Time-Independent Fully kinetic Particle-in-Cell for plasma magnetic field interactions

IEPC-2015-478p／ISTS-2015-b-478p

Presented at Joint Conference of 30th International Symposium on Space Technology and Science
34th International Electric Propulsion Conference and 6th Nano-satellite Symposium,
Hyogo-Kobe, Japan
July 4 – 10, 2015

Toshihiro Matsuguma[1], Takaya Inamori[2], Rei Kawashima[3]
Department of aeronautics and astronautics, The University of Tokyo, Tokyo, 113-8656, Japan

Shinichi Nakasuka [4]
Department of aeronautics and astronautics, The University of Tokyo, Tokyo, 113-8656, Japan

and

Ryu Funase[5]
Department of aeronautics and astronautics, The University of Tokyo, Tokyo, 113-8656, Japan

Abstract: This study proposes a Time-Independent Full Particle-in-Cell simulation method. Generally, the plasma drag force acts on a magnetic moment of a spacecraft, because a magnetic field changes momentum of a plasma flow. This plasma drag force can provide a propellantless external force with a spacecraft. One of the valuable applications of a plasma drag force is an attitude control system especially in deep space missions. To design an attitude control system using a plasma drag force, performance analyses should be conducted in multiple different conditions: attitude, magnetic field, etc. Particle-in-Cell (PIC) method is a commonly-used method in those analyses. The problem of PIC is high calculation cost. It is nearly impossible for conventional PIC to fully calculate the drag force of parametric system design on different configurations of magnetic moments and time-variable spacecraft attitude. To solve this problem, we propose a Time-Independent simulation method, which assumes that a plasma flow will converge to a steady state, and consequently, an electromagnetic field remains static in particle trajectory calculations. We applied a proposed method to the interaction of the solar wind with a dipole magnetic field. As the result, a proposed method shows the same order of a plasma drag force of conventional PIC.

Nomenclature

$\boldsymbol{r}$	=	position vector
$\boldsymbol{v}$	=	velocity vector
m	=	mass
q	=	electric charge
μ_0	=	magnetic permeability of vacuum

[1] Graduate student, Department of aeronautics and astronautics, matsuguma@space.t.u-tokyo.ac.jp
[2] Assitant Professor, Department of aeronautics and astronautics, inamori@space.t.u-tokyo.ac.jp
[3] PhD student, Department of aeronautics and astronautics, kawashima@al.t.u-tokyo.ac.jp
[4] Professor, Department of aeronautics and astronautics, nakasuka@space.t.u-tokyo.ac.jp
[5] Associate Professor, Department of aeronautics and astronautics, funase@space.t.u-tokyo.ac.jp

c	=	speed of light
dt	=	time step
ρ	=	electric charge density
ε_0	=	permittivity of vacuum
$\boldsymbol{B}$	=	magnetic flux density
$\boldsymbol{E}$	=	electric field
$\boldsymbol{J}$	=	current density
j	=	index of particle, i: ion, e: electron
Φ	=	electrostatic potential

I. Introduction

THE solar wind is high-temperature plasma flow ejected from the sun. The solar wind consists mainly of protons and electrons, containing a small amount of heavy ions and atoms. The local composition is related to the distance from the sun. In near-Earth region, electron density $n_e = 5\text{cm}^{-3}$, electron temperature $T_e = 10^5$K, interplanetary magnetic field B=5nT. The interaction between the solar wind and strong magnetic field such as geomagnetic field forms magnetosphere, in which the inflow of the solar wind is interrupted. Bow-shock forms in front of magnetopause, the boundary between the solar wind and magnetic field. The velocity of the solar wind is decelerated to subsonic through Bow-shock, while density and magnetic field is intensified In the case of interaction between the solar wind an d satellite's magnetic actuator, same phenomenon will happen. Also, a magnetic actuator receives kinetic momentum from particles of the solar wind since the plasma flow is decelerated by magnetic field. Therefore, coils on a satellite can be utilized as external force actuators.

Plasma simulations are classified into following three scales. Full Particle-in-Cell (Full-PIC) models electrons as fluid and protons as particles. Hybrid Particle-in-Cell (Hybrid-PIC) models electrons as fluid and protons as particles. Magneto Hydrodynamics (MHD) models particles as fluid. Particle-in-Cell solves particle motion from a Lagrangian perspective and distributes physical quantity to surrounding lattices iteratively. Consequently, electromagnetic field and particle motion can be simulated mutually. Fig.1 shows the simulation result of Full Particle-in-Cell.

