

Performance Impacts of Geometry and Operating Conditions on a Low Reynolds Number Micro-Nozzle Flow

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With the rise of CubeSat usage there is a growing use-case for low Reynolds number nozzles as a component within micro-propulsion systems. However, design of such systems is difficult due to the viscous losses encountered in such low Reynolds number flow regimes. As part of a program to develop a design tool for efficient micro-nozzle design, the performance of multiple nozzle geometries is measured across a range of Reynolds numbers and temperatures to create an experimental database for model validation. The thrust and specific impulse efficiency are measured using a torsional thrust balance at flow rates covering Reynolds numbers in the range of 65 – 807 for 295-523 K nitrogen. Measured performance indicates larger nozzle angles and smaller area ratios improve nozzle efficiency.

Nomenclature

A_e	= exit area of the nozzle	MW	= molecular weight of the propellant
AR	= area ratio of the nozzle	p	= gas pressure
A^*	= throat area of the nozzle	p_0	= gas pressure inside the thrust chamber
C	= Sutherland's constant	p_a	= ambient pressure
C_d	= discharge coefficient	p_e	= pressure at the exit plane of the nozzle
C_T	= thrust coefficient	R	= gas specific constant
d_{N2}	= collisional diameter of nitrogen = 364 pm	Re^*	= throat Reynolds number
d^*	= throat diameter of the nozzle	T_0	= gas temperature in the thrust chamber
F_T	= force created by the test thruster	T_e	= exit temperature of the propellant
I_{sp}	= specific impulse	$u_{e,s}$	= isentropic exit velocity
$I_{sp,s}$	= specific impulse for an isentropic nozzle	α	= nozzle angle
k_B	= Boltzmann's constant = 1.3806×10^{-23} J/K	γ	= ratio of specific heats = 1.4
L	= characteristic length scale	η_n	= nozzle efficiency
$\dot{m}$	= mass flow rate of propellant	λ	= gas constant in Sutherland's formula
M_e	= exit mach number	μ	= gas viscosity

I. Introduction

There is growing interest in CubeSats due to increased space access made possible by launch vehicle ride-sharing. Organizations that would otherwise be inhibited by the cost barriers of dedicated launches are actively conducting space missions. Additionally, CubeSats enable low-cost multi-vehicle mission concepts, such as orbital constellations for communications networks. However, current CubeSats generally contain little to no propulsive capability, which drastically limits orbital life. As an example, the Naval Research Laboratory (NRL) launched in 2010 the QbX CubeSat, and with no propulsion system, the orbit had decayed within a month under worst-case solar conditions.¹ Internal studies have found that the use of a small resistojet propulsion system could have extended this life-span to at least four months, and lifetimes of one year are readily achievable under more favorable solar conditions.² This substantial increase in CubeSat utility is limited by the relatively small selection of flight-tested micro-propulsion options. Therefore, the development of low-power micro-propulsion systems, such as a resistojet, is vital to the continued expansion of CubeSat utility.

One of the difficulties with designing resistojet systems for micro-propulsion application is for the thrust levels desired, the nozzle dimensions and flow rates are very small. This results in a very low Reynolds number flow characterized by highly viscous behavior. In large nozzles, the boundary layer in the diverging section of the nozzle is negligible, but for micro-propulsion nozzles the boundary layer can be significant. As part of a program to develop a design tool that can optimize nozzle geometry for a given throat Reynolds number, high fidelity measurements of the resistojet performance are required to validate the modelling tools. The first phase of model validation is to use the macroscopic performance of the nozzle and compare it to aggregate model results. For this work, the chosen performance metrics are thrust and specific impulse efficiency, which is defined as the ratio of the measured specific impulse of the nozzle flow to the expected specific

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impulse assuming isentropic expansion. Previous work in this area has either been limited by relatively large uncertainties compared to nozzle performance variation,^{3,4} or did not have a sufficiently wide parameter space to cover the possible operating conditions.⁵⁻⁸ This work expands on previous efforts conducted at NRL⁹ and presents the experimental characterization of the thrust, specific impulse, thrust chamber pressure and temperature, and nozzle specific impulse efficiency for multiple conical nozzles across the Reynolds number range of 50-800 and temperatures of 300-525 K for nitrogen.

II. Experimental Apparatus

A. Facilities

All testing is performed in NRL's South Vacuum Chamber, which is 2 m diameter by 2.3 m tall. The chamber is pumped with a 48" NRC diffusion pump with a nominal pumping speed of 100,000 l/s for air. The diffusion pump is backed by a Leybold WH2500 blower and a SV-630B roughing pump with a combined pumping speed of 1530 CFM. Base pressure at high vacuum operation is 5.5×10^{-6} Torr. Pressure in the chamber is measured using a Bayard-Alpert type ion gauge in high vacuum, and rough vacuum pressure is measured using a Granville Phillips 275 convectron gauge.

B. Nozzle Test Apparatus

In order to facilitate the testing of multiple nozzle geometries, a modular test bed was designed as a large-scale resistojet with embedded diagnostics. A welded combination of a 1/4 inch Swagelok T junction and a male 3/8 inch NPT pipe union constitutes the body of the test bed with two additional 1/8 inch tube taps added for access to the thrust chamber. A 50 W, 0.25 inch diameter metal-sheathed cartridge heater is passed through the axis of the test bed to provide thermal control of the gas; the rear Swagelok fitting is used to seal the heater connection. The other 1/4 inch Swagelok connection is used as the gas feed. An annular heat exchanger is used to increase convective heat transfer from the cartridge heater to the propellant gas. The nozzle itself is machined into a NPT pipe cap that is secured onto the end of the test bed. The cap is sealed using a combination of anti-seize embedded in the threads and a 0.1 inch thick Graphoil gasket that seats against the inner face of the nozzle cap. Gas temperature is measured using a sheathed K-type thermocouple that extends into the thrust chamber just behind the nozzle cap. A Watlow EZ-Zone controller regulates cartridge heater output with a PID control loop on the thrust chamber thermocouple with a standard deviation of 0.04 K. A pressure tap connects the interior of the thrust chamber to a Varian CDG-500 capacitance pressure gauge with an uncertainty of 0.2%. Figure 1 shows a picture of the nozzle test bed disassembled.

