Simulation of Temperature Deformation of Ion Thruster Electrodes

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Abstract: An ion extraction system (IES) is the most loaded structural element of ion thrusters (IT). The IES electrodes have the form of thin-walled spherical perforated shells. The basic load on them is caused by their non-uniform heating along radius that causing the appearance of internal stresses and deformations. It is necessary to take into account the material and shape of electrodes, as well as their fixation method when designing the IES. These factors influence the electrode deformation, which should not exceed rigid limits specified by the IES design. New advanced constructive metallic materials, such as titanium-niobium alloys, annealed molybdenum of high purity, are used for the ion extraction systems of modern IT. There is a trend to use electrodes of carbon - carbon composite materials. Electrodes made of different materials may be simultaneously used in the IES, too. It follows from the above that it is necessary to develop the IES thermomechanical model, which will allow designer to make preliminary estimation, calculate stresses and heat changes for electrodes made of different structural materials. The presented model is based on the theory of thin-walled shells.

Nomenclature

ρ_a = dimensionless radius of inner contour
k_0 = auxiliary dimensionless parameter
λ_3 = dimensionless loading parameter
w = additional bending of electrode
w_0 = initial (technological) electrode bending
u = radial displacement of spherical panel
k_r = coefficient of constructive orthotropy in the cylindrical section
k_φ = coefficient of constructive orthotropy in meridian section
h = thickness of spherical panel
b = radius of external contour of spherical panel
ρ = dimensionless current radius of spherical panel
ξ = dimensionless additional bending
ϑ = dimensionless initial (technological) bending
$\lambda_{3,kp}$ = critical value of dimensionless loading parameter λ_3

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α	=	coefficient of thermal linear expansion
μ	=	Poisson coefficient
E	=	elasticity module of electrode material
ΔT_r	=	temperature gradient along the electrode radius

I. Introduction

The IES of modern IT uses profiled densely perforated electrodes shaped as the sphere segments. Such shape provides tolerance for thermal loadings and definite direction of heat deformations. The IES design is presented schematically in Fig. 1. IES comprises several perforated electrodes: emissive (EE) and accelerating (AE) ones. IES with large amount of electrodes are used also. The EE thickness is (0.3 – 0.5) mm, and that of the accelerating one is (1 – 2.5) mm. Holes in electrodes are coaxial, but the diameter of EE holes is exceeds essentially the diameter of holes in AE. The materials of electrodes may be different also. It is necessary to take into account not only the material properties and shape of the electrodes but also the features of their fastening and operating conditions caused by temperature RF electromagnetic fields [1 – 3].

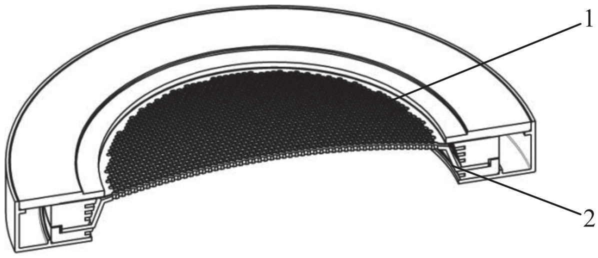

Figure 1. Ion extraction system.
1- emissive electrode; 2 - accelerating electrode.

II. Mathematical model

The electrodes are manufactured of different materials taking into account simultaneous selection of axial direction of perforated holes. Here we consider two variants of direction of orifice axes: along the normal to a surface of profiled part of electrodes and along the vertical axis of electrodes. The continuum thermomechanical model [4, 5] proved itself well for mathematical description of rigidity parameters of the IES perforated electrodes. It is based on the representation of emissive, accelerating and decelerating electrodes as structurally orthotropic flat thin-walled spherical panels [6]. The non-uniform heating of electrodes along radius and through thickness was considered as the main loading factor in general case. Consequently, the factors of electrode filling with materials, which are simultaneously the coefficients of structural orthotropy in the considered model, were directly taken into account in the initial equations of thermoelasticity [7].