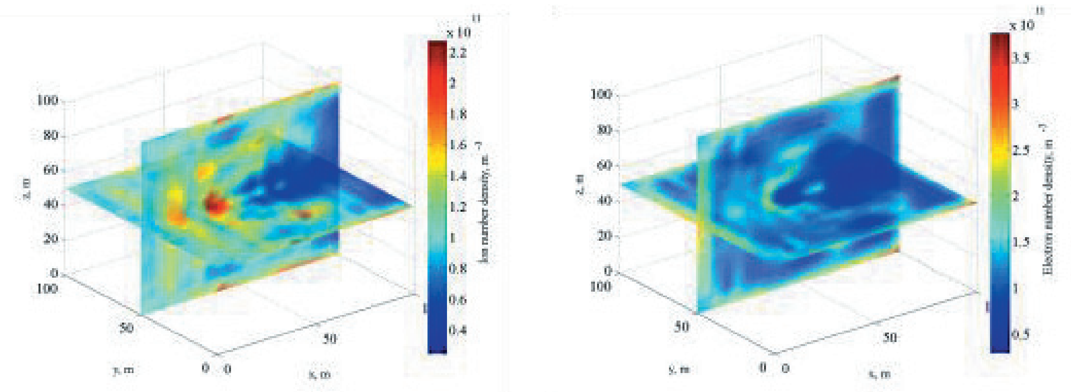

Fig.1 Plasma magnetic field interaction, Electron number density

The scale of the interaction between the solar wind and a dipole magnetic field is defined by the fractions of magnetosphere size$L$, ion Larmor radius r_{iL} and electron Larmor radius r_{eL}, L/r_{eL}, L/r_{iL}. Since the magnetosphere size of the coil which is possible to mount on a spacecraft is close to electron Larmor radius, electrons' kinetic effects have to be considered. Therefore, attitude control system using plasma magnetic field interaction should be simulated by Full-PIC. However, applying Full-PIC for the parametric design of attitude control system considering different distribution of magnetic field and time-changing attitude is unrealistic, because Full-PIC needs several days for each parameter combination. One of the solutions for the large amount of calculation is Time-Independent simulation. Time-Independent simulation focuses on the electromagnetic field which is steady when plasma flow comes to steady equilibrium, and assumes that electromagnetic field is steady state until particles outflow from the computational region. Calculating only the terminal steady state achieves great reduction of calculation time, but conventional Time-Independent simulation of PIC employs Hybrid-PIC model and cannot be adapted to Fully-kinetic model simulation.

Given these facts, the problems of previous research are that,

1) Conventional Full-PIC cannot deal with a parametric design problem such as attitude control system because of its high calculation cost.

2) Conventional Time-Independent plasma simulation is unsuitable for Fully-kinetic scale phenomena because electrons are modeled as fluid.

The purpose of this paper is to suppose Time-Independent Full Particle-in-Cell for parametric design of attitude control system utilizing plasma magnetic field interaction.

II. Theory

A. ElectroStatic Particle-in-Cell

Full-PIC treats both electrons and ions as particles in Lagrangian frame, and interactively solves Maxwell's electromagnetic equations and particles' equation of motion. Full-PIC is proximate to first-principle simulation. The fundamental equations of Full-PIC are particles' equation of motion and Maxwell's electromagnetic equations. Eq.1 and Eq.2 represent particles' equation of motion.

$$\frac{d\boldsymbol{r}_{\mathrm{j}}}{dt} = \boldsymbol{v}_{\mathrm{j}} \tag{1}$$

$$m_{\mathrm{j}}\frac{d\boldsymbol{v}_{\mathrm{j}}}{dt} = q_{\mathrm{j}}(\,\boldsymbol{E} + \boldsymbol{v}_{\mathrm{j}} \times \boldsymbol{B}\,) \tag{2}$$

We used Runge-Kutta 4$^{\mathrm{th}}$ order method in order to update particles' state quantity. The fundamental equations of an electromagnetic field are Maxwell's electromagnetic equations below.

