerline. (b) Time-resolved normalized LIF trace as a function of laser detuning frequency from the stationary ion transition at $z = 15$ mm at $t = 27.5 \mu\text{s}$. (See Fig. 2 for the corresponding phase of the current cycle.) The profile highlights the presence of a slow ion population in addition to the main accelerated one.

only a small additional increase in velocity is observed by $z = 20$ mm.

Measured ion velocity profiles of the main accelerated ion population do not significantly differ between the three radial points investigated at $z = 15$ mm. However, strong changes in the IVDF at low velocities are observed as multiple ion populations arise at some spatial locations. A low velocity structure in the IVDF appears at $z = 15$ mm on the channel centerline (after being noticeably absent at $z = 5$ mm) while the current is minimum. An example LIF trace from this location taken at $27.5 \mu\text{s}$ is shown in Fig. 4b. The intensity of this structure increases further downstream at $z = 20$ mm, where it temporarily overtakes that of the main accelerated population. Moving radially at $z = 15$ mm, we observe that the low ion velocity population appears stronger towards the thruster axis (see panel at $x = 23.5$ mm in Fig. 3) and is hardly visible above the level of the noise away from the axis ($x = 33.5$).

The sporadic appearance of this low velocity ion class over a time interval of only $10 \mu\text{s}$ suggests its origin is tied to a temporally and spatially-dependent physical process in the thruster. Low ion velocity populations have appeared previously in time-averaged LIF measurements of Hall thruster plumes^{14,15} and different hypotheses explaining their presence have been suggested, including residual ionization outside the thruster channel, charge-exchange collisions with neutrals, and geometrical effects attributed to the divergent plume originating from the opposite side of the annular channel. External ionization is a possible option, and additional studies are needed to justify the spatial and temporal dependence of the feature. The possibility of charge-exchange collisions is investigated in the following section, seeking evidence of correspondingly high velocity neutrals using the same time-resolved LIF approach. Geometrical effects would imply a very large divergence angle (~ 70 – 72°) for a short period of time, but primarily radially-directed ions leaving the center of the opposite channel at 15 km/s would arrive at the measurement locations $\sim 3.5 \mu\text{s}$ later, within the time frame to be recorded here as a low axial velocity population of ions. The observed axial velocities are too slow to traverse 15 – 20 mm downstream in this amount of time, however, a fact that may point towards residual ionization in the plume instead.

Figure 4a shows the time dependence of the excitation fluorescence intensity (peak value) at two different axial locations on the centerline. Peak LIF signal intensity (proportional to the probed excited state density of the most probable ion velocity class) follows the general shape of the current trace quite closely; signal intensity increases during the upward ramp in discharge current, peaks around the current maximum, and falls off substantially in the trough between current peaks. This behavior may be interpreted using standard models of the breathing mode in which an ionization front propagates back and forth inside the discharge channel.^{3,16} Electrons trapped in azimuthal closed drifts in the high magnetic field region near the exit plane contribute to a baseline level of propellant ionization. As the current ramps up, a propagating ionization front moves upstream, consuming neutral propellant and increasing the density (and subsequent fluorescence

intensity) of ions and free electrons. The front moves back downstream as the discharge current falls to low values, and the ion density and LIF intensity necessarily decrease as the ions accelerate out of the channel. Note that while the relationship between the probed metastable state and ground state ion densities depends on the complicated time-dependent energy flow between excited states, we assume that the ground state ion density follows the same trends described here. The fact that the signal intensity and discharge current are in phase supports this claim.

B. Time-Resolved Neutral Velocity Measurements

This section describes time-resolved measurements of the neutral velocity distribution function in the Hall thruster plume. The operating conditions are the same as used with ions. We interrogate two different spatial points along the channel centerline ($x = 28.5$ mm), one at the exit plane, $z = 0$ mm, and one downstream at $z = 15$ mm. Time-resolved measurements of the ion velocity distribution function at the latter point (Fig. 2) show the first appearance of the low velocity ion population. This structure appears over a narrow $10 \mu\text{s}$ window in time, and the presence of a high velocity neutral population in the same location during the same time interval would be symptomatic of charge-exchange collisions.