The relations between geometric sizes of circular holes regularly disposed according to a triangular scheme, and factors of electrodes filling with materials were calculated by ad hoc algorithm, which has simultaneously enabled correct accounting for not only the approbated active force loading of electrodes, but also the thermal action on IES as a whole. A diagram of dependence of structural orthotropy coefficients on the ratio of perforation hole diameter to the distance between holes became a result of adoption of such thermomechanical model for electrodes [8, 9].

For immediate calculation of deformed state of spherical electrodes at their non-uniform heating along radius we used the mathematical apparatus of matrix boundary integrated and integrodifferential equations represented in the convenient dimensionless recording format [10, 11]. Solution of these equations obtained in the geometrically nonlinear statement of problem on the bending of spherical densely perforated panel, can be effectively based on iterative methods of calculation in combination with the process of step-wise temperature loading of considered electrodes. It is reasonable to use the described above algorithm for solving the obtained integral equations at the stage of simulation and at the crucial stages of confirmatory tension and stiffness analysis. It is more efficient to make preliminary calculations by simplified algorithm while conducting research multivariate retrieval calculations.

It is the first approximation of iterative solving of respective integral equations not in the numerical, but in the analytical form. The obtained resulting nonlinear algebraic cubic equation of the third degree relative to the dimensionless additional bending on the inner contour of electrode or in its central point is solved rather well by the mathematical application software package. Multivariate study of the deformation process was made based on models RIT-16 and RIT-45 in accordance with the described above new simplified algorithm for the computation of additional bendings of spherical panels with low degree of curvature representing the above emissive electrodes manufactured of the common molybdenum alloy VM-2, new titanium-niobium alloy TV-36, as well as of the carbon-carbon composite material CCCM. The molybdenum alloy VM-2 and carbon-carbon composite material CCCM were considered for the accelerating electrode.

For increasing the efficiency and complexity of calculations, the resulting algebraic cubic equation was represented in the following dimensionless form:

$$\xi^3(\rho_a)+3\cdot\vartheta(\rho_a)\cdot\xi^2(\rho_a)+\left[2\cdot\vartheta^2(\rho_a)-\frac{\lambda_3-\lambda_{3,kp}}{k_o}\right]\xi(\rho_a)-\lambda_3\cdot\frac{\vartheta(\rho_a)}{k_o}=0 \qquad (1)$$

Where:
$$k_0 = 12\cdot(1-\mu^2\cdot k_r\cdot k_\varphi) \qquad (2)$$

$$\lambda_3 = k_o\cdot\alpha\cdot\Delta T_r\cdot\frac{b^2}{h^2} \qquad (3)$$

$$\rho=\frac{r}{b};\ \xi=\frac{w}{h};\ \vartheta=\frac{w_o}{h} \qquad (4)$$

Temperature fields of electrodes are written as follows:

$$T(\rho)=T(1)+\Delta T_r\cdot f_{T0}(\rho) \qquad (5)$$

As of fundamental law of temperature variation along the electrode radius there was chosen a simple quadratic dependence of the following form:

$$f_{T0}=1-\rho^2 \qquad (6)$$

The obtained integral equations in the dimensionless form made it possible to introduce some additional similarity criteria, which make up the complexes of dimensionless parameters of geometry, rigidity, loading, boundary conditions and so on, by which the transition to calculation of IES certain electrode is carried out.

The introduction to consideration of the inner contour of electrodes, formed by the central hole of perforation, is determined by constant need for the evaluation of indeterminate forms in the singular point at the axis of spherical segment by taking central hole into consideration as the second contour of the electrode. If in some problems such singular point does not exhibit itself in an explicit form, it is possible to omit consideration of such conditional inner contour. For any method of electrode fixation in IES, it is necessary to calculate critical values of temperature gradient along the electrode radius when conducting the entire complex of calculations for buckling and bending in linear and nonlinear statements. This will enable to define in which scope of loads the spherical electrode is, because when solving the algebraic cubic equations it is necessary to forecast initial values of the sought variable. Assignment of such initial value requires a priori information about approximate localization of the solvable algebraic equation root.