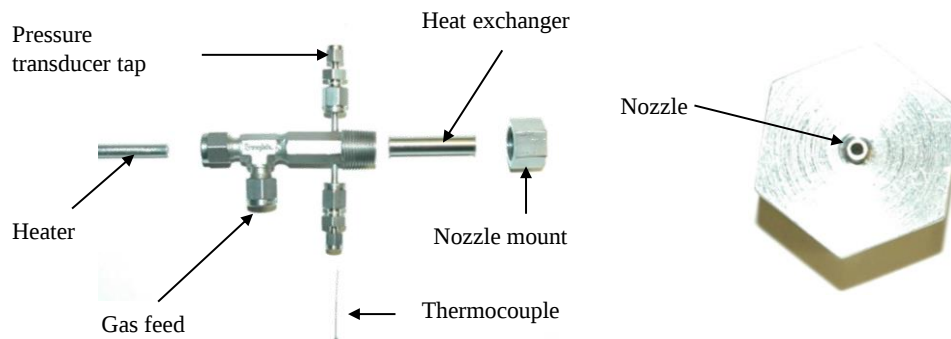


Figure 1. Nozzle test bed (left) and nozzle machined into nozzle mount (right).

C. Nozzles

The modular nature of the test bed allows for multiple nozzles to be tested while maintaining all other conditions constant. In this work, four nozzles were tested to examine the impact of changing the area ratio and nozzle angle. To accommodate the modular nature of the test bed, each nozzle machined into a 3/8-18 female NPT pipe cap that mates to the body of the test bed. The nozzles are fabricated using electrical discharge machining (EDM), with the nozzle cone fashioned through sinker EDM and the throat by wire EDM. An example CAD drawing of the nominal nozzle design is shown below in Figure 2.

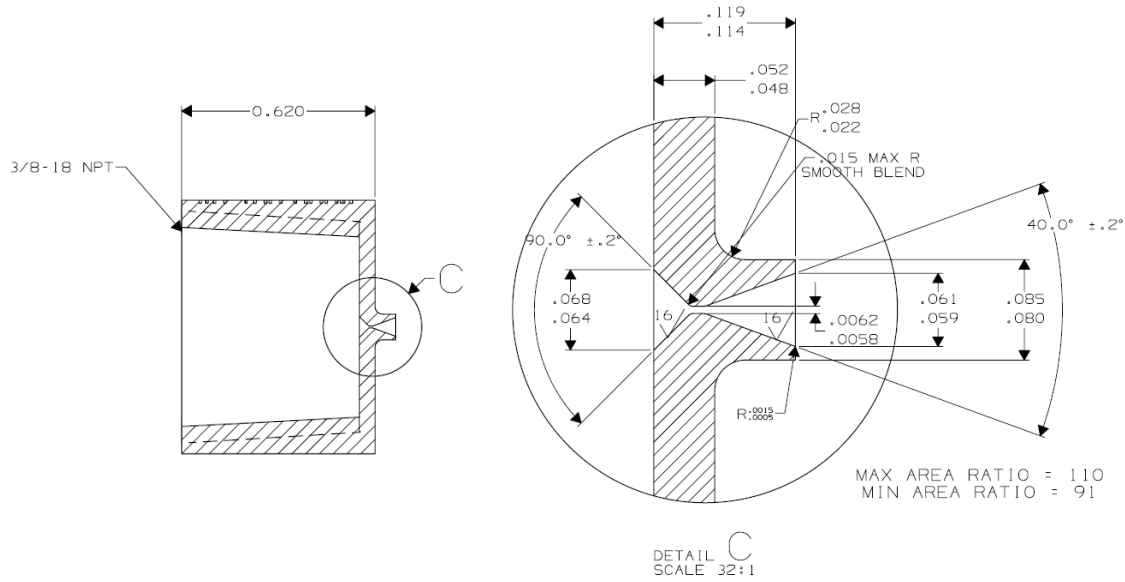


Figure 2. Nozzle cap cross section (left) and nominal dimensions (right) for nozzle 40-100-0006.

While the nozzles were designed with nominal specifications for their key design parameters, manufacturing tolerances at these small scales needed to be verified. The nozzles are individually inspected using a Keyence VHX digital microscope to measure the actual throat diameter, d^* , and exit diameter, from which the area ratio, AR, can be calculated. Three-dimensional depth imaging is used to measure the profile of the nozzle cone to determine the nozzle angle, α . An example of the inspection results are shown below in Figure 3. A full summary of the inspection is listed in Table 1.

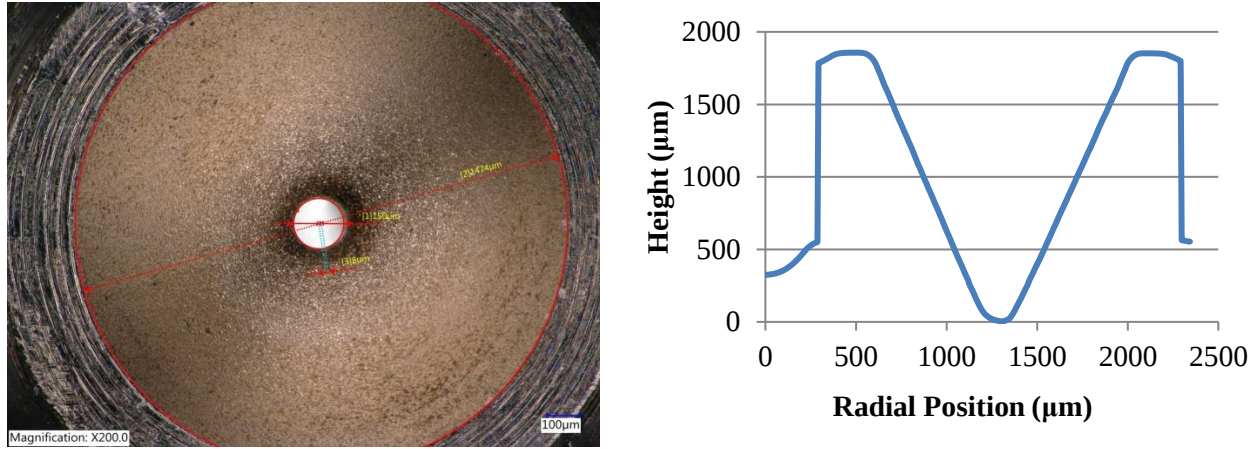


Figure 3. Example microscope image at 200X magnification and nozzle profile. Nominal characteristics are 40° nozzle angle and 100 area ratio.

