TRANSVERSE EXPANSION OF PLASMA PLUMES AND PLASMOIDS IJECTED FROM ELECTRIC THRUSTERS

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ABSTRACT

The results of the dynamics problem solution describing parameters in plasma flows injected from stationary and pulsing electric thrusters are peresented. Self-similar distributions of parameters in axis symmetrical plasma jets and in three-dimentional non-stationary plasmoids, which expand into vacuum, are obtained.It is shown that internal self-agreed E-field and heat flow significaly influence on flow expansion and plasma plumes parameters distribution.

INTRODUCTION

For the description of a supersonic ($M^2 \gg 1$) rarefied plasma flow the model of free molecular outflow from a point source [1] is often used For more dense flows the simple Robert's approximation [2] is used The continual flow models, for example [3] and [4], are developed. Transport processes in gases are taken into the account in the models. Such models as well as flow models for freely expanding gas based on kinetic equations require tedious numerical calculations [8]. All these dynamics flow models are good enough to describe the dense core of a flow, but not so accurate for description of the peripheral regions of plasma plumes or plasmoids. This suggestion is confirmed by direct comparison between theoretical and experimental distribution of parameters for certain plasma and gas sources [5].

However these peripheral regions are extremely important since they determine the actual boundaries of arising inhomogeneities. These regions are essentially responsible for RF wave refraction/scattering, plasma plumes interaction on a spacecraft and for electron collection by tether plasma contactors.

The objective of the work is development and testing of mathematical models of exhausted plasma plumes and plasmoids in space with due regard for actions of inner electric fields and thermal conductivity in plasma.

PROBLEM FORMULATION ON FREE PLASMA EXPANSION

The combined Braginsky's equations for two- components fully ionized plasma are used to describe steady state plasma jet or temporal plasmoid evolution [12]. This equations system can be simplified if dissipative effects due to relative motions of electrons and ions i.e. due to electric current flowing and viscosity, are rather weak.Under the said conditions of free plasma expansion all the dissipative effects mentioned above can be evaluated using a dimensionless parameter Π. For the case of a srtetched supersonic flow the parameter is as follows :

$$\Pi = \frac{б T_e a}{e^2 N}$$

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The analogous parameter can be received for the plasmoid. The numerical evaluation suggests that equation terms listed above can be neglected when the value of $\Pi < 1$ corresponding to rather dense plasma of moderate temperature.

In gently expanding jet $a/l \ll 1$, the relative impacts of transversal and longitudinal thermal fluxes are quite different since: $l_{Tx} \sim 1$; $l_{Tr} \sim a$; $\Pi_{qr} = \Pi \cdot l/a$; $\Pi_{qx} = \Pi \cdot a/l = \Pi_{qr} \cdot a^2/l^2$. Thus different thermal modes of jet expansion are possible :

when $\Pi_{qx} \ll 1$ and $\Pi_{qr} \ll 1$ - adiabatic expansion;
when $\Pi_{qx} > 1$ and $\Pi_{qr} > 1$ - isothermal one ;
when $\Pi_{qx} < 1$ and $\Pi_{qr} > 1$ - the temperature is uniform transversely and adiabatically changes along the jet axis.

The initial set of the equation can be simplified by composing the equations for components. For the condition of

$$\Pi \cdot a/l \ll 1$$

it can be described as :

$$\frac{\partial n}{\partial t} + \nabla n V = 0 ;$$

$$mn \left(\frac{\partial V}{\partial t} + \vec{V}\nabla\vec{V} \right) = - \nabla(nT) ; \qquad T = T_e + T_i$$

$$\frac{3}{2} n \left(\frac{\partial T}{\partial t} + V\nabla T \right) + nT\nabla\vec{V} = -\nabla(\vec{q}_i + \vec{q}_e) ;$$

$$\nabla\vec{j} = 0 ;$$

$$\vec{j} = б\,(-\nabla\Phi + \nabla nT_e/ne + \beta \cdot \nabla T_e/e).$$

The first three equations do not depend on electric quantities j and Φ. All the effects of mutual interaction between electron and ion species including the influence of self consistent electric field finally resulted in substitution of T_i by $T=T_i+T_e$ and appearance of term (q_i+q_e). Since often $T_e \gg T_i$ and practically always $q_e \gg q_i$ the influence of electron component on the ion motion is rather sufficient. Formally the three equationsare analogous to equations for neutral gas so that the results presented below are also valid for gas flows.