When calculating dimensionless additional bending on the inner contour of electrode $\xi(\rho_a)$ or in its geometric center $\xi(0)$, the selection of these initial values is simplified due to the small local differences in the input data used. Computation results for additional bendings of electrodes $\Delta w(0)$ are presented in tables 1 - 7. Calculations of nonlinear deformation of emissive and accelerating electrodes in the central point ($\rho_a=0$) were made based on the considered variants of input data. For the correct comparison of results obtained, all considered electrodes, which were loosely connected at the external contour in relation to the radial movements and angular turns, were heated nonuniformly along the radius with the temperature distribution according to the parabolic law.

The following parameters were varied in calculations:

- The electrode materials (molybdenum VM-2 (Rus.), as well as the carbon-carbon composite material were used for emissive and accelerating electrodes; the titanium alloy with niobium of TV-36 (Rus.) grade was used for emissive electrodes);
- Radiuses of external contours of emissive and accelerating electrodes;
- Temperature at the external contour of electrodes in combination with the temperature gradient along radius;
- Three variants of electrodes thicknesses specified for subsequent analysis;
- Geometric dimensions of perforation hole diameter and distances between them;
- Values of factors of electrodes filling with material;
- Elastic modulus of applied materials;

- Coefficient of thermal linear expansion of materials;
- Poisson coefficient;
- Critical values of dimensionless load parameter $\lambda_{3,kp}$.

For the convenience of identification and analysis of obtained calculation results for the sought additional bendings $\Delta w(0)$, taking place in the poles of spherical segments, there have been introduced certain numbers for each electrode in combination with respective numbers of the items (lines) in final tables.

Calculation Number	h, mm	$w_0(0)$, mm	Δw(0), mm
1	0. 3	5. 0	1. 529
2	0. 3	10. 0	0. 848
3	0. 3	15. 0	0. 578
4	0. 3	20. 0	0. 437
5	0. 3	25. 0	0. 351
6	0. 3	30. 0	0. 293
7	0. 4	5. 0	1. 525
8	0. 4	10. 0	0. 847
9	0. 4	15. 0	0. 578
10	0. 4	20. 0	0. 437
11	0. 4	25. 0	0. 351
12	0. 4	30. 0	0. 293
13	0. 5	5. 0	1. 521
14	0. 5	10. 0	0. 846
15	0. 5	15. 0	0. 578
16	0. 5	20. 0	0. 437
17	0. 5	25. 0	0. 351
18	0. 5	30. 0	0. 293

Table 1. Input data and results of calculation of additional bendings of emissive spherical electrodes of 450 mm in diameter made of Molybdenum alloy VM-2 (Rus.).
At b = 247 mm; d_o = 4.0 mm; S_o = 5.0 mm; d_o/S_o = 0.8; k_r = 0.3; $E = 32{,}5 \cdot 10^4\ MPa$;
T(1)=330 C° ; $\Delta T_r = 50\ C^\circ$; $\lambda_{3,kp} = 35{,}5$; $\alpha = 5{,}8 \cdot 10^{-6}\ 1/C^\circ$; $\mu = 0{,}3$.

Calculation Number	h, mm	$w_0(0)$, mm	Δw(0), mm
1	0. 3	5. 0	0. 1111
2	0. 3	10. 0	0. 0562
3	0. 3	15. 0	0. 0376
4	0. 3	20. 0	0. 0282
5	0. 3	25. 0	0. 0226
6	0. 3	30. 0	0. 0188
7	0. 4	5. 0	0. 1106
8	0. 4	10. 0	0. 0561
9	0. 4	15. 0	0. 0375
10	0. 4	20. 0	0. 0282
11	0. 4	25. 0	0. 0226
12	0. 4	30. 0	0. 0188
13	0. 5	5. 0	0. 1106
14	0. 5	10. 0	0. 0561
15	0. 5	15. 0	0. 0375
16	0. 5	20. 0	0. 0282
17	0. 5	25. 0	0. 0226
18	0. 5	30. 0	0. 0188

Table 2. Input data and results of calculation of additional bendings of emissive spherical electrodes of 450 mm in diameter made of carbon-carbon composite material.
At b = 247 mm; d_o = 4.0 mm; S_o = 5.0 mm; d_o/S_o = 0.8; k_r = 0.3; $E = 2{,}7 \cdot 10^4\ MPa$;
T(1)=330 C° ; $\Delta T_r = 50\ C^\circ$; $\lambda_{3,kp} = 35{,}2$; $\alpha = 0{,}37 \cdot 10^{-6}\ 1/C^\circ$; $\mu = 0{,}1$.

