Plume Structure Study of Hollow Cathode in Bias Voltage Conditions

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Abstract: In this study, a spherical symmetric theoretical model is chosen to predict the plume structure of a hollow cathode. An isothermal model regardless of neutrals is employed to simulate the plume structure in the working condition of high voltage and low electron emission current. Both models' results tell that the potential distribution mainly depends on the ratio of plasma potential and electron temperature while the density changes little when other parameters vary. Plasma potential and density distribution of cathode plume calculated by the numerical model is approximated to that calculated by the theoretical model in axial direction but it drops faster in radial direction. In addition, it is concluded that the sheath layer of the vacuum chamber cannot be ignored when the voltage near the chamber is high and density is low based on the classical collisionless sheath model.

Nomenclature

ϕ	=	plasma potential
ϕ_0	=	plasma potential at initial position
V_P	=	maximum plasma potential
V_k	=	keeper voltage
V_C	=	cathode voltage
r	=	axis distance
r_0	=	axis distance value at initial position
r_s	=	axis distance value at sheath boundary
r_W	=	axis distance value at tank wall

I. Introduction

HOLLOW cathode is an important technique which has been widely used in electric propulsion[1], plasma contactor[2,3] and electrodynamic tether system[4,5]. It can provide enough electron current at a relatively low voltage and power for those devices so that current-voltage characteristic has always been the most important performance of a hollow cathode production[6,7]. With more applications and more problems on hollow cathode, the plume structure is getting more attentions[8]. The plume of hollow cathode is quite different from electric prolusion for its high plasma density and slow ion velocity and the distribution of the plasma parameters is totally self-consistent. It is necessary and meaningful to understand the plume structure of hollow cathode in bias voltage

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conditions. If the plume structure can be predicted, the thrust could be inferred when hollow cathode is applied to a thruster, analysis on potential oscillation of hollow cathode will be easier and so on. In addition, the credibility of the date measured in the experiment can be discussed by the predicted plume structure.

Lots of experiments have been done to get a better understanding of the plume structure[8-11] of a hollow cathode. The spherical theoretical model presented by Parks and Katz[12] which will be chosen to discuss in this paper made it possible to predict the plume structure of a hollow cathode. In addition, many numerical models have been established to predict the plume structure of the hollow cathode[9,13-15]. There are mainly two typical kinds of models, one is two-fluid model, the other one is hybrid model (including particle in cell and fluid). Many of them are on the hypothesis of quasinetual and are used to describe the plume mode regardless of electron drift velocity and external ionization. In our study, a method same to the plume prediction of ion or hall thruster[16,17] will be established on the hypothesis of isothermal. This method can only be used in the working condition like the numerical model we mentioned. The external ionization is ignored like the models, too. Comparison is made between results from the theoretical model and numerical model we employed. Besides, the classical collisionless sheath model is used to discuss the influence of the vacuum chamber on plume structure.

II. Method

A. Theoretical model

Fig.1 shows the potential distribution we considered. We focus on the plasma in the region $r > r_0$ where external ionization can be ignored, but r_0 is hard to choose. It is supposed that in the region $r > r_0$, ions are accelerating to the far field to form the ion current. At the position $r = r_0$, electrons thermalize to a distribution characterized by some temperature $T_e (= \theta)$ and saturation emission current I_s. Depending on their energy, they escape to the far field or are trapped in the potential field. The ions and electrons neutralize one another to form quansineutral plasma.

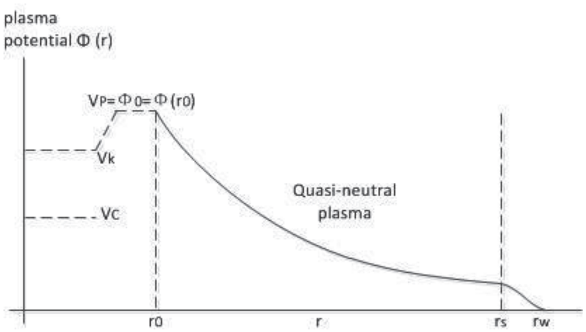


Figure 1. Schematic form of contactor plasma potential in a tank.

We consider a spherically symmetric potential of the general form as we described in Fig.1, which means the potential decreases monotonically with increasing r from a value ϕ_0 at $r = r_0$. The ions density flowing rate is given,

$$\frac{I_{\mathrm{i}}}{q} = 4\pi r_0^2 N_0 V_0 \tag{1}$$

where the ion velocity is determined based on the Bohm-like condition which is the same in the numerical model, $V_0 = \sqrt{\theta/M}$, M is the ionic mass.