Table 1. Nozzle Inspection Summary

Designation	Nominal Parameters			Actual Parameters		
	d^* (μm)	AR	α (deg)	d^* (μm)	AR	α (deg)
40-050-0006	152	50	40	153	47.6	39.5
40-100-0006	152	100	40	150	96.6	38.9
40-200-0006	152	200	40	150	193.4	39.2
60-100-0006	152	100	60	154	95.9	59.7

D. Diagnostics

The thrust generated by the nozzle is measured using a torsional thrust balance developed in collaboration with Busek Co., Inc. The thrust stand has a nominal range of 0.1-10 mN with a demonstrated resolution of 4.8 μN and an average uncertainty of ±14.6 μN. In situ calibration of the thrust stand is performed using capacitive force generation (CFG) plates across a range of voltages and comparing the calculated capacitive force to the torsional displacement as measured by a

Philtec laser displacement sensor. The capacitive force is estimated based on measurements of the CFG plate capacitance using a BK Precision 889B LCR meter. Leveling of the thrust stand is verified using two piezoelectric tilt sensors arranged orthogonally along one of the base beams of the torsional stand. Two linear actuators connected to two corners of the thrust stand base provide fine control of the alignment. The particular details of the thrust stand, including its operational characteristics and the method of deriving uncertainty, are available in another publication.¹⁰

III. Results

A. Performance Parameters

The performance of the nozzles is characterized by four parameters: thrust, specific impulse, nozzle efficiency, and thrust coefficient. Thrust is the only performance parameter that is directly measured, while the other three are derived from measured and calculated quantities. The specific impulse, I_{sp} , is determined from the thrust, F_T , and the propellant mass flow rate $\dot{m}$,

$$I_{sp} = \frac{F_T}{\dot{m}g} \quad (1)$$

where g is the acceleration due to gravity at sea level. The mass flow can be calculated from the volumetric flow rate by the equation below,

$$\dot{m} [mg / s] = \frac{q [sccm]}{1344} MW [g / mol] \quad (2)$$

where q is the volumetric flow rate as measured from the flowmeter, and MW is the molecular weight of the propellant used, each with the units denoted. The factor of 1344 is a conversion constant for the given units.

The nozzle efficiency is defined specifically as the specific impulse efficiency, which is the ratio of the actual specific impulse to the theoretical specific impulse assuming isentropic expansion.

$$\eta_n = \frac{I_{sp}}{I_{sp,s}} \quad (3)$$

The isentropic specific impulse is given by

$$I_{sp,s} = \frac{u_{e,s}}{g} + \frac{p_e - p_a}{\dot{m}g} A_e \quad (4)$$

where $u_{e,s}$ is the isentropic exit velocity, p_e and p_a are the exit and ambient pressures, respectively, and A_e is the exit area of the nozzle. The isentropic exit velocity is defined as

$$u_{e,s} = M_e \sqrt{\gamma R T_e} \quad (5)$$

where γ and R are the ratio of specific heats and gas specific constant, respectively, of the propellant, and M_e is the exit mach number, which can be found by solving the equation below, where A^* is the nozzle throat area.

$$\frac{A_e}{A^*} = \left(\frac{\gamma+1}{2} \right)^{-\frac{\gamma+1}{2(\gamma-1)}} \frac{1}{M_e} \left(1 + \frac{\gamma-1}{2} M_e^2 \right)^{\frac{\gamma+1}{2(\gamma-1)}} \quad (6)$$

The gas conditions at the exit plane of the nozzle can be calculated using the relations below.

$$p_e = p_0 \left(1 + \frac{\gamma-1}{2} M_e^2 \right)^{-\frac{\gamma}{\gamma-1}} \quad (7)$$

$$T_e = T_0 \left(1 + \frac{\gamma - 1}{2} M_e^2 \right)^{-1} \quad (8)$$

The nozzle thrust coefficient is defined in terms of the measured thrust and thrust chamber conditions.

$$C_T = \frac{F_T}{p_0 A^*} \quad (9)$$

B. Thrust

Prior to testing, several steps are taken to prepare the diagnostics to minimize uncertainty. While the chamber is still at ambient pressure, the CFG plates are aligned and the capacitance is measured across a range of gap distances. The chamber is then evacuated, during which the thrust stand electronics, as well as the other diagnostic controllers, are run to reach thermal equilibrium and reduce drift. Once the vacuum chamber has reached high vacuum, the thrust stand is calibrated using the established procedure.¹⁰ The thruster operating conditions are then set, and the thruster given time to reach equilibrium as determined by the thrust chamber pressure and temperature (this required approximately 10 minutes). The thruster was allowed to operate at steady state for at least 3 minutes during which data was collected at 4 Hz. The nozzle parameter space was mapped by operating across a flow rate range of 10-80 sccm in 10 sccm increments for a given temperature, then by incrementing temperature. This was then repeated for each nozzle in turn in subsequent tests. The data is plotted against the throat Reynolds number, given by

$$\text{Re}^* = \frac{4\dot{m}}{\pi \mu d^*} \quad (10)$$

where μ is the viscosity of the gas. The viscosity is calculated using Sutherland's formula,

$$\mu = \lambda \frac{T^{3/2}}{T + C} \quad (11)$$

where λ and C are constants, which for nitrogen are: $\lambda = 1.4067 \times 10^{-6}$ Pa-s-K^{1/2} and $C = 111$ K. Note that because the Reynolds number is dependent on viscosity, which in turn varies with temperature, at higher temperatures the throat Reynolds number will decrease for the same propellant flow rate. For testing, the propellant flow rates in the range of 10-80 sccm were used in increments of 10 sccm; the final Reynolds numbers were calculated during data analysis.

The thrust generated by each nozzle is shown below in Figure 4. As expected, thrust is very linear with respect to the throat Reynolds number; variation between nozzle geometries is small relative to the overall scale of the thrust range. Due to the scale disparity between the thrust variation of the nozzles and the thrust throttling range, it is difficult to make any reliable performance conclusions from Figure 4. Variation in thrust is more readily apparent between gas temperatures, shown in Figure 5, where increasing the thrust chamber gas temperature from room temperature to 150 °C or 250 °C generally increased thrust by approximately 10% and 20%, respectively.