Calculation Number	h, mm	$w_0(0)$, mm	$\Delta w(0)$, mm
1	2. 5	5. 0	0. 0718
2	2. 5	10. 0	0. 0450
3	2. 5	15. 0	0. 0315
4	2. 5	20. 0	0. 0240
5	2. 5	25. 0	0. 0194
6	2. 5	30. 0	0. 0162

Table 3. Input data and results of calculation of additional bendings of accelerating spherical electrodes of 450 mm in diameter made of carbon-carbon composite material.
At b = 247 mm; d_o = 1,3 mm; S_o = 2,55 mm; d_o/S_o = 0,51; k_r = 0.65; $E = 2{,}7 \cdot 10^4\ MPa$;
$T(1)$=272 C°; $\Delta T_r = 45\ C^{\circ}$; $\lambda_{3,kp} = 35{,}2$; $\alpha = 0{,}37 \cdot 10^{-6}\ 1/C^{\circ}$; $\mu = 0{,}1$.

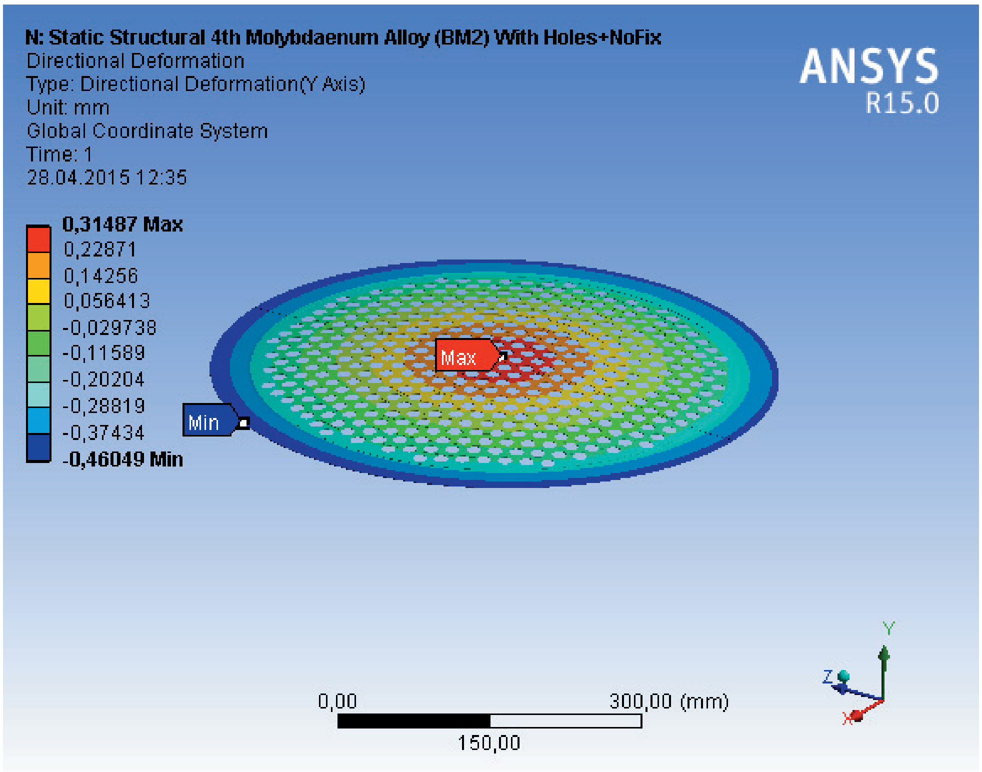

Figure 2. Simulation of directional deformation along the Y axis EE450 for free fixing.