For the ions are accelerating all the time, the steady-state density at position r can be easily got,

$$N(r) = \frac{I_{\mathrm{i}}/q}{4\pi r^2 \left[V_0^2 + 2e(\phi_0 - \phi)/M\right]^{1/2}} \tag{2}$$

We assume that at the position $r = r_0$ the distribution of the velocity of electrons is Maxwellian. For the electrons are slowing down all the time in the potential field, the electron density consists of two kinds of electrons, one with insufficient energy which will flow back to the source, the other which can flow to the far field. If the density at position r_0 is known, density can be inferred through integration of the electron kinetic energy on distribution of Maxwellian. Finally, we have the relationship between electron density and the position,

$$n(r) = \frac{I_s/q}{4\pi r_0^2 \left(\dfrac{\theta}{M}\right)^{1/2}} 2\sqrt{2} e^{-\phi_0/\theta} \left[G_+\left(\frac{\phi}{\theta}\right) - \left(1 - \frac{r_0^2}{r^2}\right)^{1/2} H\left(\frac{\chi}{\theta}\right) \right] \tag{3}$$

where details of the formulation can be got through the article[12].

The relationship between the position and potential can be inferred by the quasinetrual hypothesis when the relationship of I_s and I_e is got. At the position r_0, density of ions equals to the density of electrons, them the relationship is given,

$$2\sqrt{2}e^{-\phi_0/\theta}G_+\left(\frac{\phi_0}{\theta}\right)I_s=\left(\frac{\theta}{mV_0^2}\right)^{1/2}I_i \tag{4}$$

B. Numerical model

In numerical model, only ions and electrons are considered for the collisions between ions and neutrals have little influence on plume structure in working condition of high voltage and low current. The movement of ions is treated using the explicit particle–in-cell (PIC) method for transporting the ions in electrostatic fields,

$$\begin{aligned}\mathbf{v}^{n+1/2}&=\mathbf{v}^{n-1/2}+\Delta t_i\frac{e}{M}\mathbf{E}^n\\ \mathbf{x}^{n+1}&=\mathbf{x}^{n-1}+\Delta t_i\mathbf{v}^{n+1/2}\end{aligned} \tag{5}$$

where $\mathbf{v}$ and $\mathbf{x}$ are the displacement vector and velocity vector of the ion, Δt_i is the time step for the simulation, $\mathbf{E}$ is the electric field, n is the simulation step, $\mathbf{E}^n$ means the electric field at n step.

The movement of electrons can be given by,

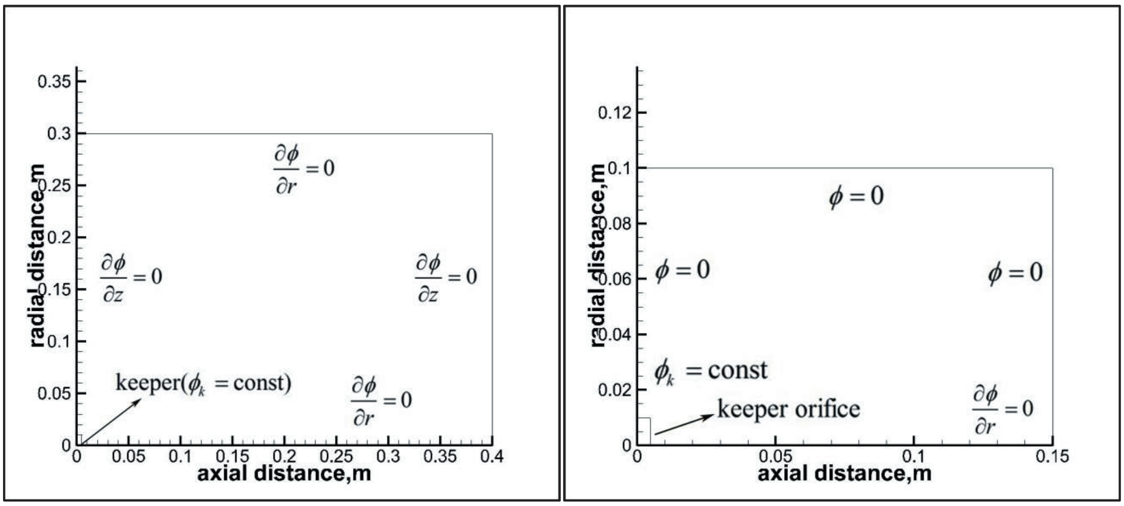
$$\frac{\partial}{\partial t}\left(m_eN_e\mathbf{v_e}\right)+m_eN_e\left(\mathbf{v_e}\cdot\nabla\right)\mathbf{v_e}=-eN_e\mathbf{E}-\nabla p_e+\mathbf{R} \tag{6}$$

where m_e is the mass of an electron, p_e is the electron temperature which can equal to $p_e = kN_eT_e$, and $\mathbf{R}$ is the friction term. The items on the left side of the equation can be ignored because this is a steady state flow and in the plume mode, the drift velocity of electrons can be ignored. There are two ways to calculate the electric field, one is using the Poisson equation, the other one is using this equation based on the hypothesis of quasineutral. The former is chosen because it is useful and easier to discuss the influence of the vacuum chamber although it is not accurate; besides, the latter is hard to calculate for its weak source term. Furthermore, we ignore and the friction term on the right sides, which is $\mathbf{R} = eN_e\mathbf{j}/\sigma$.