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} \tag{3}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{4}$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0} \tag{5}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{6}$$

These equations obey the first principle, but ElectroStatic PIC, which we applied to proposal method, assumes that magnetic field is static. For the assumption to be true, the magnetic field fluctuation induced by plasma flow has to be negligible. In the case of a satellite's coil and the solar wind, a coil mounted on a spacecraft changes particles' velocity distribution and magnetic field will be induced, but the induced magnetic field is small enough compared to the external magnetic field of a coil. Assuming that an electromagnetic field can be replaced by an electrostatic field, the electric field is updated by solving Poisson equation (Eq.7) of electrostatic potential and magnetic field is kept constant.

$$\nabla^2 \phi = -\frac{\rho}{\varepsilon_0} \tag{7}$$

B. Time-Independent Simulation

This section overviews Time-Independent plasma simulations. The word, Time-Independent, is different from the meanings of “implicit”. Implicit methods do not need to meet the Courant-Friedrichs-Lewy Condition (CFL Condition), the restriction that the information propagation speed must be higher than the speed of physical phenomena’s propagation. The time step of implicit can be set much longer time than explicit methods. However, it does not mean that implicit method is Time-Independent because implicit methods are updated by the time step and can simulate each time’s state.

Time-Independent plasma simulation is intended to calculate only the terminal steady state and reduce the computational cost drastically. Assuming that an electromagnetic field is near-terminal steady state, an electromagnetic field does not have to be solved at each time steps in which particles’ state quantities are updated. Therefore, Time-Independent methods update an electromagnetic field after calculating particle trajectories from inflow to outflow. An update of an electromagnetic field is iterated until the plasma flow converges. Consequently, the total amount of calculation is reduced because Time-Independent does not have to simulate time propagation of the plasma flow. When a simulation problem contains a lot of wide range parameters, Time-Independent simulation can estimate the results quantitatively with a certain level of precision. The attitude control system utilizing plasma magnetic field interaction has many parameters which are time-changing attitude and distribution of magnetic field. It is not feasible to solve each parameter combination by Time-dependent simulation.

Time-independent simulation of Hybrid –PIC model has already been theorized. However, Time-Independent Hybrid-PIC cannot simulate satellites magnetosphere scale rightly, since Hybrid model treats electrons as mass-less fluid and ignores the kinetic effects of electrons. Therefore, in this study, we extended Time-Independent simulation to Full particle model.

C. Particle Trajectory Map and Particle Insertion

Time-Independent Full-PIC has a novelty that electrons are modeled as particles. In addition, we introduced two original concepts below to Time-Independent Full-PIC.

1) Particle Trajectory Map
2) Particle Insertion

These concepts are based on the fact that particle trajectories are independent with each other because Time-Independent simulation calculates particle trajectories in a static electromagnetic field. So, if a set of particles has close initial state vector, those particles’ trajectories are considered to be close. From this perspective, a certain particle’s trajectory can be estimated from other particles’ trajectories which have close initial state vector.

Particle Trajectory Map is a set of test particle trajectories. The proposed method calculates test particle trajectories instead of whole trajectory calculation, and then estimates whole particles’ state vector at arbitrary time using the Particle Trajectory Map. This concept can reduce the calculation cost for particle trajectory and deal with more particles than conventional method because the calculation cost for each particle become much lower. Fig.2 shows conceptual diagram of our concepts. Y-axis and Z-axis represents the surface of plasma inflow.