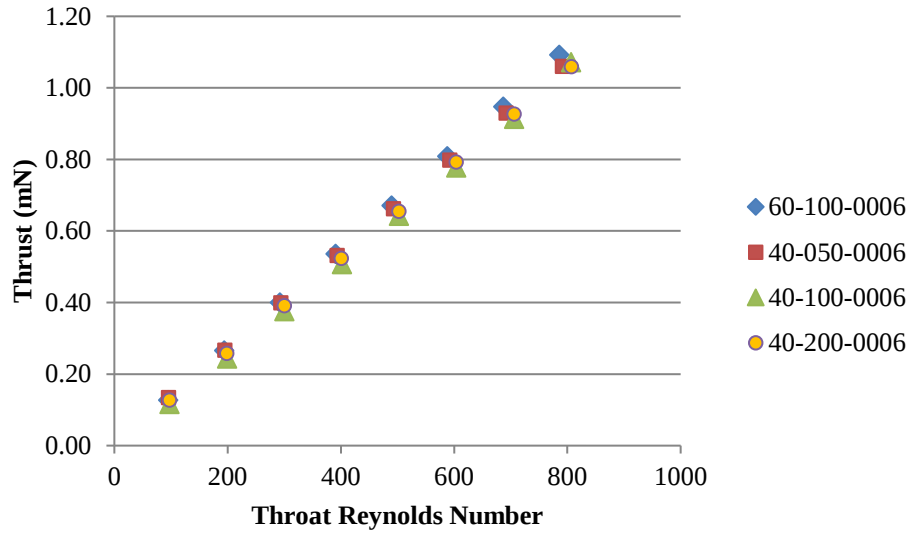


Figure 4. Thrust as a function of throat Reynolds number for each nozzle at 295 K.

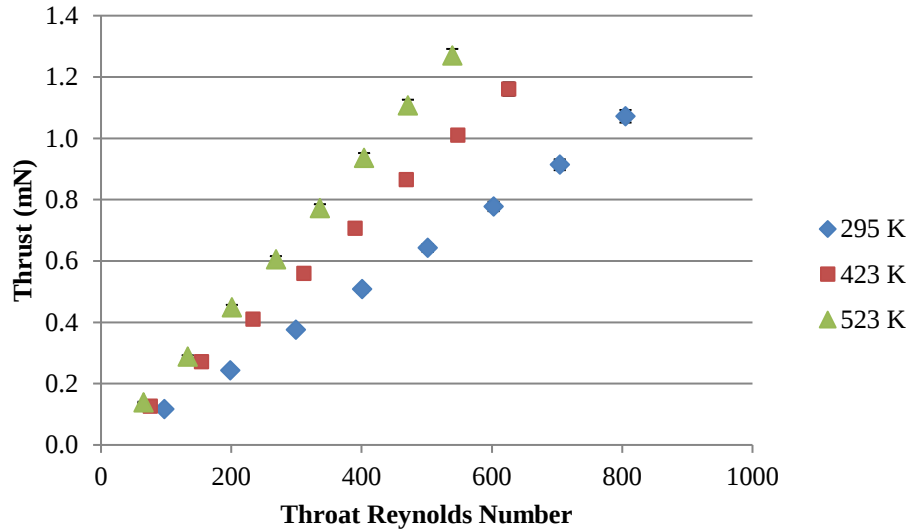


Figure 5. Thrust as a function of throat Reynolds number and thrust chamber temperature for nozzle 40-100-0006.

C. Specific Impulse

Turning our attention now to propellant utilization, the specific impulse of the four nozzle geometries are compared at two different thrust chamber gas temperatures. As can be seen in Figure 6, increasing the gas temperature tends to improve overall specific impulse. Increasing gas temperature also appears to shift the relative performance of the nozzle configurations: nozzle 40-200-0006 drops to the lowest in the ranking at 425 K, while nozzle 60-100-0006 rises to top. This trend continues to 525K where the 60° nozzle angle is demonstratively the superior choice, shown below in Figure 7. While it is within the margin of error, within the three 40° nozzles increasing the area ratio seems to decrease the specific impulse.

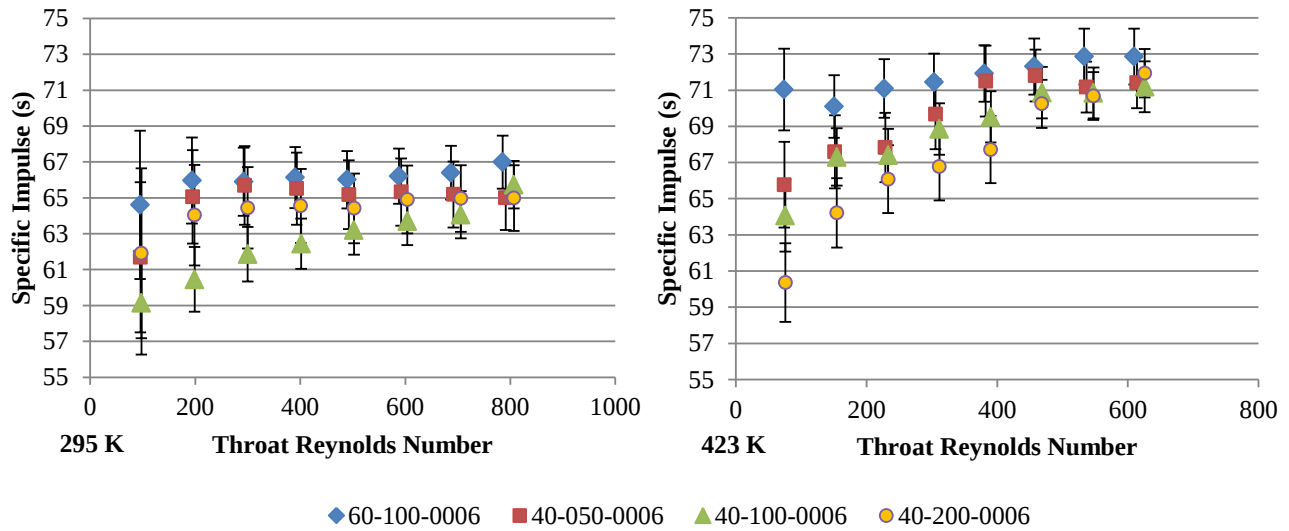


Figure 6. Specific impulse as a function of throat Reynolds number and nozzle configuration for thrust chamber gas temperatures of 295 K (left) and 423 K (right).

