

Performance and Vibration Characterization of a Low-Thrust Torsional Thrust Balance

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As research into micro-propulsion at the Naval Research Laboratory grows, there is an increasing need for high-resolution thrust measurement with high-accuracy and low noise. Several research projects are on-going that require thrust resolution below 50 μN ; some also require high flow rates at high vacuum, which prevent the use of smaller vacuum chambers mounted on pneumatic vibration isolation tables. To meet these research needs, a torsional thrust balance system has been developed with Busek Co. to provide thrust measurements with an average resolution of 4.8 μN and average uncertainty of $\pm 14.6 \mu\text{N}$. A low-resonant-frequency vibration isolation system is used in-vacuum to limit noise transmission from the vacuum pumps and surrounding environment.

Nomenclature

A	= idealized area of capacitor plate	n_p	= number of parameters to fit in a curve fit
A_{eff}	= effective area of the capacitor plate	N	= number of recursion or calibration points
C	= capacitance	q	= volumetric flow rate of propellant
CI	= confidence interval	r_e	= radial position of the CFG plates
d	= distance between capacitor plates	r_o	= radial position of the ODS
d_0	= initial CFG plate separation distance	r_T	= radial position of the test thruster
d^*	= distance to the balance arm from ODS	R_{ss}	= residual sum of the squares
F	= force on a parallel plate capacitor	U_i	= uncertainty of the parameter i
F_{CFG}	= force on the arm by the CFG plates	V	= voltage of a capacitor/CFG plates
F_T	= force created by the test thruster	Z_s	= zero shift to the CFG plate gap
H	= total drift of the null position observed	δd	= absolute uncertainty of position
I_{sp}	= specific impulse	ϵ_0	= permittivity of free space = $8.85 \times 10^{-12} \text{ A}^2\text{-s}^4/\text{kg-m}^4$
k_x	= effective linear stiffness	θ	= angular position of the balance arm
k_θ	= torsional stiffness of the balance arm	σ	= standard error
$\dot{m}$	= mass flow rate of propellant	τ	= applied torque on the balance arm
MW	= molecular weight of the propellant		

I. Introduction

With the growth of interest in micro-propulsion at the Naval Research Laboratory (NRL), whether to provide propulsive capabilities to CubeSats or explore the phenomenology of alternative thruster configurations, there has been increased need for low thrust measuring capability in ongoing research.^{1,2} For projects such as measuring low Reynolds number nozzle thrust, a high-accuracy, high-precision thrust stand is essential for ensuring meaningful results that can be applied for research and design purposes. Programs of this nature require the ability to resolve the difference in thrust output for small operational parameter changes, which places high demands on the accuracy of the device as well as the vibrational characteristics of the test setup. One of the key challenges with maintaining high precision in thrust stands is minimizing the impact of external vibrations, whether they are from the vacuum chamber system, intermittent shocks from nearby personnel, or background facility noise (e.g. air handlers).

There are three categories of thrust stands: hanging, inverted pendulum, and torsional. A hanging thrust stand operates as a pendulum, where the linear force of the thruster is converted to a rotational displacement measured from a linear perspective. The stiffness, and thus sensitivity, of a hanging thrust stand is set by the length of the pendulum and the mass of the payload. Thus while stable, the hanging configuration is not conducive for space-limited vacuum chambers. Inverted pendulum thrust stands balance the thruster on a platform supported by flexures with the position controlled by actively controlled electromagnets. While very sensitive, these platforms are unstable and require very fine control. Torsional thrust stands consist of a lateral arm suspended on a vertical rotational axis bound by rotational flexures. The low torsional stiffness of the flexures convert linear force to azimuthal displacement; the use of a balance arm amplifies the torque on the balance

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and the resulting linear displacement. This configuration is ideal for small, low mass thrusters that require both stability and sensitivity.

To that end, NRL with the help of Busek Co. has created a torsional thrust balance system that combines a high-accuracy thrust stand with a low-stiffness vibration isolation system from Minus-K. Standard practice for high-precision thrust stands in vacuum applications is to use a small cylindrical vacuum chamber mounted on a pneumatic vibration isolation table for noise suppression. However, in order to test thrusters in high vacuum conditions at higher propellant flow rates, a larger vacuum pump and chamber is required. The vacuum chamber used is too large for external vibration isolation, thus an internal, in-vacuum isolation system is required. This work outlines the performance capabilities and vibrational characteristics of the developed thrust balance system in terms of thrust range and resolution, as well as the system noise floor and vibration transmissibility.