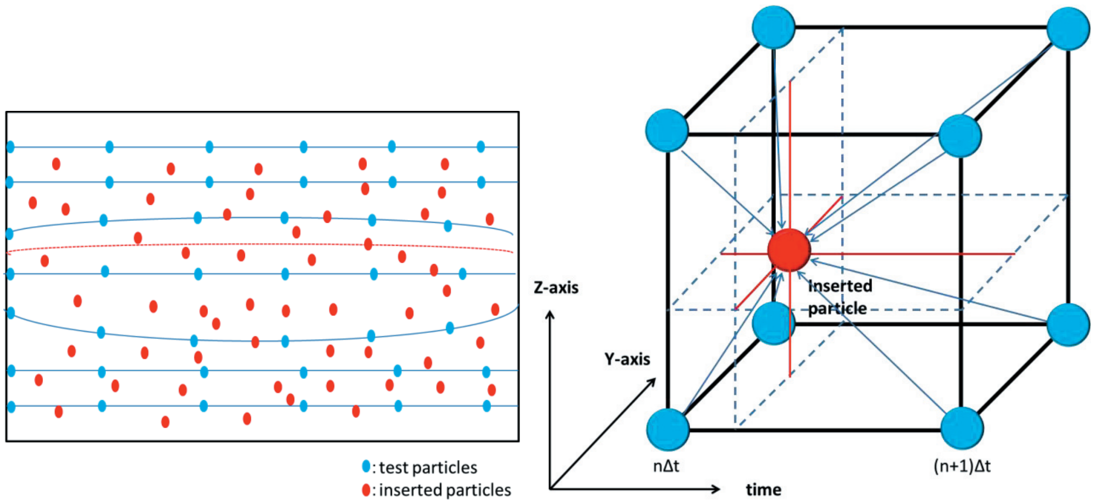

Fig.2 Conceptual diagram of Particle Trajectory Map and Particle Insertion

D. Algorithm

Fig.3 represents the algorithm of Time-Independent Full Particle-in-Cell

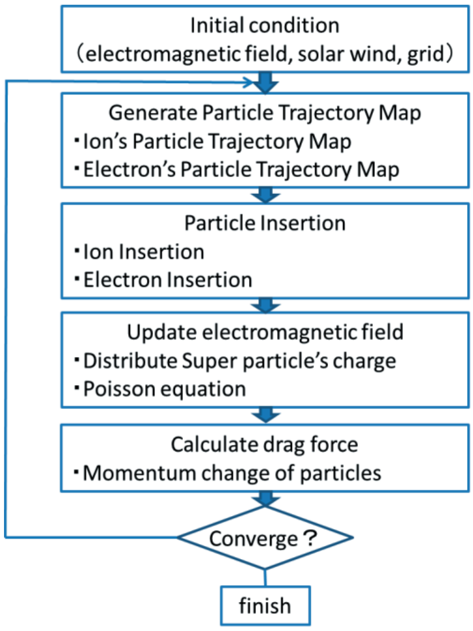

Fig3. The algorithm of Time-Independent Full Particle-in-Cell

III. Simulation Results and Discussions

A. Computational Settings

We defined computational settings as follows. Fig.4 represents the configuration that dipole moment is parallel to the solar wind which ignores thermal motion.

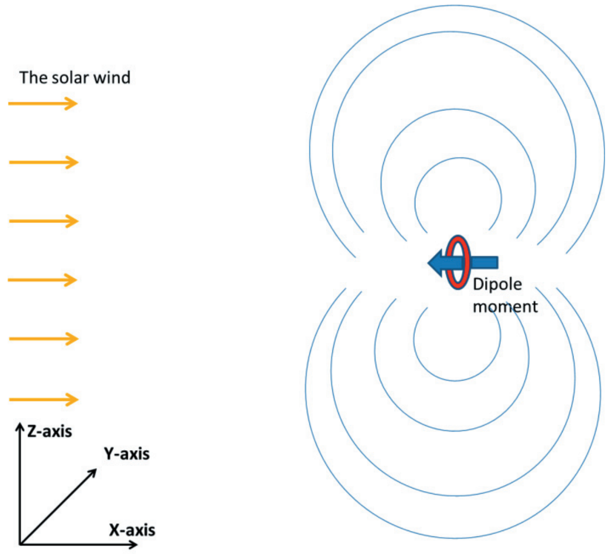

Fig4. The configuration of the solar wind and the coil

We simulated electron density, ion density, electromagnetic field and drag force acting on the coil in the following conditions. For each conditions, conventional Full-PIC (Time-Dependent) and proposed method (Time-Independent) are calculated.

1) Dipole moment $M=0.5\pi \times 10^4[Am^2]$, Time-Dependent : case1, Time-Independent : case2
2) Dipole moment $M=0.5\pi \times 10^8[Am^2]$, Time-Dependent : case3, Time-Independent : case4

Plasma dipole magnetic field interaction has other parameters, but we changed only the magnetic moment in order to simplify the problem.