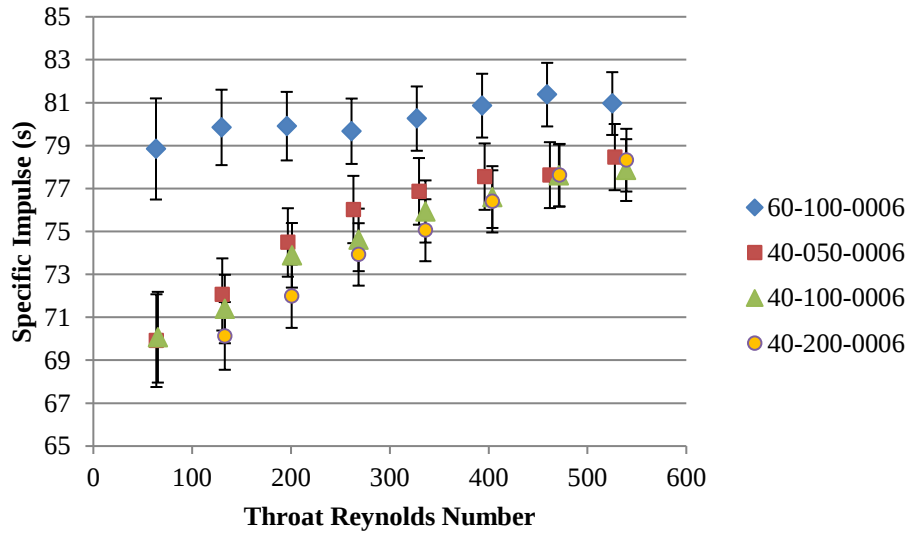


Figure 7. Specific impulse as a function of throat Reynolds number and nozzle configuration at 523 K thrust chamber gas temperature.

Looking individually at each nozzle over multiple gas temperatures, the immediate observation is that specific impulse increases with gas temperature, as shown in Figure 8. All four nozzles displayed this behavior where each increase in gas temperature resulted in a clear increase in specific impulse regardless of Reynolds number. From a design standpoint, this suggests that while a certain nozzle configuration may be better suited for a given operating condition, there is always an absolute increase in thrust and specific impulse to be gained by increasing gas temperature.

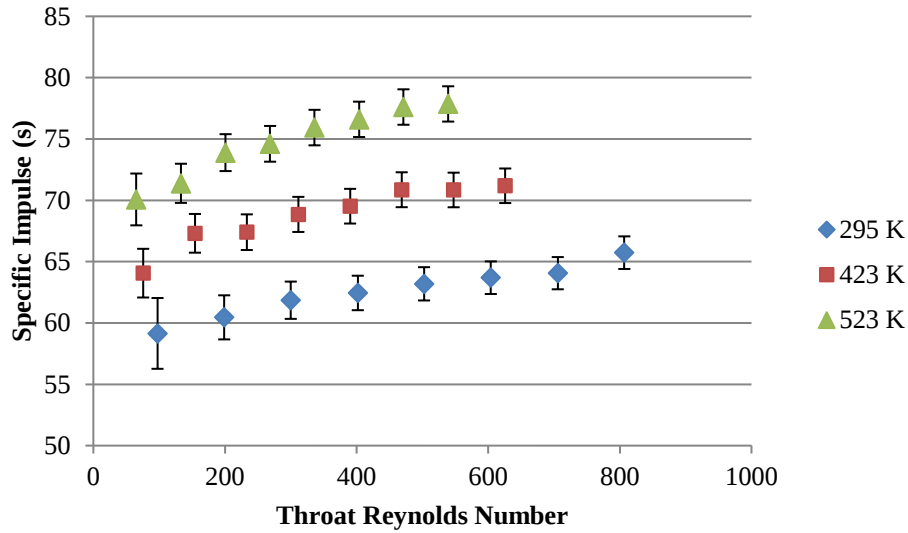


Figure 8. Specific impulse for nozzle 40-100-0006 as a function of throat Reynolds number and temperature. Increasing temperature is found to increase specific impulse across each nozzle.

D. Efficiency

Since the nozzle specific impulse efficiency is the ratio of the measured specific impulse to the isentropic efficiency, and as there is only minor variation of the isentropic specific impulse between nozzles, the overall behavior of efficiency versus nozzle configuration will mirror the behavior seen previously. As an example, the plot of efficiency as a function of the throat Reynolds number and nozzle configuration at 423 K in Figure 9 looks very similar to the plot of specific impulse at 423 K in Figure 6. The more interesting observation is found when observing the efficiency of a single nozzle at multiple temperatures. As shown in Figure 10, despite the observed increase in specific impulse, increasing the gas temperature actually decreases nozzle efficiency. Additionally, the efficiency of the nozzle does not appear to perceptibly change between 423 K and 523 K, despite the marked change between 295 K and 423 K. This behavior is consistent across all four nozzles, but the relative change at higher temperature is difficult to explain.

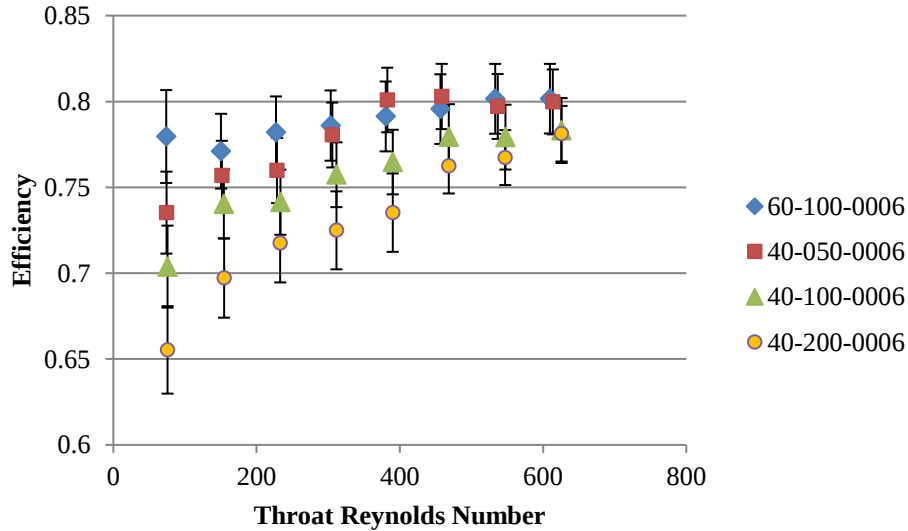


Figure 9. Specific impulse efficiency as a function of throat Reynolds number and nozzle configuration at 423 K.