The upper contour plot in Fig. 5 shows the time evolution of the neutral velocity at the thruster exit plane during a discharge current cycle. We measure a time-averaged peak velocity of ~ 350 m/s, and the velocity profiles at this location do not seem to change appreciably over time. An interesting feature is observed in the excitation fluorescence intensity profile, shown at the upper right, in which the intensity fluctuation leads that of the current by about a quarter oscillation period (90° phase lead). The LIF intensity peaks about $10 \mu\text{s}$ before the current maximum, while the current is still ramping up. As mentioned above, the fluorescence signal intensity is representative of the lower state density of the probed transition, whose time evolution results from a combination of several processes, including electron impact ionization and excitation, radiative decay, heavy particle collisions and local electron and ion transport mechanisms. These various processes interact in complex ways, but a possible reason behind the observed decrease in fluorescence intensity when the discharge current peaks is excitation of the lower probed neutral state to higher excited levels and the continuum.

This hypothesis is supported by the emission data shown in the top right plot of Fig. 5, collected with the photomultiplier tube from the same 823 nm transition (recall that a resonant LIF scheme is used for neutrals). Processing the PMT signal through a lock-in amplifier discriminates laser induced fluorescence photons from the overall background emission, yielding information about the lower state density in the localized LIF measurement volume. The raw PMT signal, however, contains the intensity of the prevalent background emission, providing information about the upper state population (over the entire optical collection path). As shown in the figure, the emission intensity closely tracks the current profile. We also know that metastable ion density increases along with the current from Fig. 4a, together indicating an energy flow towards higher xenon neutral excited states and the continuum (ionization) when the current is high. This view is supported by the observed corresponding decrease in population density of the lower neutral probed state during this time.

The lower contour plot in Fig. 5 shows the time-dependent neutral velocity over a discharge current cycle at a location $z = 15$ mm downstream of the exit plane along the channel centerline. The measured time-averaged neutral velocity is ~ 400 m/s, and again we do not see any coherent evolution of the velocity profile throughout the breathing mode cycle. The fluorescence intensity profile (shown at the lower right) also does not show any distinct behavior, with only a slight decrease observed moving from the high current to low current interval. This point is investigated mainly for evidence of fast neutral atoms produced by possible charge-exchange collisions with accelerated ions. The LIF spectrum for velocities above 2 km/s does not show any signs of additional populations, and consequently we cannot report any evidence for charge-exchange in this study.

Charge-exchange collisions cannot be completely excluded as a possible mechanism behind the observed low ion velocity population, since the signal intensity originating from the presumed high neutral velocity population may be significantly weaker than the baseline neutral signal. The resonant neutral LIF scheme used in this study is much stronger than that used for ions, and it is possible that strong background emission, persisting even through the current trough, overshadows the signal originating from any additional low density populations.