B. Simulation Results

1) Time-Dependent, Dipole moment $M=0.5\pi \times 10^4[Am^2]$

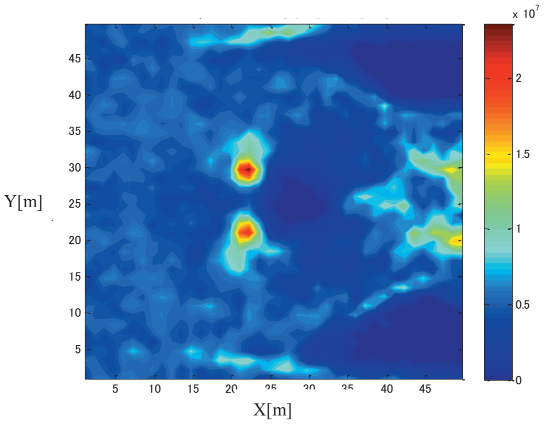

Fig5. Electron density case1

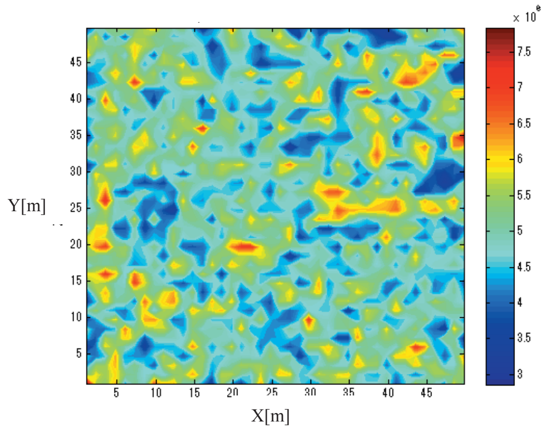

Fig6. Ion density case1

2) Time-Independent., Dipole moment $M=0.5\pi \times 10^4[Am^2]$

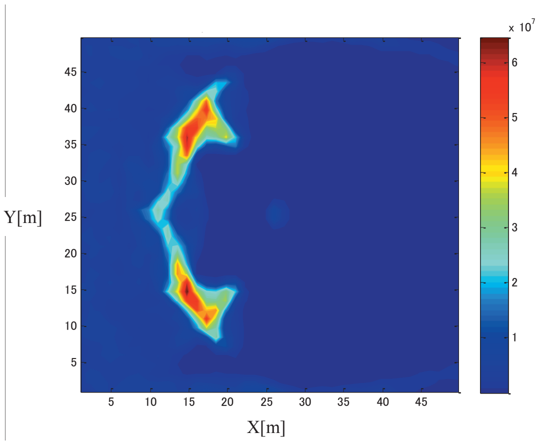

Fig7. Electron density case2, oscilation-1

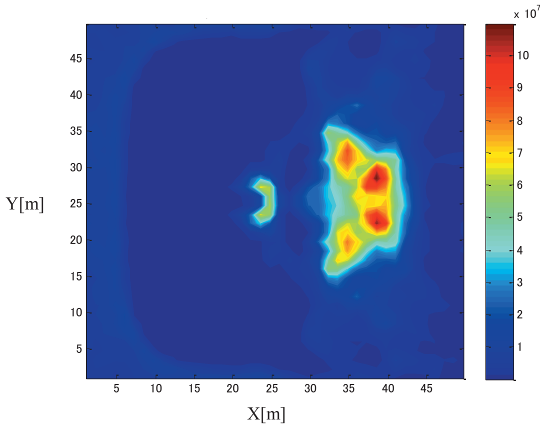

Fig8. Electron density case2, oscilation-2

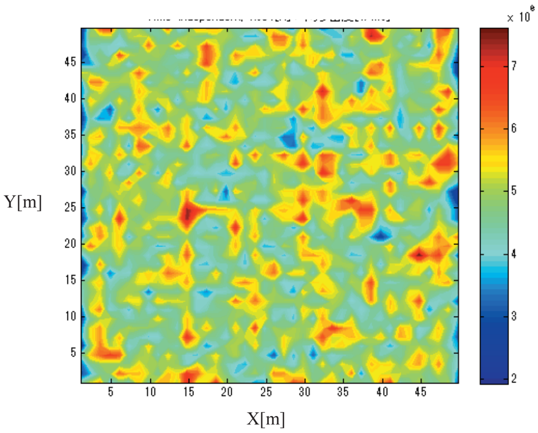

Fig9. Ion density case2

3) Time-Dependent, Dipole moment M=0.5π $\times$ 10^8[Am^2]

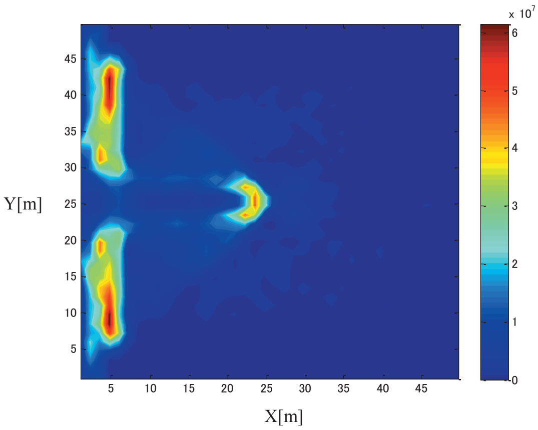

Fig10. Electron density case3

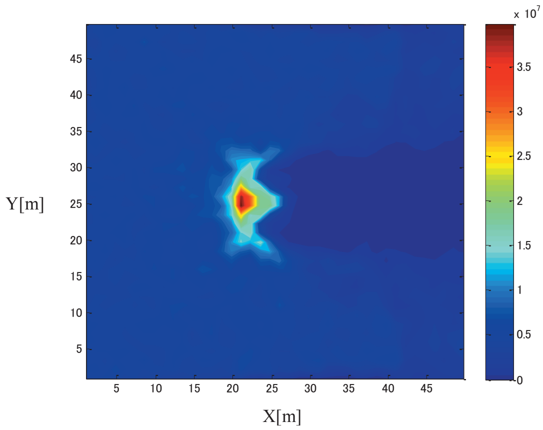

Fig11. Ion density case3

4) Time-Independent., Dipole moment M=0.5π $\times$ 10^8[Am^2]