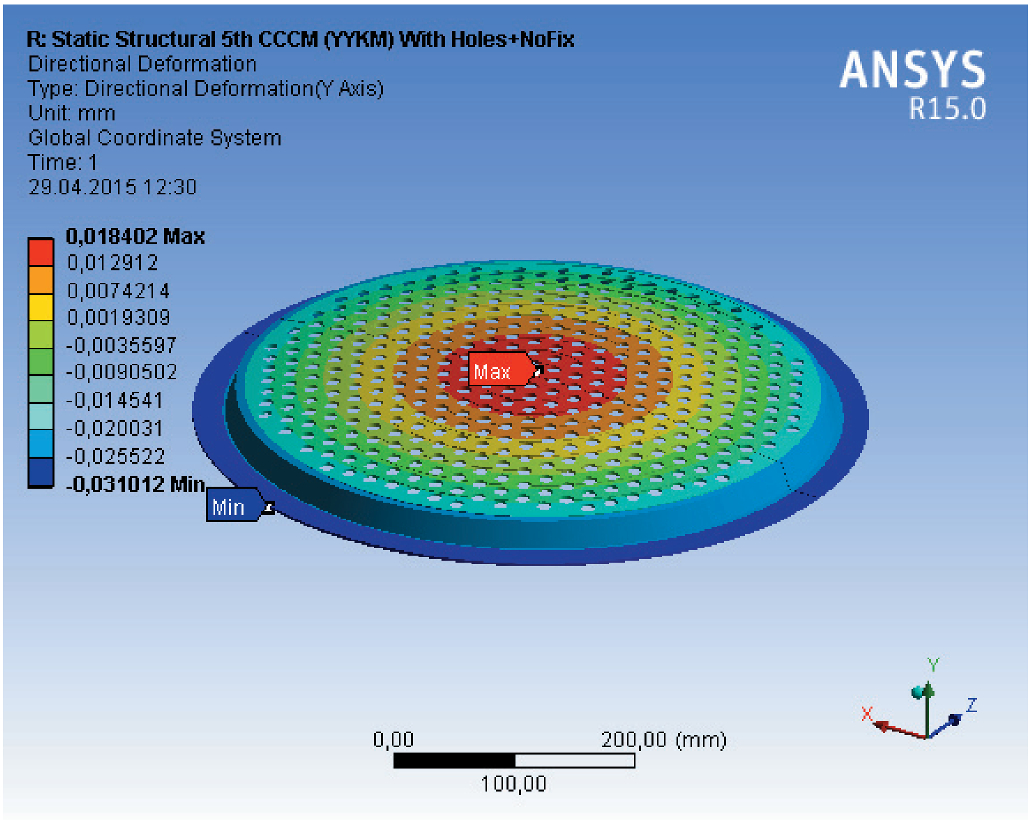

Figure 3. Simulation of directional deformation along the Y axis AE450 for free fixing.

Calculation Number	h, mm	$w_0(0)$, mm	Δw(0) mm
1	0. 3	5. 00	0. 14560
2	0. 3	7. 50	0. 09813
3	0. 3	9. 50	0. 07773
4	0. 3	12. 50	0. 05920
5	0. 3	15. 00	0. 04940
6	0. 5	5. 00	0. 14430
7	0. 5	7. 50	0. 09770
8	0. 5	9. 50	0. 07750
9	0. 5	12. 50	0. 05910
10	0. 5	15. 00	0. 04933

Table 4. Input data and results of calculation of additional bendings of emissive spherical electrodes 160 mm from Molybdenum alloy VM-2.
At $b = 80$ *mm;* $d_o = 2{,}3$ *mm;* $S_o = 2{,}9$ *mm;* $d_o/S_o = 0.793$*;* $k_r = 0.31$*;* $E = 32{,}5 \cdot 10^4\ MPa$*;*
$T(1)=330\ C^{\circ}$; $\Delta T_r = 40\ C^{\circ}$; $\lambda_{3,kp} = 35{,}2$; $\alpha = 5{,}8 \cdot 10^{-6}\ 1/C^{\circ}$; $\mu = 0{,}3$.