The distribution of electrons is then given by the form of Boltzmann,

$$n_e=n_{e_0}\exp\left(\frac{\phi-\phi_0}{T_e}\right) \tag{7}$$

Boltzmann distribution of electrons density has been widely used in many plume structure calculations of ion thruster[16] and hall thruster[17]. In their calculations, ϕ_0 and n_{e0} is usually determined by the data from experiment. Here, ϕ_0 is given the value same to the keeper voltage (equals to initial plasma potential) and n_{e0} is given the value



Big field **Small field**

Figure 2. Schematic form of computational field.

same to the plasma initial density. Because the velocity of ions is much smaller than that in electric propulsion device, the choice of the value in Boltzmann equation is limited. This is the main defect of this model.

Poisson equation is used to solve the electric field,

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0} \tag{8}$$

where ρ is net charge density and ε_0 is the vacuum dielectric constant.

Two computational fields showed in Fig.2 are used in the paper. The big one is used to calculate the plume structure of hollow cathode and the small one is used to discuss the influence of vacuum chamber on plume structure whose grids are smaller in order to describe the detail of the sheath. The exit of the keeper is the entrance of the simulation. The inner diameter of keeper is 3mm and outer is 20mm, and only 5mm length is chosen because its influence can be ignored. When PIC is used, the time step and grid dimension are strictly limited. In our simulation, the maximum initial plasma density will be a bit lower than normal condition to meet the limitation which will not influence the conclusion.

III. Discussion of result

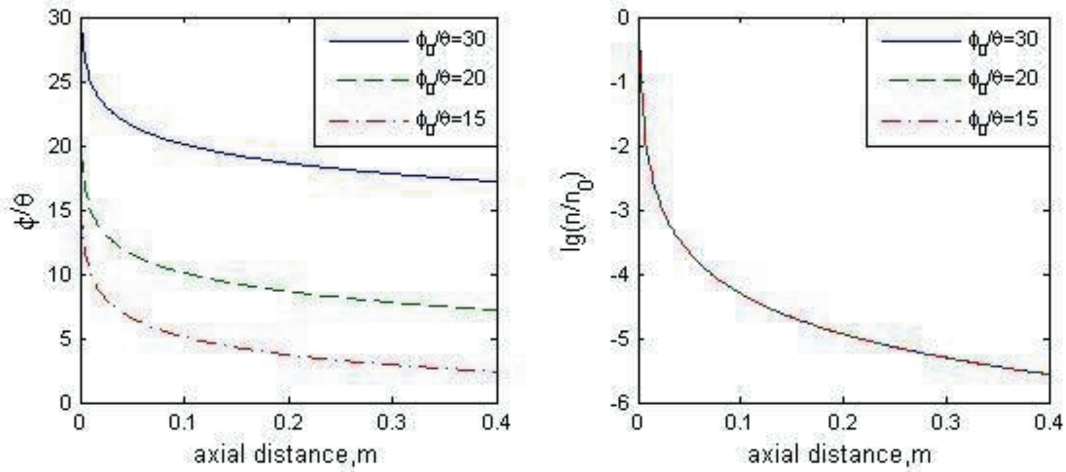
As claimed before, the isothermal model is only appropriate for the working condition that the keeper voltage is high and the electron emission current is low which is called the plume mode of a hollow cathode. There is no condition limitation in theoretical model. Numerical model can get more information to find the deficiency in the theoretical model and the theoretical model can help prove the rationality of the numerical model in this particular working condition. Two important parameters, plasma potential and density are discussed in the plume structure. The influences of electron temperature, plasma potential and the plasma density at the keeper on plume structure are calculated and made comparison separately.