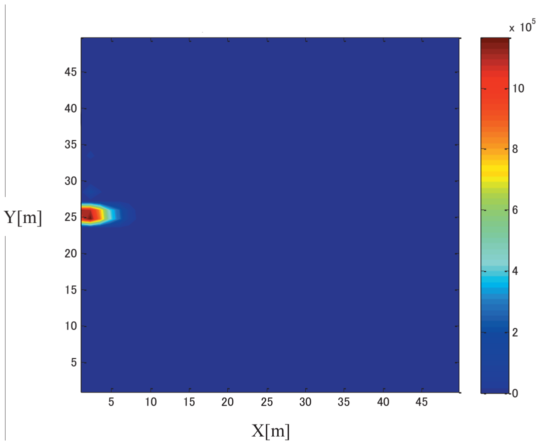

Fig11. Electron density case4

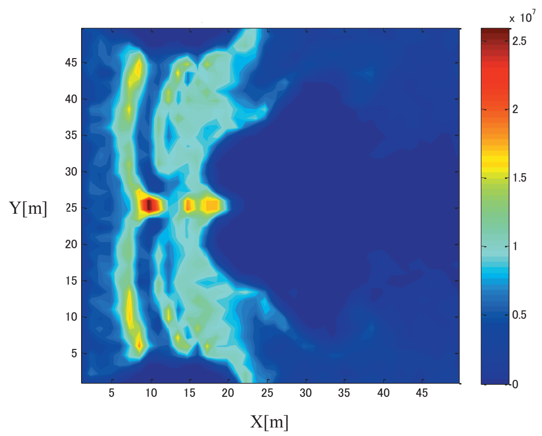

Fig12. Ion density case4

5) Drag force and convergence time

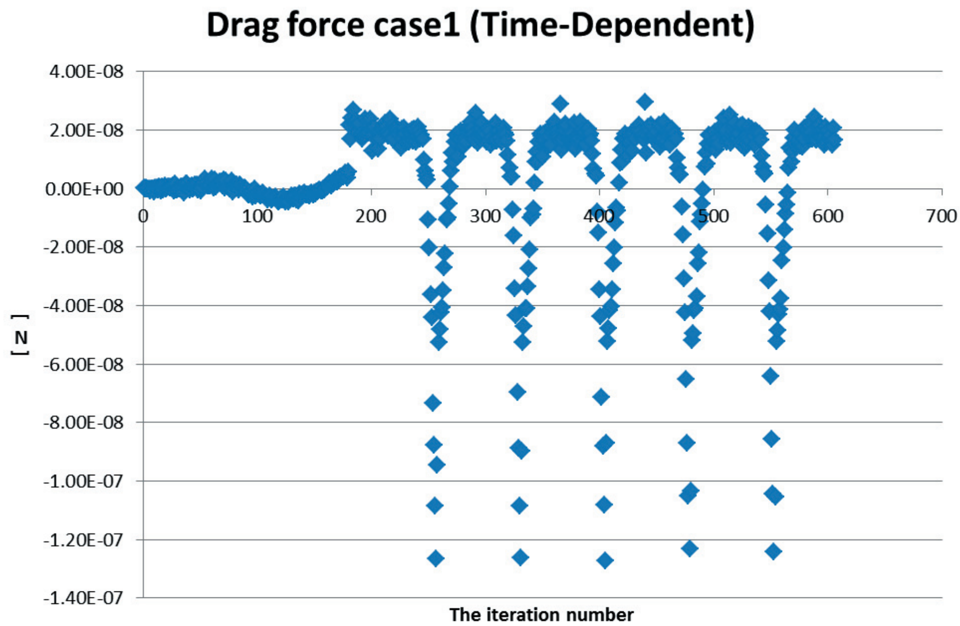

Fig13. Drag force case1

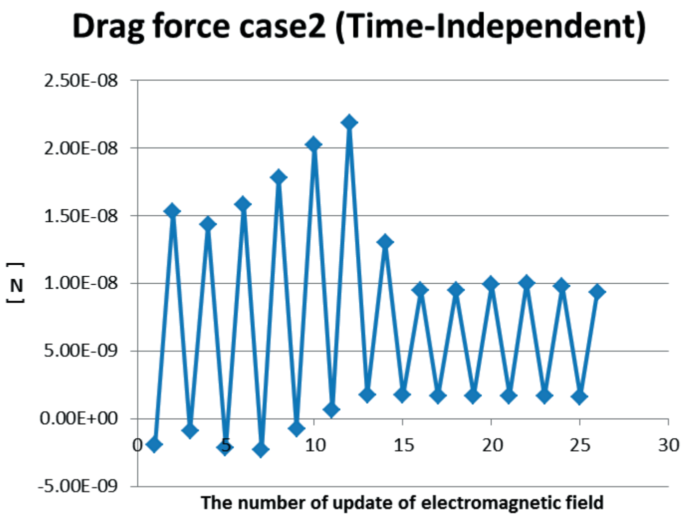

Fig14. Drag force case2

C. Discussions

1) Qualitative analysis of Electron density and ion density

In case1, magnetosphere of ion did not form because the strength of the magnetic moment is not enough. In case3, magnetosphere of electron ran off the edge of the computational region because the strength of the magnetic moment is too strong, but the condition meets the formation of the ion's magnetosphere. In addition, electron density distribution of Time-Independent simulation seems to be oscillating. One reasonable cause of the oscillation is that electron motion is susceptible to the change of the electromagnetic field because of its minuscule mass. High sensitivity of electrons to electromagnetic field causes dramatic changes of electron density distribution in the iteration, and that dramatic change of electron density directly reforms electromagnetic field. These phenomena do not appear in Time-Independent Hybrid-PIC because Hybrid model ignores electron's kinetic effects.

2) Drag force and convergence time.

The convergence time is 17 hours at case1 and 30 minutes at case2. These results have shown that the proposed method can reduce the computational cost drastically. On the other hand, although the drag force of the proposed method is the same order of magnitude as that of a conventional method, the drag force of the proposed method is about half of the conventional method's numerical value.

3) The effect of mass of electron

We found that the electron density distribution oscillates in Time-Independent Full-PIC. In order to analyze this phenomenon, we controlled mass of electron artificially. Fig.15~Fig18 represent electron densities of different artificial mass conditions. The artificial mass ranges from 5 times to 40 times of real mass. When decreasing the mass of electron gradually, electron density's oscillation is restrained while the mass is larger than 10 times of real mass. However, electron density began to oscillate when the mass is 7~8 times of real mass.