The latter two equations of the system describe the influence of the flow dynamics on the inner distribution of electric fields and currents.

EXPANSION OF SUPERSONIC PLASMA JET IN VACUUM

The set of equations when cylindrical coordinates x and r are used descriubes steady state plasma jet or a gas :

$$\frac{\partial nu}{\partial x} + \frac{\partial rn\upsilon}{r\partial r} = 0;$$

$$mn \left(u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial r} \right) = - \frac{\partial nT}{\partial x} ; \qquad (1)$$

$$mn\left(u\frac{\partial \upsilon}{\partial x} + \upsilon\frac{\partial \upsilon}{\partial r}\right) = -\frac{\partial nT}{\partial r};$$

$$\frac{n}{\gamma-1}\left(u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial r}\right) + nT\left(\frac{\partial u}{\partial x} + \frac{\partial r\upsilon}{\partial r}\right) = \nabla q,$$

where u and υ are velocity components along coordinates x and r; γ is adiabat index.

These equations can be applied under the following conditions:

$$K_n = \frac{\lambda}{a} \ll 1 \;;\quad \Pi_q = \frac{q\cdot a}{nuT} \ll 1$$

where λ is the length of ion free path in the jet.

For neutral gas these conditions correspond only to the initial region of relatively dense flow core, whereas for adiabatically expanding fully ionized plasma these conditions are valid in much more wide range of n variation because the Coulomb cross-sectional area, S, being proportional to T^{-2} grows as temperature decreases e.i.

$$K_n \sim a^{-5/3},\ \Pi_q \sim a^{-7/3},\ \text{if } \gamma = 5/3 \qquad (2)$$

The self-similar form of solution for the system (1) on the analogy with [9] can be written as:

$$u = u_c(x)\cdot y(\eta);\ \upsilon=u\cdot da/dx\cdot\eta;\ T=T_c(x)\cdot\tau(\eta);\ nu=\dot{N}\nu/\pi a^2\cdot f(\eta), \qquad (3)$$

where $\eta=r/a$ is a self-similar variable quantity;
$r=a=a(x)$ is a flow current line (see Fig.1);
"c" is an index corresponding to the flow axis

Figure 1 illustrates the expected flow current line and expected view of functions f, y and τ. Assume that $f(0) = y(0) = \tau(0) = 1$, and ν is a numerical multiplier defined from the condition:

$$\int_0^\infty nu2\pi r\, dr = \dot{N} \quad \text{or} \quad \nu^{-1} = \int_0^\infty f2\eta\, d\eta.$$

Substitution of relationship (3) in the set (1) reduces the equation of continuity to an identity and rearranges other equations to the following forms:

$$fy = \frac{1}{(1 + \eta^2\cdot C_1/C_2)^{C_2/2+1}}$$

$$\frac{\tau}{y^2} = 1 + \frac{C_1}{C_2}\eta^2$$

$$\frac{mu_c'}{a^2} + \left(\frac{T_c}{u_c a^2}\right)' = 0$$

$$\frac{mu_c a}{T_c} (u_c a')' = C_1$$

$$\frac{N\nu}{\pi} \frac{T_c}{a^2} \left(T_c^{1/(-1)} \cdot u_c a^2 \right)' f\tau = \nabla q$$

$$\frac{T_c}{T_0} = \frac{u_c}{u_0} \left(\frac{a}{a_0} \right)^{2-C_2}$$

The self-similar variable quantities are separeted if the two equalities are valid :

$$\frac{mu_c'}{a^2} = -\left(\frac{T_c}{u_c a^2} \right)' = \frac{T_c}{u_c a^2} \frac{a'}{a} C_1$$

The first relationship is obvious when η=0, and the second one is additional that is necessary for self-similar flow in two dimentions.

As a result we have got 4 equations for three functions $u_c(x)$, $T_c(x)$ and $a(x)$:

$$\frac{mu_c^2}{2} + \frac{\gamma}{\gamma-1} T_c = \text{const}; \qquad \frac{mu_c a}{T_c} (u_c a')' = C_1;$$

$$T_c^{1/(\gamma-1)} \cdot u_c a^2 = \text{const}; \qquad \frac{T_c}{u_c} a^{(C_2-1)} = \text{const};$$

where C_1 and C_2 are constsnts of variable quantities separation x and η.

The additional relationship can be identical to the adiabat equation if u_c = const,i.e. the plasma jet reached it maximum velocity while expanding.

$$u_m^2 = u_0^2 [1 + 2 \cdot M_0^{-2} \cdot (\gamma - 1)^{-1}] .$$

In this case the separation constsnt $C_2 = 2\gamma$ is defined.