A. Plume Structure from theoretical model

In the theoretical model, the point r_0 is hard to choose. To make it simple, we just assume that r_0 is the radius of the keeper orifice. The plume structure calculated by theoretical model is got first. Equation (9) can be inferred based on the hypothesis of quasinetrual.

$$\frac{I_i/q}{4\pi r^2\left[V_0^2+2e\left(\phi_0-\phi\right)/M\right]^{1/2}} = \frac{I_s/q}{4\pi r_0^2\left(\dfrac{\theta}{m}\right)^{1/2}}2\sqrt{2}e^{-\phi_0/\theta}\left[G_+\left(\frac{\phi}{\theta}\right)-\left(1-\frac{r_0^2}{r^2}\right)^{1/2}H\left(\frac{\chi}{\theta}\right)\right] \tag{9}$$

Fig.3 shows the plume structure calculated by the theoretical model. The dimensionless plasma potential (ratio of plasma potential and electron temperature) distribution is only determined by the ratio of plasma potential and electron temperature at the exit of the keeper orifice. While the dimensionless plasma density (common logarithm of

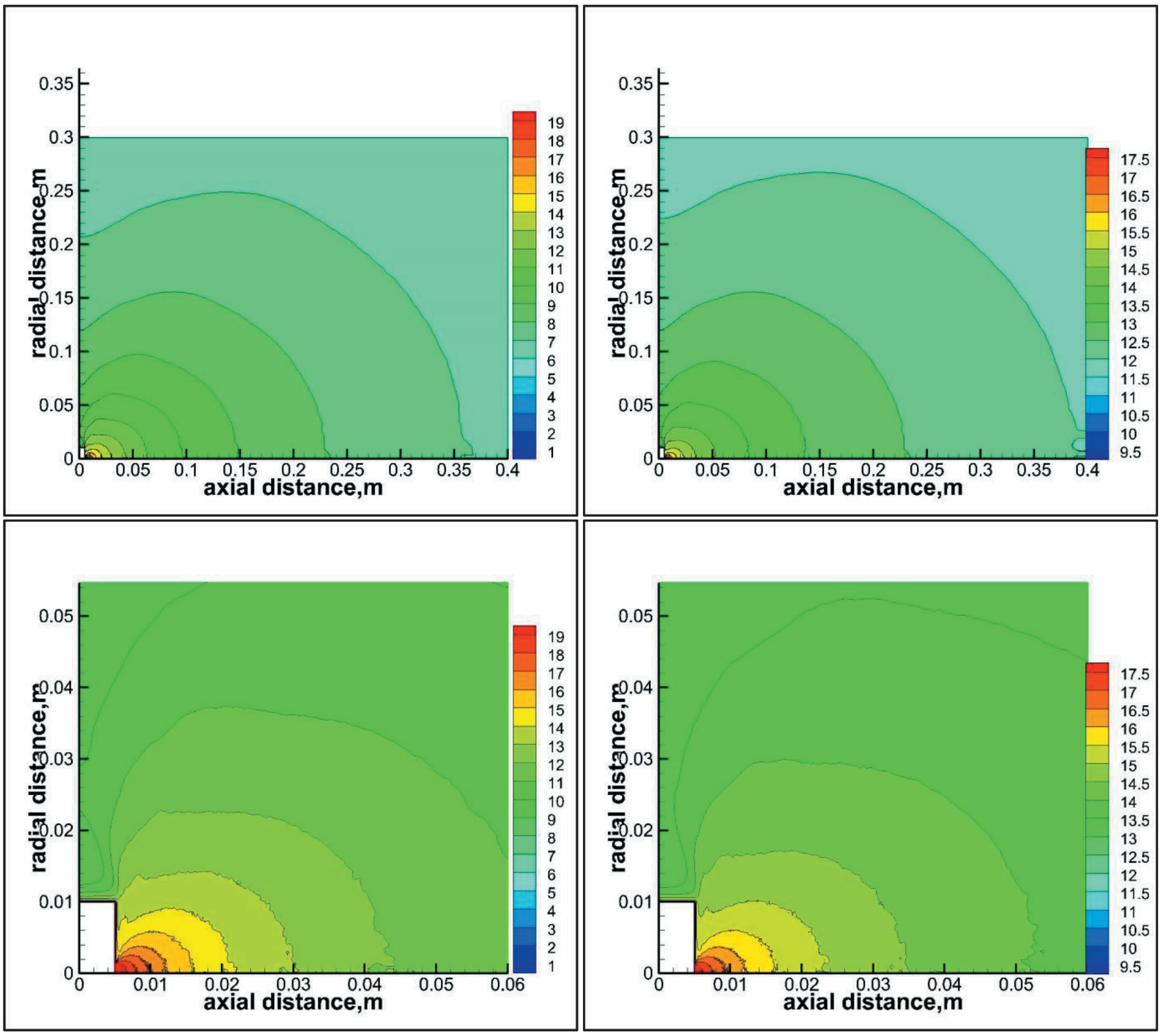


Plasma potential **Plasma density**

Figure 3. Plume structure calculated by theoretical model.

the ratio of density and keeper orifice density) stays in the same trend.