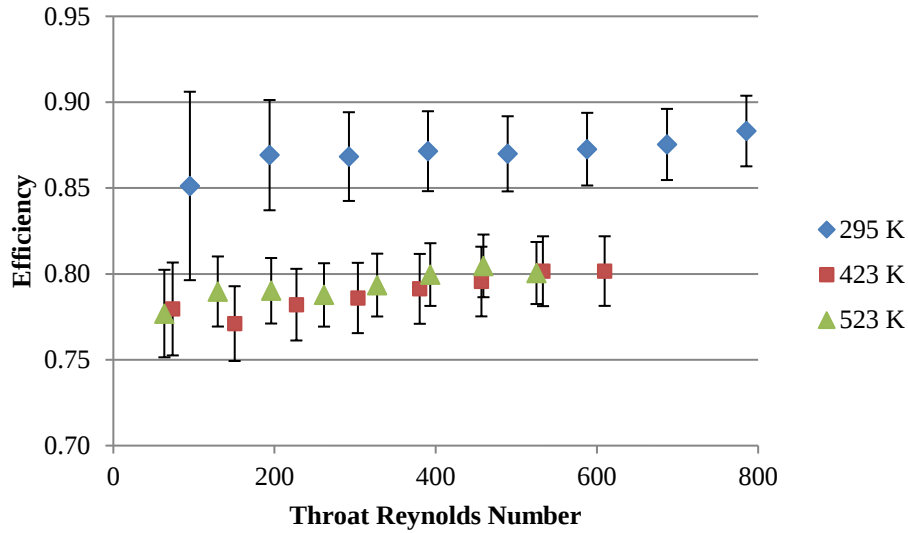


Figure 10. Specific impulse efficiency as a function of throat Reynolds number and thrust chamber gas temperature for nozzle 60-100-0006.

E. Thrust Coefficient

Comparing the thrust coefficient of the four nozzle configurations does not immediately yield any obvious significant conclusions. As can be seen in Figure 11, the four thrust coefficient curves are all within their mutual error bars, with the exception of the smallest area ratio nozzle at higher Reynolds number. The four nozzles share the general trend that below a throat Reynolds number of 200 the thrust coefficient is less than one. Examining each nozzle data set individually, two trends are observed. The first is that increasing the gas temperature decreases the thrust coefficient beyond merely lowering the Reynolds number, as it shifts the entire thrust coefficient curve down. The second observation is that the 423 K and 523 K K curves are in close agreement despite the marked change from 295 K to 423 K. The sole exception to this is nozzle 60-100-0006, which has three discrete curves. These trends can be seen below in Figure 12.

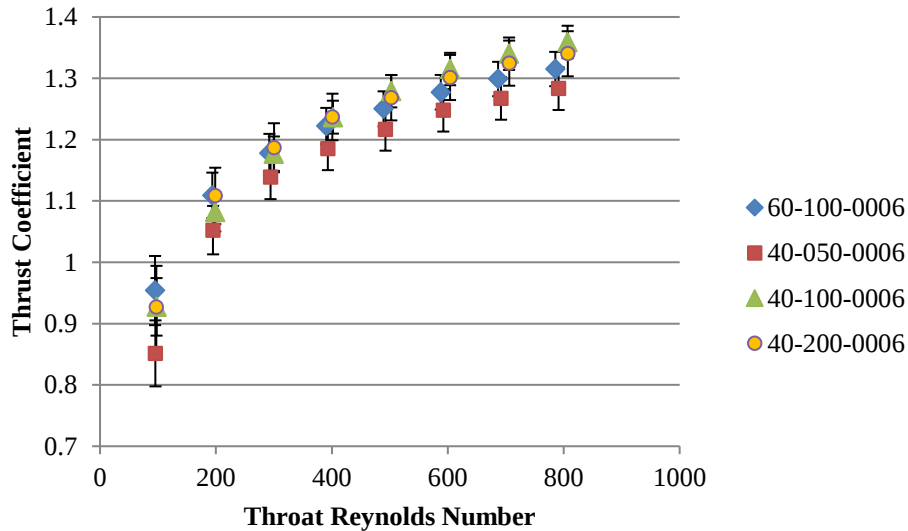


Figure 11. Thrust coefficient as a function of throat Reynolds number and nozzle configuration at 295 K.