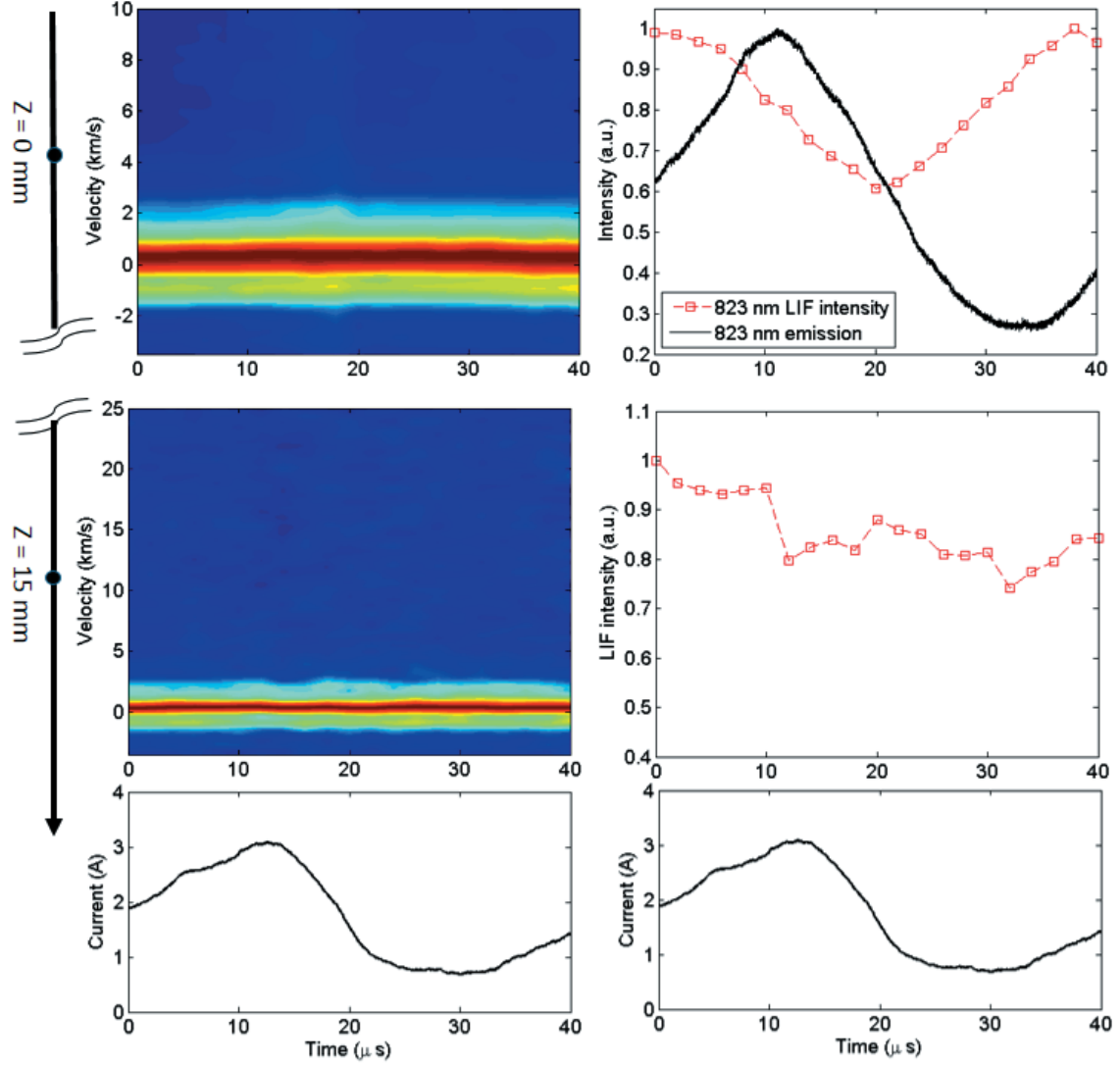


Figure 5. (Left) Contour maps of measured xenon neutral resonant fluorescence lineshapes at the exit plane ($z = 0$) and $z = 15$ mm downstream along the channel centerline. A single laser scan forms a vertical strip at a given time delay. Each trace is normalized by its maximum value to show structure despite variations in the overall signal intensity, shown at right. (Right) Normalized peak LIF signal intensity for each case, in addition to spontaneous plasma emission intensity at 823 nm detected by the PMT at $z = 0$.

IV. Summary and Future Work

This paper reports continuous wave laser induced fluorescence measurements of a 400 W laboratory Hall thruster operating on xenon, synchronized to the 23 kHz breathing mode using a sample-hold technique. Time varying ion and neutral velocity distribution functions are obtained in the near-field plume throughout the discharge current oscillation. Time series taken at several spatial points along the channel centerline show ion acceleration through a potential drop located (at least for some times) downstream of the exit plane. Fluctuations in measured axial ion velocity throughout the discharge current cycle are observed at each location, with the maxima appearing in the low current interval. Excitation fluorescence signal intensity (proportional to the probed lower excited state ion density) is greatest during the upward ramp in discharge current, peaks around the current maximum, and falls off substantially in the trough between current peaks.

The appearance of a low velocity ion population 15 mm into the plume is further investigated with time-synchronized LIF of excited xenon neutrals, seeking evidence of charge-exchange collisions. This feature is inherently spatially and temporally dependent, demonstrating the utility of this type of measurement that can resolve the velocity distribution functions in both time and space. Neutral measurements do not show any evidence of charge-exchange collisions, although we cannot completely exclude this hypothesis due to strong background emission that may overpower a weak signal originating from a low density population. Finally, time-resolved LIF measurements on xenon neutrals show that the probed excited neutral population at the exit plane populates and de-populates 90° out of phase with the discharge current. Evidence is presented suggesting this effect is associated with stronger excitation to higher energy states and ionization during the current peak.

Future work will involve further analysis of excited neutral dynamics within the thruster channel and improving the spatial resolution of the time-resolved ion velocity measurements to capture the time-dependent ion acceleration front. Radial ion LIF measurements will complete the physical picture of the dynamic near-field plume region, showing how the total ion velocity vectors change with time throughout the breathing mode cycle.

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