In order to simplify the theoretical model to make the analysis, it is assumed that the ions are moving to the all directions at the same velocity. But in actual situation, only the axial direction is the main flow direction of the hollow cathode, while the radial velocity of the particles is much smaller than that of axial. It may not be accurate if the plume structure is applied to the radial direction.



Plasma potential **Plasma density**

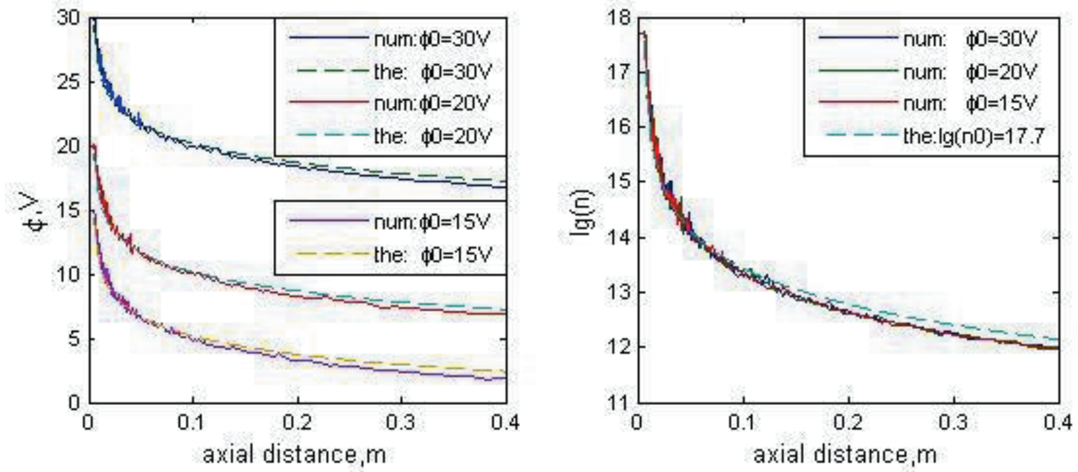
Figure 4. Plume structure distribution of numerical model.

B. Comparison of plume structure

Fig.4 shows the plume structure calculated by the numerical model when $\phi_0 = 20\text{V}$, $\theta = 1\text{eV}$ and $n_0 = 10^{17.7}\text{m}^{-3}$. For the speed of ions is too slow, its movement is influenced much by the self-consistent potential. The radial distribution cannot be ignored.

Electron temperature, plasma potential and density at the exit of keeper plate are changed to see the plume structure in the numerical simulation. All the axial data is listed and put in one figure with theoretical model results for each comparison. To see the radial distribution, data from the line deviating a certain angle (0^o, 30^o, 60^o, 80^o chosen) from the axis are got in only one working conditon following and only plasma potential is showed.

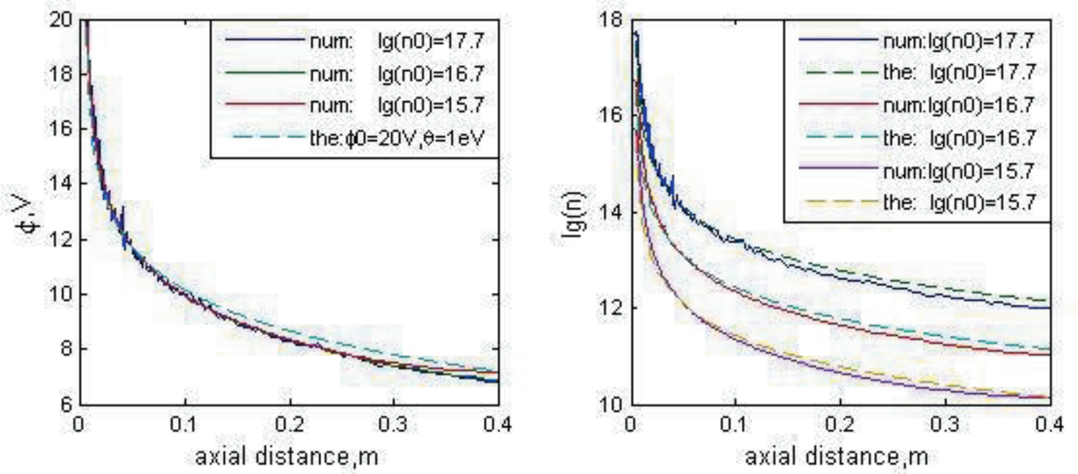
1. Changing the initial plasma potential ϕ_0



Plasma potential **Plasma density**

Figure 5. Plume structure distribution of numerical model (changing potential).