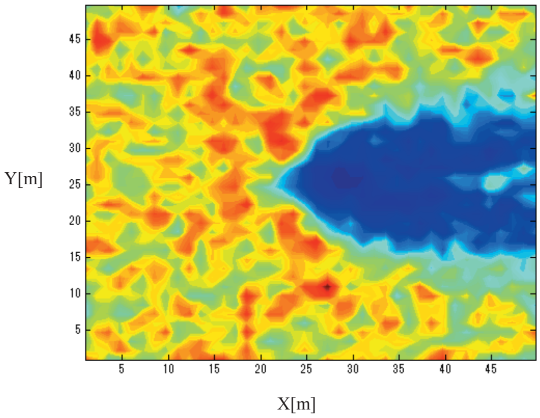

Fig15. Electron density, 40 times mass

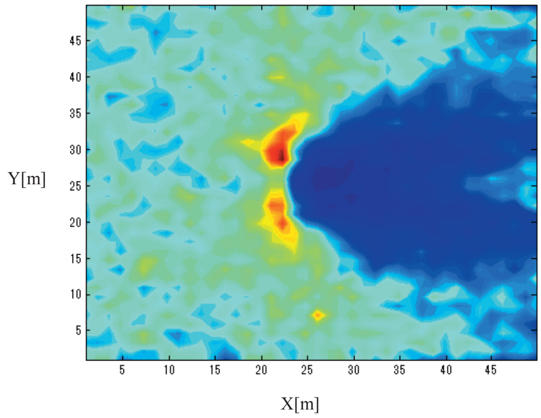

Fig16. Electron density, 20 times mass

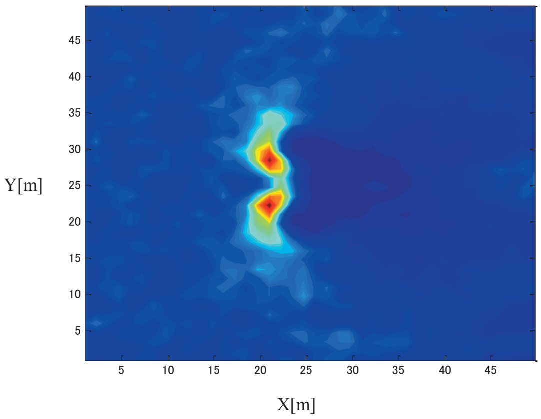

Fig17. Electron density, 10 times mass

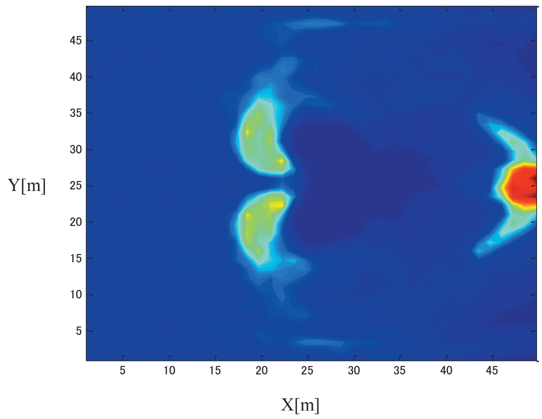

Fig18. Electron density, 5 times mass

IV. Conclusion

This study proposes a Time-Independent Full Particle-in-Cell simulation method for the parametric design of an attitude control system for interplanetary spacecraft. Time-Independent Full-PIC simulation aims to reduce calculation cost drastically. To assess the proposed method, we conducted numerical simulations using both proposed Time-Independent and conventional Time-Dependent Full-PIC methods. As the result, the plasma drag force of a proposed method is the same order of that of a conventional PIC. However, the electron density distribution of the proposed method does not correspond completely with that of the conventional method because of the electron density's oscillation. These oscillations are not found in simulation results with larger aritificial electron mass. Therefore, we concluded that the oscillations are caused by the small mass ratio of electron to ion in the simulations. Based on the simulation results, the oscillations appear when artificial electron mass is lower than 7~8 times of real electron mass.

Acknowledgments

I wish to express my gratitude to Prof. Nakasuka, Mr. Inamori and my friends in Intelligent Satellite System Laboratory. I gratefully acknowledge the support of Rei Kawashima in Komurasaki Laboratory, The University of Tokyo.

References