For two functions $f(\eta)$,$\tau(\eta)$ and $y(\eta)$ there are only two relationships :

$$\frac{\tau}{y^2} = 1 + \frac{C_1}{C_2}\eta^2 \quad ; \qquad (fy)^{-1} = (1 + \frac{C_1}{C_2}\eta^2)^{\gamma+1} \tag{4}$$

To make the equations expression more compact $C_1 = C_2$ is chosen which is an equvalent to "a" as the radius index when η=1

$$\frac{\tau}{y^2} = 2 \ ; \qquad \frac{nu_c^2}{n_0 u_0^2} = fy = \frac{1}{2^{\gamma+1}}$$

Thus the jet parameters change along the axis in the following way :

$$u_c = u_m \; ; \quad T_c a^{2(\gamma-1)} = T_0 a_0^{2(\gamma-1)} \; ; \; mu_m^2 aa'' = 2T_0 (a_0/a)^{2(\gamma-1)} \qquad (5)$$

The cross dependences for adiabatic flow can be described as :

$$\frac{nu^2}{n_c u_c^2} = fy = (1 + \eta^2)^{-\ -1} \quad ;$$

$$\frac{nT}{n_c T_c} = \frac{f\,\tau}{y} = (1 + \eta^2)^{-} \quad ;$$

$$\frac{M_x^2}{M_c^2} = \frac{y^2}{\tau} = (1 + \eta^2)^{-1} \quad ;$$

Two particular cases for adiabatic flow are representative ($\tau=1$ and $y^2(1+a'^2\eta^2) = 1$.

The former situation ($dT/dr = 0$) is typical for the initial part of rarefied plasma flows and suggests that temperature is uniform in the flow cross sectional plane in vicinity of the flow pole due to high thermal conductivity of the plasma ($\Pi qr>1, \Pi qx<<1$).

The latter case is inherent to the dense flow when enthalpy is constant in the cross sectional plane of the plasma jet.

With $\tau = 1$ the problem solution can be written as:

$$n = \frac{\dot N\cdot(\gamma-1/2)}{\pi a^2 u_m}\;\frac{1}{(1+r^2/a^2)^{\gamma}} \; ; \qquad u = u_m\cdot\frac{1}{(1+r^2/a^2)^{1/2}} \; ;$$

$$T = T_{co}\cdot(a_0/a)^{2(\gamma-1)} \; ; \qquad \upsilon = u\,\frac{r}{a}\,a' \qquad (6)$$

$$a \approx a_0 + a_m'(x-x_0) \; ; \quad (a'_m)^2 = (a'_0)^2 + 2/((\gamma-1)\cdot M_0^2) \; ;$$

For dense flow with V^2 = const we obtain:

$$n = \frac{\dot N\cdot \nu}{\pi a^2 u_m}\;\frac{1 + a'\cdot r^2/a^2}{(1+r^2/a^2)^{\nu+1}} \quad ; \qquad u = \frac{u_m}{(1 + a'^2\cdot r^2/a^2)^{1/2}} \quad ;$$

$$T = T_{co}\cdot\left(\frac{a_0}{a}\right)^{2(\gamma-1)}\;\frac{1 + r^2/a^2}{1 + a'^2\cdot r^2/a^2} \; ; \quad \upsilon = u\,\frac{r}{a}\,a' \qquad (7)$$

$$a \approx a_0 + a'_m(x-x_0) \; ; \quad (a'_m)^2 = (a'_0)^2 + 2/((\gamma-1)\cdot M_0^2).$$

These formulas suggest that the distributions of plasma flow parameters in the peripheral region of the jet where $a \gg a_0$ are defined by the particles flow rate, N, the ultimate plasma velocity, u_m, and the factor of flow divergency, $k = 1/(a'_m)^2$.

The curved lines R corresponding to Robert's approximation [2] and A-self-similar distribution are presented in Fig.2

They coinside with an experimental curved line near the axis quite well. But far from the axis the density indexes recieved by using Robert's approximation become less than experimental ones, whereas the indexes received by self-similar problem solution coinside with the results of experiments [5] in the wide range of plasma concentration changing.

Dependance a(x) calculated for the experiment in accordance with (5) is presented in Fig.2.Calculated quantities a' and k' for cross sectional planes x=0.7m and x=1.2m are also presented in Fif.2.