In this comparison, the constant variables are $\theta = 1\text{eV}$ and $n_0 = 10^{17.7}\text{m}^{-3}$, while the initial plasma potential is changing to $\phi_0 = 30\text{V}, 20\text{V}, 15\text{V}$. The results are showed in Fig.5. When the initial plasma potential changes, the plasma potential changes also but it is approximated to that of the theoretical model. The plasma density does not change.



Plasma potential **Plasma density**

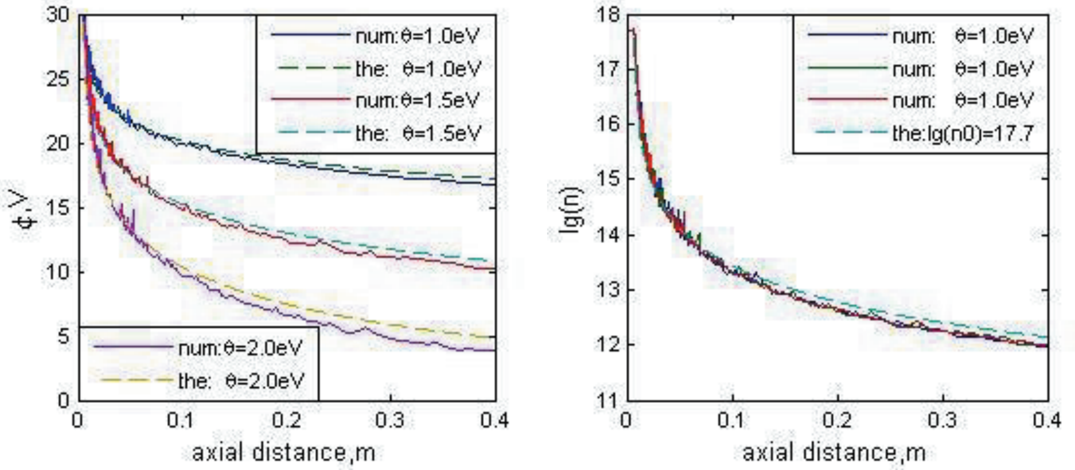
Figure 6. Plume structure distribution of numerical model (changing density).

2. Changing the initial plasma density n_0

In this comparison, the constant variables are $\theta = 1\text{eV}$ and $\phi_0 = 20\text{V}$, while the initial plasma density is changing to $n_0 = 10^{17.7}\text{m}^{-3}, 10^{16.7}\text{m}^{-3}, 10^{15.7}\text{m}^{-3}$. The results are showed in Fig.6. When the initial plasma density changes, the plasma density changes also but it is approximated to that of the theoretical model. The plasma potential does not change.

3. Changing the electron temperature θ

In this comparison, the constant variables are $\phi_0 = 30\text{V}$ and $n_0 = 10^{17.7}\text{m}^{-3}$, while the electron temperature is changing to $\theta = 2.0\text{eV}, 1.5\text{eV}, 1.0\text{eV}$. The results are showed in Fig.7. When the electron temperature changes, the plasma potential and density change like the situation which changes the initial plasma potential.

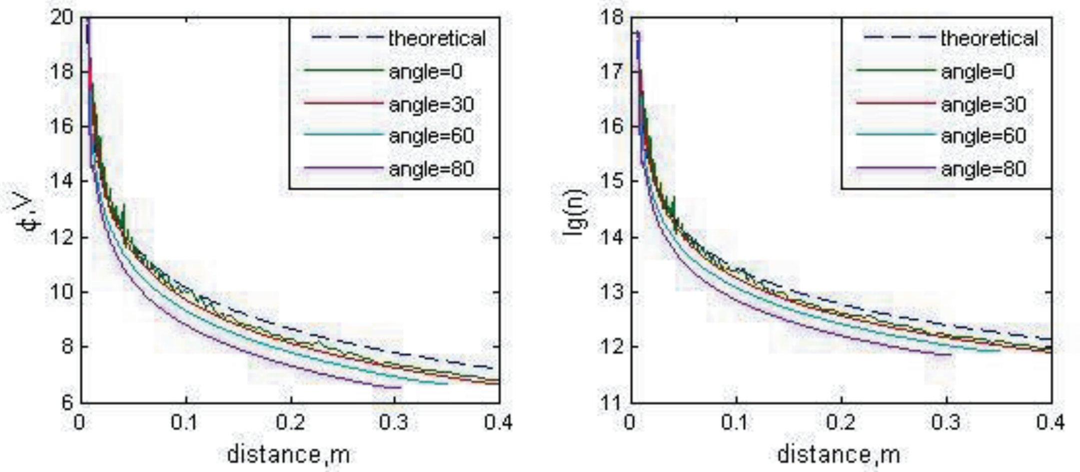


Plasma potential **Plasma density**

Figure 7. Plume structure distribution of numerical model (changing temperature).