Calculation Number	h, mm	$w_0(0)$, mm	Δw(0), mm
1	0. 3	5. 00	0. 01272
2	0. 3	7. 50	0. 00851
3	0. 3	9. 50	0. 00673
4	0. 3	12. 50	0. 00512
5	0. 3	15. 00	0. 00426
6	0. 5	5. 00	0. 01260
7	0. 5	7. 50	0. 00847
8	0. 5	9. 50	0. 00671
9	0. 5	12. 50	0. 00511
10	0. 5	15. 00	0. 00426

Table 5. Input data and results of calculation of additional bendings of emissive spherical electrodes of 160 mm in diameter made of titanium alloy with niobium (TV-36 grade).
At $b = 80$ *mm;* $d_o = 2{,}3$ *mm;* $S_o = 2{,}9$ *mm;* $d_o/S_o = 0.793$*;* $k_r = 0.31$*;* $E = 9{,}8 \cdot 10^4\ MPa$*;*
$T(1)=330\ C^{\circ}$; $\Delta T_r = 40\ C^{\circ}$; $\lambda_{3,kp} = 35{,}2$; $\alpha = 0{,}5 \cdot 10^{-6}\ 1/C^{\circ}$; $\mu = 0{,}32$.

Calculation Number	h, mm	$w_0(0)$, mm	$\Delta w(0)$, mm
1	0. 3	5. 00	0. 009411
2	0. 3	7. 50	0. 006297
3	0. 3	9. 50	0. 004977
4	0. 3	12. 50	0. 003786
5	0. 3	15. 00	0. 003156
6	0. 5	5. 00	0. 025870
7	0. 5	7. 50	0. 017400
8	0. 5	9. 50	0. 013780
9	0. 5	12. 50	0. 010500
10	0. 5	15. 00	0. 008750

Table 6. Input data and results of calculation of additional bendings of emissive spherical electrodes of 160 mm in diameter made of carbon-carbon composite material.

At $b = 80$ *mm;* $d_o = 2{,}3$ *mm;* $S_o = 2{,}9$ *mm;* $d_o/S_o = 0{,}793$*;* $k_r = 0.31$*;* $E = 2{,}7 \cdot 10^4\ MPa$*;*
T(1)=330 C° ; $\Delta T_r = 40\ C^\circ$; $\lambda_{3,kp} = 35{,}2$; $\alpha = 0{,}37 \cdot 10^{-6}\ 1/C^\circ$; $\mu = 0{,}1$.

Calculation Number	h, mm	$w_0(0)$, mm	$\Delta w(0)$, mm
1	2. 0	5. 00	0. 11780
2	2. 0	7. 50	0. 08860
3	2. 0	9. 50	0. 07278
4	2. 0	12. 50	0. 05696
5	2. 0	15. 00	0. 04806

Table 7. Input data and results of calculation of additional bendings of accelerating spherical electrodes of 160 mm in diameter made of molybdenum alloy VM-2.

At $b = 80$ *mm;* $d_o = 1{,}6$ *mm;* $S_o = 2{,}9$ *mm;* $d_o/S_o = 0{,}55$*;* $k_r = 0{,}6$*;* $E = 32{,}5 \cdot 10^4\ MPa$*;*
T(1)=300 C° ; $\Delta T_r = 40\ C^\circ$; $\lambda_{3,kp} = 37{,}0$; $\alpha = 5{,}8 \cdot 10^{-6}\ 1/C^\circ$; $\mu = 0{,}3$.