PLASMOID EXPANSION IN VACUUM

For three dimensional non-steady-state plasmoid this system in orthogonal coordinates x, y, z is written as:

$$\frac{\partial n}{\partial t} + \frac{\partial nu}{\partial x} + \frac{\partial n\upsilon}{\partial y} + \frac{\partial nw}{\partial z} = 0 \qquad (8)$$

$$mn\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) + \frac{\partial nT}{\partial x} = 0 \qquad (9)$$

$$mn\left(\frac{\partial \upsilon}{\partial t} + u\frac{\partial \upsilon}{\partial x} + \upsilon\frac{\partial \upsilon}{\partial y} + w\frac{\partial \upsilon}{\partial z}\right) + \frac{\partial nT}{\partial y} = 0 \qquad (10)$$

$$mn\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + \upsilon\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) + \frac{\partial nT}{\partial z} = 0 \qquad (11)$$

$$\frac{1}{\gamma-1}\, n\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right) + nT\cdot\nabla\vec{V} = \nabla\vec{q} \qquad (12)$$

Here u, υ, w - velocity components along ccordinates x, y, z respectively.

By analogy with the self-similar solution of non-steady-state one-dimensional set of gas dynamics equations [9] it is reasonable to suppose that velocity dependencies on coordinates are linear and can be written as:

$$u = a\frac{x}{a}\ ;\quad \upsilon = b\frac{y}{b}\ ;\quad w = c\frac{z}{c} \qquad (13)$$

where a = a(t), b = b(t) c = c(t) are characteristic sizes of the plasmoid along axes x,y,z respectively.Concreate equations for these unkown functions will be specified later.

Taking into consideration the results obtained for the one-dimensional gas expansion [9] and the two-dimensional plasma jet expansion described above it is possible to find the expression for plasma concentration within the plasmoid in the following :

$$n = \frac{N\,\nu}{8abc}\cdot f_1(\eta)f_2(\xi)f_3(\zeta)\ ; \qquad (14)$$

$$T = T_c(x)\,\tau\,(\,\eta,\ \xi,\ \zeta\,)$$

where" ν" is a normalization factor defined from the integral

$$\iiint_{\infty}^{\infty} n\ dxdydz = N$$

and f_1, f_2 and f_3 are functions of self-similar variables

$\eta=\dfrac{x}{a}$; $\xi=\dfrac{y}{b}$; $\zeta=\dfrac{z}{c}$ such that $f_1(0) = f_2(0) = f_3(0) = 1$.

Substitution of Eqs. (13) and (14) reduce the equation of continuity from system (8) to an identity. Motion equations (9,10,11)are rearranged into the equation with separated variables by substitution of (13) and (14) :

$$\frac{ma}{T_c}\,\frac{d^2a}{dt^2} = -\,\frac{d(f_1\tau)}{d\eta}\,\frac{1}{f_1\cdot\eta} = C_1$$

$$\frac{mb}{T_c}\,\frac{d^2b}{dt^2} = -\,\frac{d(f_2\tau)}{d\xi}\,\frac{1}{f_2\cdot\xi} = C_2 \qquad (15)$$

$$\frac{mc}{T_c}\,\frac{d^2c}{dt^2} = -\,\frac{d(f_3\tau)}{d\zeta}\,\frac{1}{f_3\cdot\zeta} = C_3$$

where C_1, C_2, C_3 are constants of separation.

The most simple and visual solution corresponds to the case of uniform temperature throughout the plasmoid, e.i. $\tau = 1$.

This case is realized for a "hot" plasmoid when the role of heat conductivity in plasma is sufficient, especially under condition of no energy exchange between the plasmoid and the environment. In this case:

$$f_1 = \exp\left(-\,\frac{x^2}{2a^2}\cdot C_1\right);\quad f_2 = \exp\left(-\,\frac{y^2}{2b^2}\cdot C_2\right);\quad f_3 = \exp\left(-\,\frac{z^2}{2c^2}\cdot C_3\right);$$

Now the characteristic sizes of a, b, c can be specified as those corresponding to "knee" in dependence of n on coordinates:

$$f_1''(1) = f_2''(1) = f_3''(1) = 0$$

The separation constants can be also determined from these conditions:

$$C_1 = C_2 = C_3 = 1 \qquad (16)$$

For a "heat insulated" plasmoid with uniform temperature the energy equation (12) is rearranged into the adiabatic relationship:

$$T_C = T_0 \left(\frac{n}{n_0}\right)^{\gamma-1} = T_0 \left(\frac{a_0 b_0 c_0}{a\ b\ c}\right)^{\gamma-1} \tag{17}$$

Substitution of Eqs. (16) and (17) rearranges the system (15) to a set of non-linear differential equations for definition of time dependent sizes of the plasmoid a(t),b(t),c(t) :

$$maa'' = mbb'' = mcc'' = To \left(\frac{a_0\ b_0\ c_0}{a\ b\ c}\right)^{\gamma-1} \tag{18}$$

Here dots denote derivatives with respect to time, t.