[1]Alexander, S. L., The Hybrid Multiscale Simulation Technology: An Introduction with Application to Astrophysical and Laboratory Plasmas, Springer, New York, 2002, pp. 6-12.

[2]Ashida, Y., Funaki, I., and Yamakawa, H., “Thrust Evaluation of a Magnetic Sail by Flux-Tube Model,” Japan Society for Aeronautical and Space Sciences, Vol. 59, No.668, pp.126-131, 2011
[3]Zubrin, R. M., and Andrews, D.G, “Magnetic Sails and Interplanetary Travel,” Journal of Spacecraft and Rockets, Vol. 28, No. 2, pp.197-203, 1991.
[4]Khazanov, G., Delamere, P., Kabin, K., and Linde, T.J., “Fundamentals of the Plasma Sail Concept: Magnetohydrodynamic and kinetic Studies,” Journal of Propulsion and Power, Vol. 21, No. 5, pp.853-861, 2005.
[5]Russell, C. T., “The Dynamics of Planetary Magnetospheres,” Planetary and Space Science, Vol. 49, pp.1005-1030, 2009.
[6]Funaki, I., and Yamakawa, H., “Research Status of Sail Propulsion Using the Solar Wind,” J Plasma Fusion Res, Vol. 8, pp.1580-1584, 2009.
[7]Matsumoto, M., Kajimura, U., Usui, H., “Two-Dimensional Hybrid Particle-In-Cell Simulation of Solar Wind Plasma Flow around Magnetic Sail,” Transactions of the Japan Society for Aeronautical and Space Sciences, Aerospace Technology Japan, vol.10, pp.Pb43-Pb50, 2008.
[8]Paes, A., Abe, M., and Passaro, A., “Simulations of Plasmas with Electrostatic PIC models Using the Finite Element Method,” Brazilian Journal of Physics, Vol.33, No. 2, pp. 411-417, 2003.
[9]Fujita, K., “Particle Simulation of Moderately Sized Magnetic Sails,” Journal of Space Technology and Science, Vol. 20, No. 2, pp.26-31, 2005.
[10]Omidi, N., Blanco-Cano, X., and Russell, C. T., “Dipole Magnetospheres and their Characterization as a Function of Magnetic Moment,” Advances in Space Research, Vol. 33, pp.1996-2003, 2004.