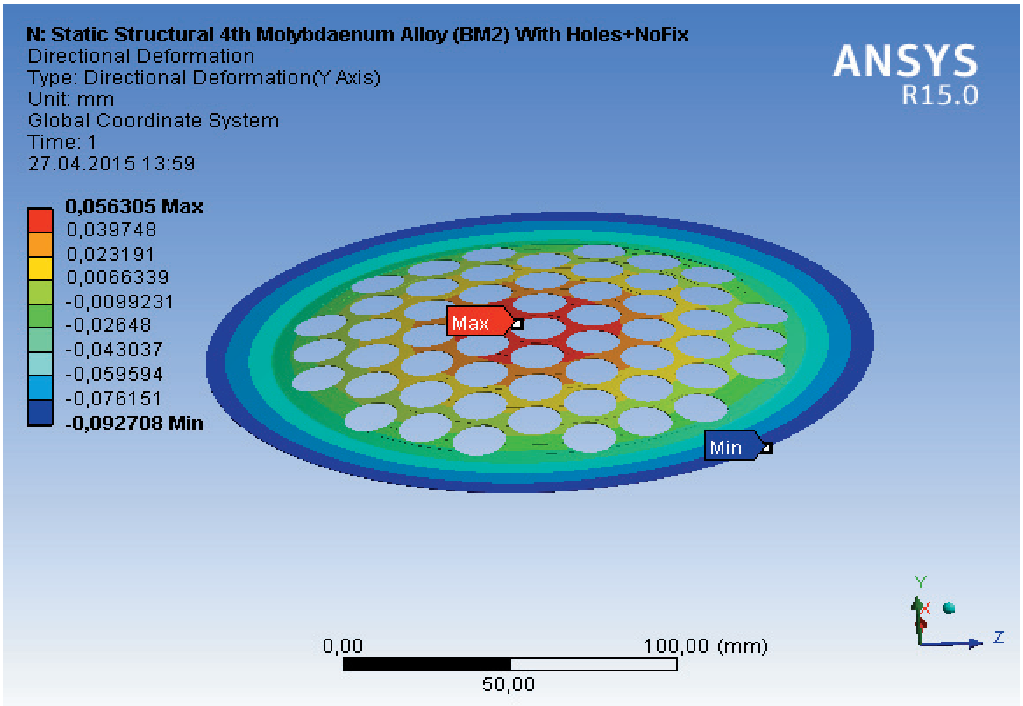

Figure 4. Simulation of directional deformation along the Y axis EE160 for free fixing.

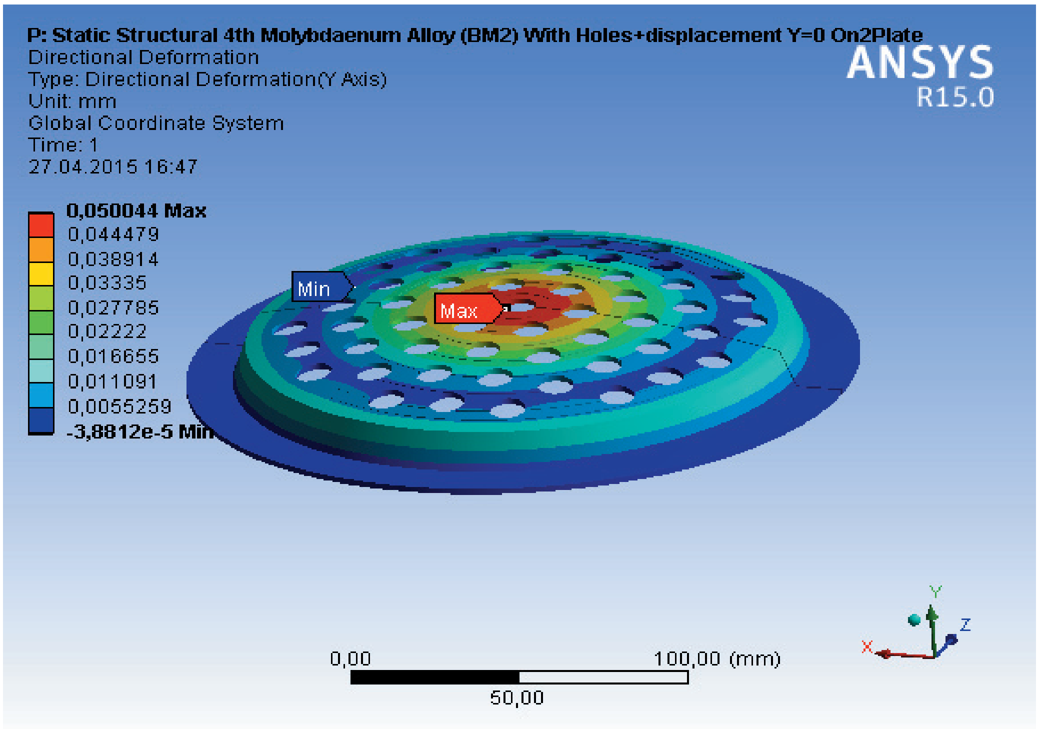

Figure 5. Simulation of directional deformation along the Y axis AE160 for free fixing.