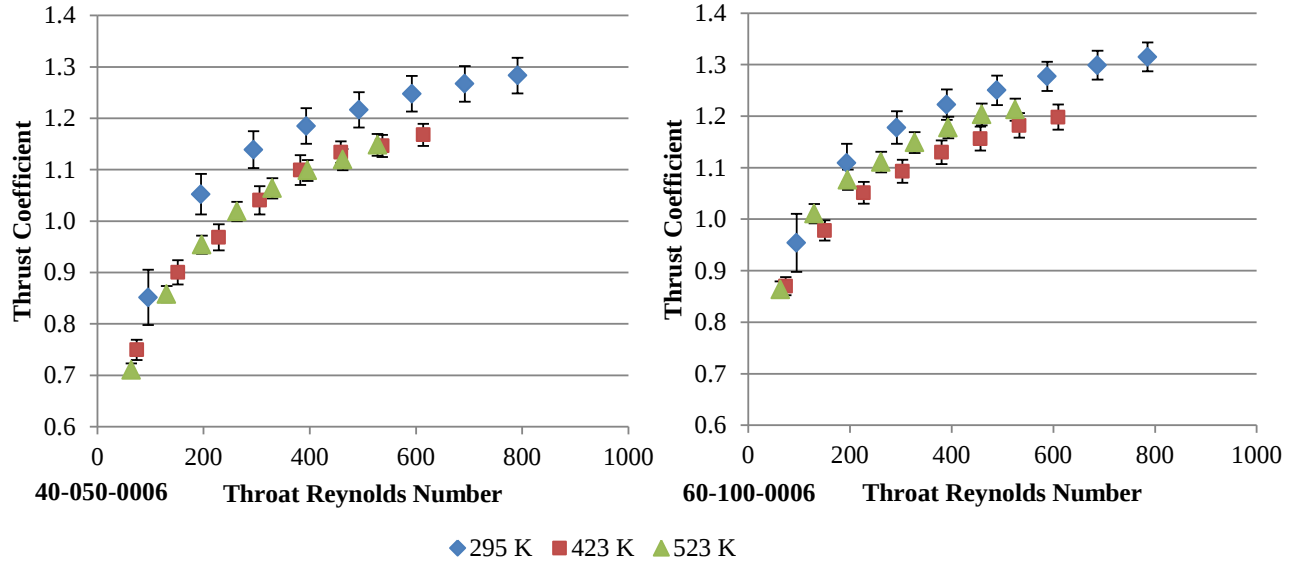


Figure 12. Thrust coefficient as a function of Reynolds number and gas temperature for nozzles 40-050-0006 (*left*) and 60-100-0006 (*right*). For all three nozzles with a 40° angle, the thrust coefficient curve at 423 K overlaps the curve for 523 K, while the 60° nozzle has three distinct curves.

IV. Discussion

From a design standpoint, nozzle 60-100-0006 is the best performing nozzle of the four tested, as it consistently demonstrated the highest specific impulse while generally maintaining the highest efficiency. It is worth noting that nozzle 40-050-0006 closely matched nozzle 60-100-0006 in terms of efficiency. This is consistent with the idea that nozzles that minimize boundary layer interactions with the core flow maximize efficiency. A larger nozzle angle can offer the same area ratio with a smaller overall length, and thus less space for boundary layer growth. A nozzle with a smaller area ratio likewise limits boundary layer growth, but at the cost of maximum specific impulse achievable. Thus, nozzle 40-050-0006 has a higher nozzle efficiency for the most part than nozzle 40-200-0006, but both have similar values of specific impulse. The former has a lower cap on theoretical performance, but is able to more closely match it than the latter, which has a higher isentropic performance but greater viscous losses.

Beyond simple performance comparisons between nozzles, a key observation of interest is that while increasing gas temperature increases the specific impulse, it does not seem to always reduce nozzle efficiency. Transitioning from a cold gas to 423 K increases specific impulse on average by approximately 6 seconds, but reduces nozzle efficiency on average by 7%. Conceptually, the increased gas temperature provides additional internal energy that can be extracted for bulk flow velocity, but the increase in temperature also raises the viscosity of the gas and increases losses. As with increasing area ratio, increasing the gas temperature raises the theoretical performance while at the same time decreasing the ability of the nozzle to reach it. The net result is an increase in specific impulse, but one that is smaller than what would have been achieved in an inviscid system. The surprising find is that the transition from 423 K to 523 K produces a similar increase in specific impulse without any comparable decrease in nozzle efficiency. This raises the question whether additional increases in temperature would provide further improvements in specific impulse without incurring efficiency costs. It also requires some verification of the observed trends to gain confidence in our test and measurement methods.

The central question of verification is this: is the interior thermocouple accurately measuring the gas temperature prior to expansion in the nozzle, or is it capturing the body or heater temperature? Rerunning the tests with additional invasive diagnostics would risk creating leaks and thus introduce more chance for error, so we investigated an alternative verification utilizing the discharge coefficient. The mass flow rate through a choked orifice can be written as

$$\dot{m} = C_d A \sqrt{\gamma \frac{p_0^2}{RT_0} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}} \quad (12)$$

where C_d is the discharge coefficient. Assuming the discharge coefficient is independent of gas temperature, this means that an accurate measure of the gas temperature can be confirmed by comparing the ratio of the pressure at one gas temperature to the pressure at another for the same nozzle and mass flow rate. The calculated discharge coefficients were found to be

relatively similar across the three temperatures, especially when plotted against flow rate rather than Reynolds number, as can be seen in Figure 13.

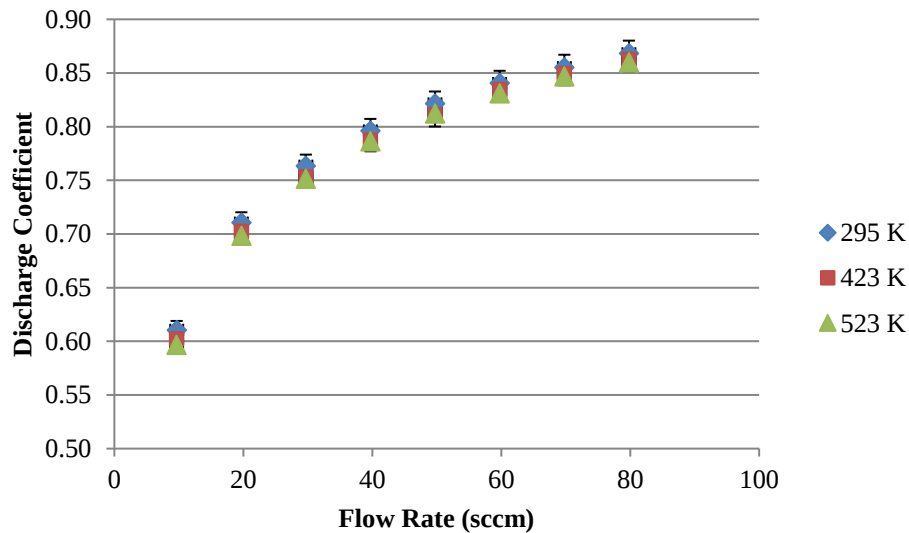


Figure 13. Discharge coefficient as a function of nitrogen flow rate and temperature for nozzle 40-050-0006.

With the assumption of limited variability with temperature validated, the thrust chamber stagnation pressure is compared between temperatures at each flow rate. As can be seen in Figure 14, the stagnation pressures are highly linear with constant proportionality between them. From Equation (12) we know that for constant mass flow rate, the ratio of the pressures must equal the square root of the ratio of the gas temperatures. This is found consistently across all four nozzles. As an example, for nozzle 40-100-0006 the average pressure ratio between 423 K and 295 K is 1.199 with less than 1% variability across the range of flow rates compared to an expected ratio of 1.198. Similarly, the pressure ratio between 523 K and 295 K is 1.332 compared to an expected 1.333. This suggests that any error in the gas temperature measurement would have to be matched by a mathematically compatible error in the pressure measurement, which is highly unlikely. Therefore, we can conclude that the gas temperature measurement is accurate and that nozzle efficiency does not fall as temperature increases from 423 K to 523 K.