If the size and time scale are defined as a_0 and $t_0 = \sqrt{m a_0^2 / T_0}$, respectively,the set can be rewritten in the dimensionless form:

$$X\ X'' = Y\ Y'' = Z\ Z'' = \left(\frac{B_0\ C_0}{X\ Y\ Z}\right)^{\gamma-1} \tag{19}$$

where $X = a/a_0$; $Y = b/a_0$; $Z = c/a_0$; $B_0 = b_0/a_0$; $C_0 = c_0/a_0$.

Fig.3 illustrates a sample of this problem solution.

The characteristic property of the three-dimensional expansion is non-uniform increase of plasmoid acceleration along different axes.

If the initial velocities are low as compared with the mean random velocity, $\upsilon_T \sim \sqrt{T_0/m}$, the resulting velocity is maximal along the coordinate where the initial size of the plasmoid is minimal.Consequently the resulting size of the plasmoid along this coordinate finally occurs to be maximal.So the inversion of sizes takes place [10].

For example, a plasmoid initially generated as a "knitting needle" stretched along axis x, later transforms into a disk with minimal x dimension.On the contrary a dense disk can transform in a long "knitting needle".The phisical reason for such inversion is clear enough: the thermal energy initially accumulated by the plasmoid, gradually transforms into the kinetic energy of expansion predominantly in the direction of maximal gradient of electric potential or which is an equivalent, along the maximal gradient of total pressure $\nabla(nT_i + nT_e) \approx \nabla nT_i + en\nabla\Phi$.

CONCLUSION

The dynamical problem describing evolution of both fully ionized supersonic plasma jet and ellipsoid plasmoid injected from a spacecraft is formulated and solved in a self-similar form.

For some interesting cases for plasma jet with uniform temperature or enthalpy in the flow cross sectional plane the solution was obtained in a simple analytic form.

The comparison between the obtained theoretical results and the experimental data proves that the self-similar solution describes the peripheral regions of the flow more correctly than the often used Robert's[2] and Narasimba's[1] approximations.

The most noticeable feature of three dimensional plasmoid expansion is the inversion of its shape.

It is believed that self-similar solutions can be obtained for some other thermal modes of plasma expansion.