The results presented verified efficiency of proposed procedure for making short-cut calculations for additional bending of spherical elements of electrodes. This will enable making mathematically substantiated forecast for the variation of IES interelectrode gap. The results of calculation of additional bendings of electrodes of molybdenum and titanium-niobium alloy revealed possibility for the ion extraction systems manufacturing using these alloys as a material for emissive electrodes, which meet the requirement to small thickness of shell plate from 0.3 mm up to 0.5 mm. Primary factors influencing such electrodes:

- Temperature fields variable in radial direction with temperature gradient of some dozens of Centigrade between the centre of electrodes and outer contour;
- Initial technological bendings of electrodes in the central point of spherical segment $w_o(0)$;
- Coefficients of temperature linear expansion of material;
- Coefficients of structural orthotropy of electrodes k_r and k_φ, depending on relation of perforation hole diameters and distance between them.

For ensuring of minimal change in the IES interelectrode gaps during the ion thruster operation, a necessity has started up in the minimization of tolerances. Within the frames of the project on the development of radio-frequency ion thrusters with the ionizer diameter of 160 and 450 mm, the tolerance for the interelectrode gap was specified as not exceeding 0.2-0.4mm. The use of carbon-carbon composite material for emissive and accelerating electrodes was studied independently. Calculated values of additional bendings $\Delta w(0)$ of emissive electrodes and accelerating electrodes thickened up to 2.5 mm are presented in the tables. Low values of such bendings are first of all explained by low value of the coefficient of temperature linear expansion of certain material. Decision on the shape and lifetime of IES, on materials used, manufacturing technology and operation should be made by taking into consideration of all problems related to the development of radio-frequency ion thrusters.

III. Conclusion

On the grounds of the conducted research a conclusion can be made on the efficiency and simplicity of simultaneous use of the thermomechanical model of IES electrodes and new simplified analytical method for calculation of additional bending of profiled electrodes. Additional bending of electrodes made of different materials and having different initial bending was determined numerically at the thermal loading with the radial temperature gradient of 5 grad/cm.

The following the results were obtained:

1. For the EE with the thickness h = 0.3-0.5 mm made of the molybdenum alloy VM-2 at the initial bending of 5-15 mm the expected additional bending is 0.15-0.05 mm at h = 0.3 mm and 0.4-0.14 mm at h = 0.5 mm. Based on this, the recommended parameters of molybdenum EE with the working interelectrode gap of 0.8 mm are: thickness – 0.3 mm, initial bending - 15 mm.
2. For the EE made of the titanium-niobium alloy of invar grade TV-36 at the initial bending of 5-15 mm the expected additional bending is 0.013-0.005 mm at h = 0.3 mm and 0.035-0.012 mm at h = 0.5 mm. Based on this, the recommended parameters of titanium-niobium alloy EE with the working interelectrode gap of 0.8 mm are: thickness – 0.3 mm, initial bending - 5 mm.
3. For the EE made of the carbon-carbon composite materials at the initial bending of 5-15 mm the expected additional bending is 0.009-0.004 mm at h = 0.3 mm and 0.026-0.009 mm at h = 0, 5 mm. Based on this, the recommended parameters of EE of carbon-carbon composite material with the working interelectrode of 0.8 mm are: thickness – 0.3-0.5 mm, initial bending - 5 mm.
4. For AE made of the molybdenum alloy VM-2 with the thickness of 2 mm at the initial bending of 5-15 mm the expected additional bending is 0.119-0.057 mm at h = 0.3 mm and 0.035-0.012 mm at h = 0.5 mm. Based on this, the recommended parameters of AE made of the titanium alloy VM-2 are: thickness – 2.0 mm, initial bending - 5 mm.
5. For AE made of the carbon-carbon composite material with the thickness of 2 mm at the initial bending of 5-15 mm the expected additional bending is 0.007-0.003 mm. Based on this, the recommended parameters of AE made of the carbon-carbon composite material are: thickness – 2.0 mm, initial bending - 5 mm.

The already developed thermomechanical model of IES unit was used for calculation of electrodes differing in diameter and thickness, which were made of different construction materials.

Acknowledgments

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