4. Radial distribution

In this comparison, only working condition of $\phi_0 = 20\text{V}$, $\theta = 1\text{eV}$ and $n_0 = 10^{17.7}\text{m}^{-3}$ is chosen to understand the difference of plume structure between main flow direction and other flow direction. The results are showed in Fig.8. As the data line angle deviating from the axis becomes larger, the difference is getting more obvious. In the main flow direction, the direction of the velocity is relatively single and the particles gather together easily so that the plasma potential and density drops which is opposite to the situation in the other flow direction. For the theoretical model can only give the distribution of main flow direction, it is necessary to use the numerical model to get the plume distribution of other direction which is not the main flow direction.



Plasma potential **Plasma density**

Figure 8. Plume structure distribution comparison between main flow direction and other direction.

As we can see, the results of numerical model are similar to those of theoretical model in axial direction but have negligible differences in radial direction. Below a certain axial distance, the potential calculated by numerical model is the same to the theoretical model, but lower after that if it is the main flow direction. In the direction which is not the main flow direction, the distribution parameters calculated by numerical model drop faster than that in the main flow direction. The distribution of density is similar to the potential. In addition, when any one of these two models is employed, if the ratio of plasma potential and electron temperature is the same, the distribution trend of the plasma potential and density will not change, especially in the main flow direction. Through our analysis, the numerical model has the same conclusion to the theoretical model when the working condition is the plume mode.

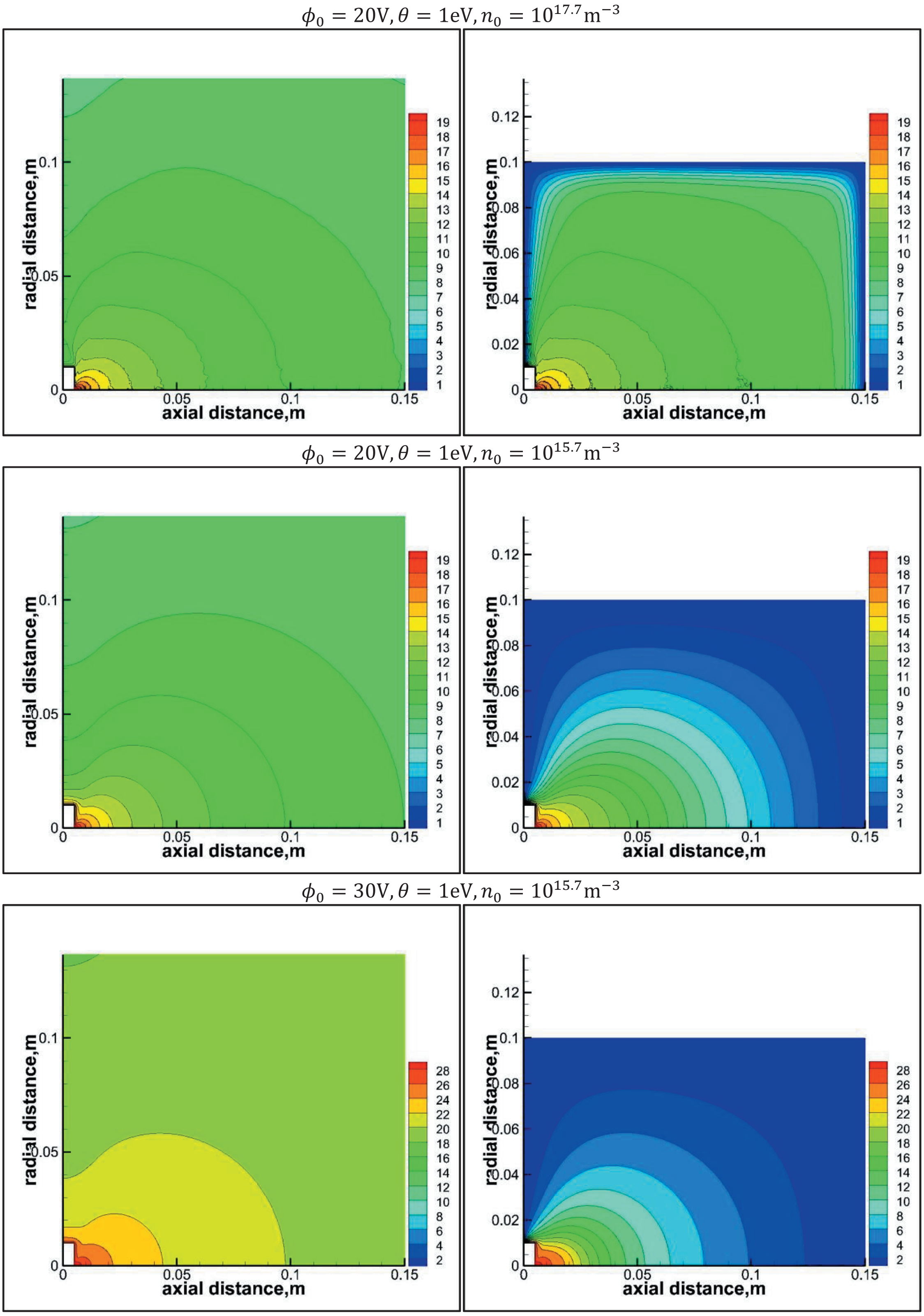


Figure 9. Influences of vacuum chamber on plume structure (plasma potential): free expansion (left) limited expansion (right).