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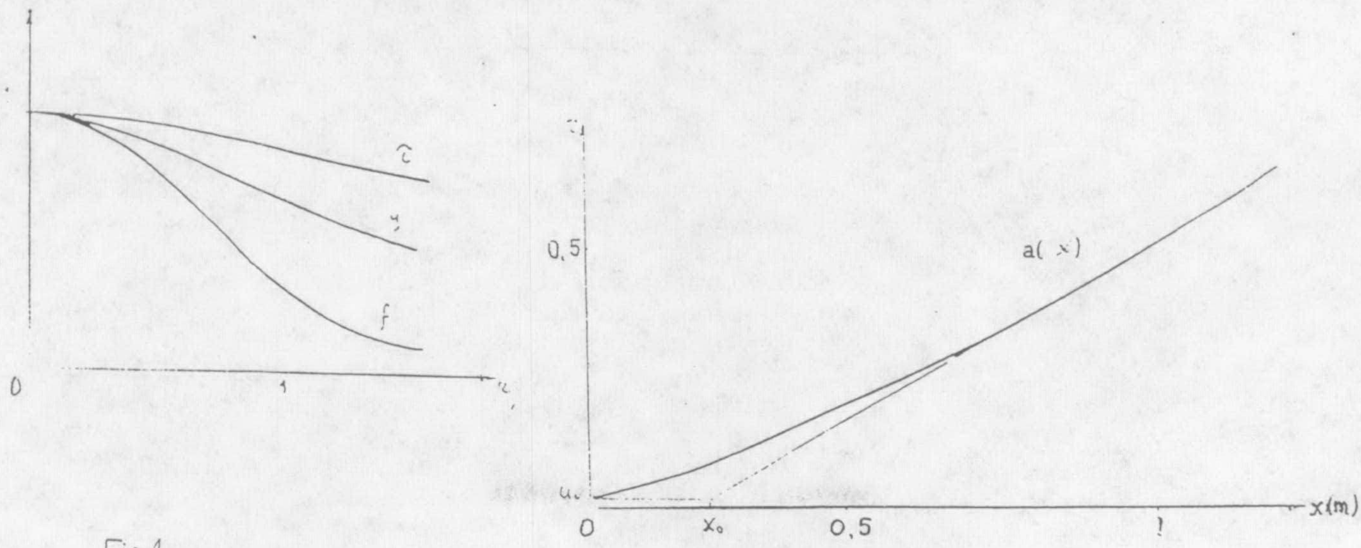
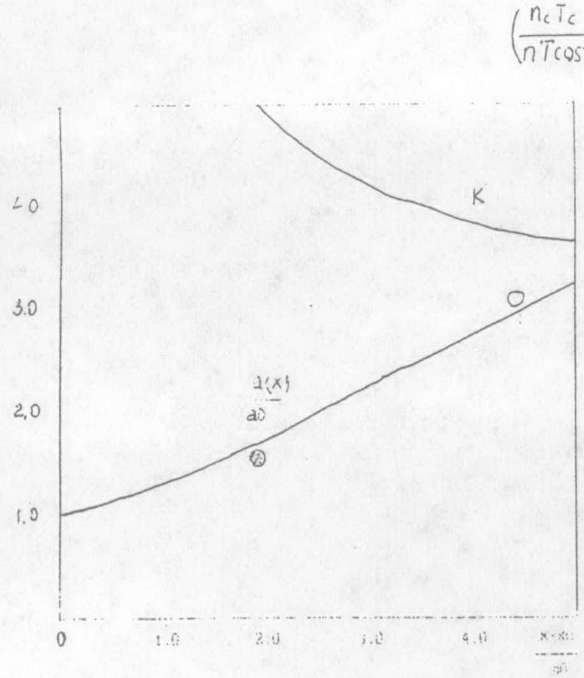


Fig.1



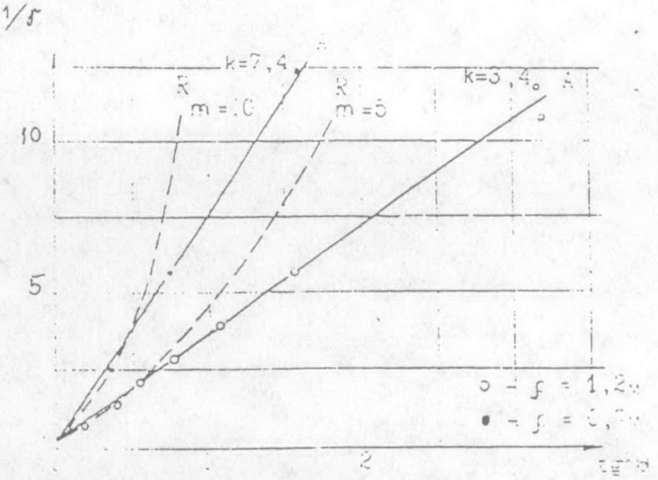


Fig 2 [illegible]
$\dot{N} = 2\cdot 10^{20}$ 1/s; $u = 2\cdot 10^{3}$ m/s; $T_0 = 0.3$ ev;
$M_0^2 = 11$, $\Pi = 3\cdot 10^{1}$; $x_0 = 0{,}3$ m; $a_0 = 0{,}2$ m $a_0' = 0{,}25$

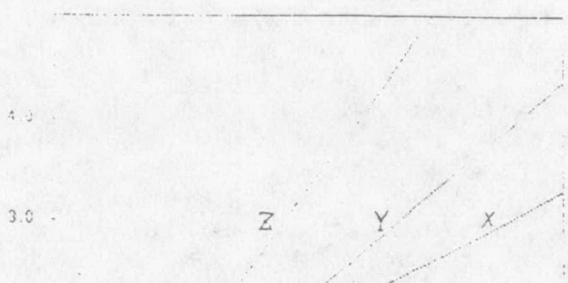


Fig.3 Tree-dimensional expansion of plazmoid.
Initial data (t=0): X=1, Y=0.5, Z=0.2,
$\dot{X}$=0, $\dot{Y}$=0, $\dot{Z}$=0.