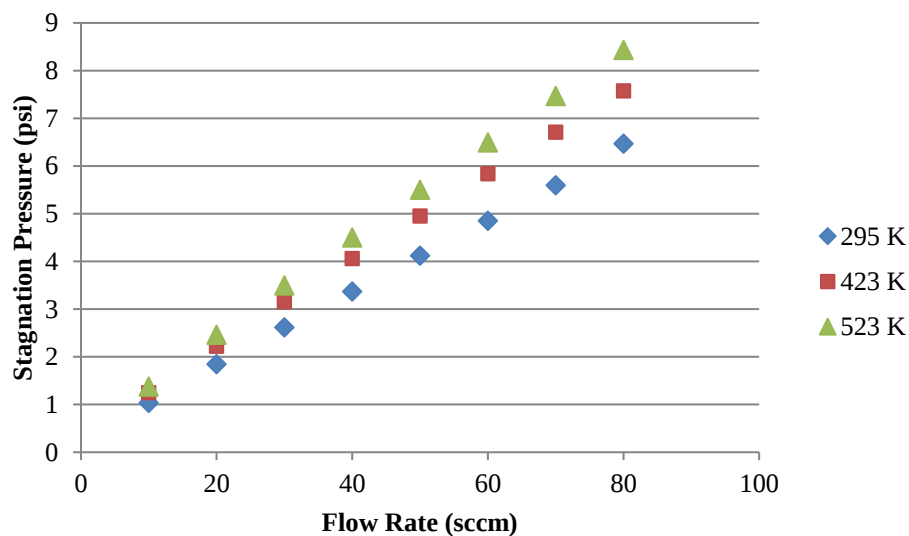


Figure 14. Thrust chamber stagnation pressure as a function of flow rate and temperature for nozzle 40-100-0006.

The assumption going into the use of the throat Reynolds number as the independent parameter is that the nozzle performance is driven by the relative impacts of viscosity and momentum. Thus, lower flow rates or higher gas temperatures would favor viscous effects and increase losses within the nozzle. However, this would predict that increasing gas

temperature from 423 K to 523 K would result in a further decrease in nozzle efficiency, which was not observed. One possibility is that viscosity and momentum are not the only factors in play. During expansion through the nozzle, the gas transitions from a continuum flow to a free molecular regime where gas-surface interactions become more important. The Knudsen number is the ratio of the mean free path of a molecule to the characteristic length scale, L ,

$$Kn = \frac{k_B T}{\sqrt{2} \pi d_{N_2}^2 p L} \quad (13)$$

where k_B is Boltzman's constant, d_{N_2} is the collisional diameter of nitrogen, and p is the pressure of the gas. As can be seen in Figure 15, the Knudsen number at the exit plane of the nozzles are no longer negligible, particularly for Reynolds numbers below 200. The Knudsen number at the throat is shown for comparison.

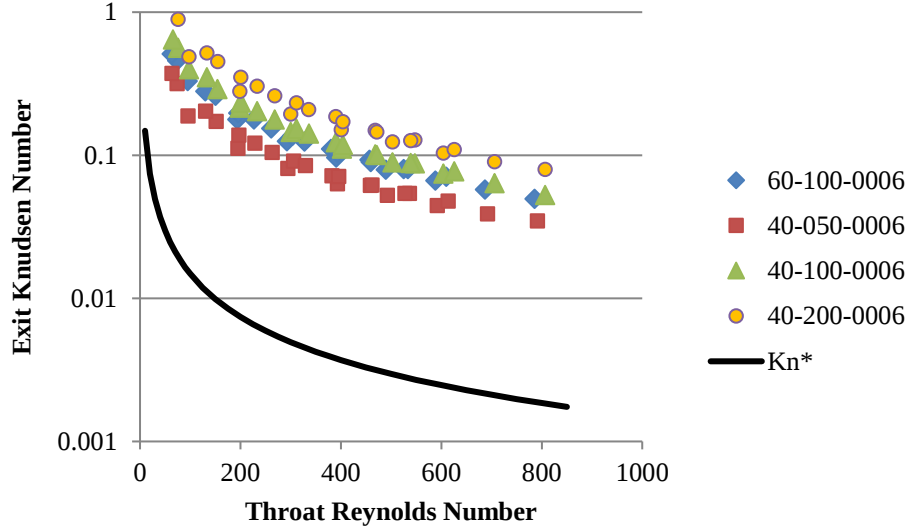


Figure 15. Knudsen number at the exit plan of the nozzle as a function of throat Reynolds number and nozzle configuration across all three temperatures. Note that for each nozzle the Knudsen number curve is independent of temperature. Throat Knudsen number shown as the black line for comparison.

The large increase in Knudsen number from the throat to the nozzle exit plane confirms that while the flow begins in the continuum regime, free molecular flow regime effects are possible, especially at low Reynolds numbers. Therefore, there may be additional wall interactions beyond the continuum no-slip condition or other phenomena to consider that might explain the observed behavior of the nozzle efficiency. Concurrent work to establish a combined CFD-DSMC computational tool to model nozzle performance has also encountered difficulty in accurately capturing the observed behavior at low Reynolds numbers.¹¹ Together, this suggests that future work should focus on exploring this and other possible explanations for the observed behavior and to determine if further increases in gas temperature do not result in decreased nozzle efficiency. If this is the case, it would suggest prioritizing higher gas temperature in thruster designs, as it would provide increased specific impulse without degrading nozzle efficiency and may represent a general design guideline.

V. Conclusion

The Naval Research Laboratory has tested the thrust performance of four micro-nozzles in a modular resistojet test bed. Thrust was produced in the range of 0.117-1.28 mN across a throat Reynolds number range of 65-807. In general, nozzles with higher nozzle angles or smaller area ratios provided high nozzle efficiency, as their geometry limited the impact of boundary layer growth. The observed performance shows that the temperature dependency of the nozzle efficiency is more complex than initially expected, and may suggest additional factors impacting nozzle performance at this scale beyond viscosity and flow rate. Additional testing at higher temperatures is required to confirm this observation and to identify the phenomena responsible.

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