C. Discussion of influence of the vacuum chamber

The influence of vacuum chamber on plume structure is discussed. Classical collisionless sheath layer model is used. Due to the isothermal model, the distribution of electron fits well to the sheath model. As we know in part B, the ratio of potential and temperature at keeper orifice can be put into one changeable parameter, so only potential and density is changed. In order to capture the details, the small calculation field is employed. Although in reality the vacuum chamber is much larger than this dimension, it is useful to make analysis quickly. Three working conditions are given: 1) $\phi_0 = 20V, \theta = 1eV, n_0 = 10^{17.7}m^{-3}$; 2) $\phi_0 = 20V, \theta = 1eV, n_0 = 10^{15.7}m^{-3}$; 3) $\phi_0 = 30V, \theta = 1eV, n_0 = 10^{15.7}m^{-3}$.

Fig.9 shows the influences of vacuum chamber on plume structure in different working conditions. Figures on the left side are calculated by the large computation field to simulate the plume structure when the plasma flow is free. Figures on the right side are calculated by the small computation field to simulate the plume structure when the flow is limited by vacuum chamber. Fig.10 gives the position of the sheath boundary and radial axe gives the potential ratio of free expansion and limited expansion.

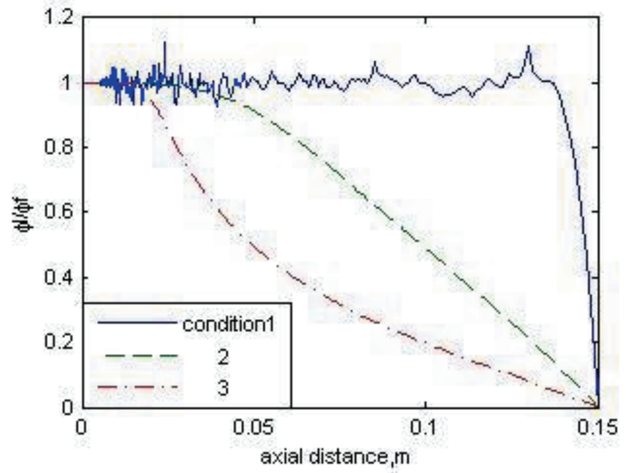


Figure 10. Axis data comparison between free expansion and limitedexpansion

If the plume is surrounded by chamber walls, there will be difference. A sheath layer is produced to overcome the potential difference and all the potential drop will be completed in several Debye lengths. If the plasma density near the wall is high (top of Fig.9), little difference will be made in the main plume structure which is the blue curve in Fig.10. When the plasma is low, the Debye length becomes non-neglectful and the abnormal distribution will influence the main plume what can be seen in middle and bottom of Fig.11. Besides, higher voltage near the wall will make it worse. From the previous results, potential distribution only depends on ratio of potential and temperature at the keeper plate while the gradient of density changes little. For example, if the axis length of a vacuum chamber is 1m, while initial potential is 20V and density is as low as $10^{16}\mathrm{m}^{-3}$ when a large anode is used with the hollow cathode, the voltage and density near the wall will be 5V and $10^{9}\mathrm{m}^{-3}$. The sheath layer may be about 0.2m. So, the influence of vacuum chamber on plume structure will be huge when the plasma density is low and the potential is high.

IV. Conclusion

A theoretical model which is established by Parks and Katz is chosen to calculate the hollow cathode plume structure. A numerical model with ions using PIC and electrons using isothermal model is used to calculate the plume structure in the working condition of plume mode. Plume structure calculated by the numerical model is approximated to the theoretical model in the main flow direction. The plume parameters drop faster in other direction and the difference between numerical and theoretical model is negligible in this situation. So it is not enough to make the one-dimension simulation (like the theoretical model) to get the details of the hollow cathode plume structure. When the model employed in this paper is used, plasma potential and density distribution trend only depends on the ratio of initial plasma potential and electron temperature. The influence of vacuum chamber should be considered if the hollow cathode is working in the condition of high voltage and low density if the dimension of the chamber is not enough.

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