II. Torsional Thrust Stand

The NRL torsional thrust stand was designed by Busek for use in micro-propulsion testing.^{3,4} The stand consists of a horizontal torsional balance arm with a mounting platform on each end welded to a vertical pivot axis. The pivot axis is secured to a housing structure that contains two flexural pivots fabricated by the Riverhawk Company that help define the torsional stiffness of the balance. Multiple models of the flexural pivots are available to provide variable torsional stiffness. The position of the arm is measured using a coaxial Philtec model D63 optical displacement sensor (ODS); the ODS is mounted on an actuated linear traverse to position the sensor for optimum accuracy. A parallel plate capacitance force generator (CFG) is used for calibration of the thrust balance, with one plate mounted on the balance arm and the other plate fixed to another linear traverse. An eddy current damper is mounted close to the balance arm to reduce arm oscillations and minimize settling times. Two linear variable differential transformers (LVDT) are used to measure the tilt of the housing structure, and two actuators are used to adjust the leveling along the two axes of rotation.

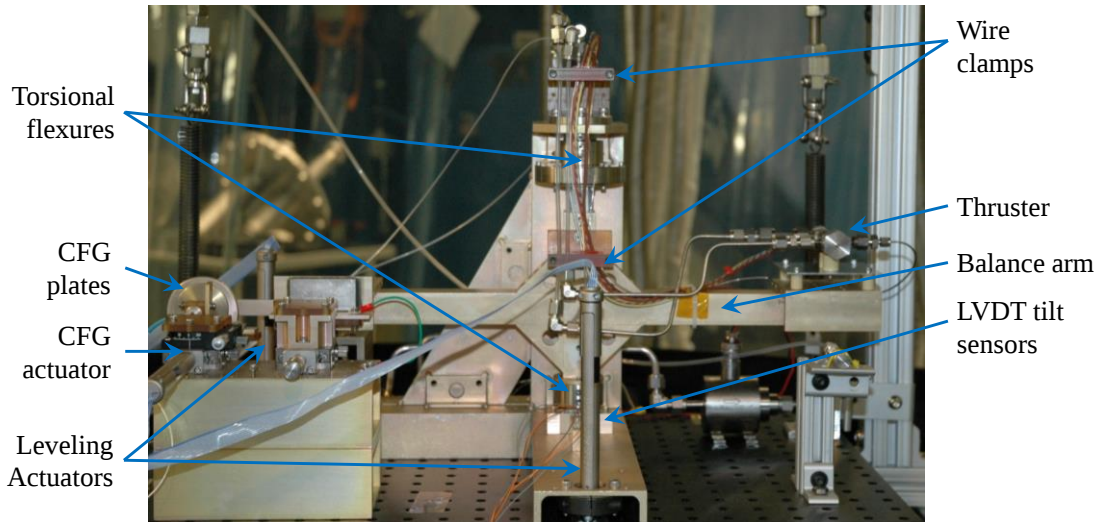


Figure 1. Torsional thrust stand and primary components.

A. Torsion Balance

In its most basic form, a torsional thrust stand is a relatively long, supported balance arm that has a low resistance or stiffness to torsional loads. The core operating principle of the torsional thrust stand is that a small force can be amplified by a long moment arm into a larger torque, and the torque in turn amplified by a low torsional stiffness into an even larger displacement of the arm. Thus, the magnitude of a very small force can be measured by observing the relatively large displacement of the thrust stand arm. Coupled with high-accuracy optical displacement sensors, the thrust stand can resolve forces in the micro-Newton range.

The key components of the thrust stand include: the CFG plates, an optical displacement sensor to measure motion of the arm, and the thruster being tested. The two CFG plates are mounted separately, with one mounted to the thrust stand arm, and one mounted to an actuator close to the arm. This allows for a variable CFG separation gap without disturbing the thrust stand arm. Due to the torsional nature of the device, the radial position of each component is important in determining the overall effect on arm displacement. Figure 2 shows a conceptual illustration of the thrust stand arm and the relevant radial positions from the armature center.

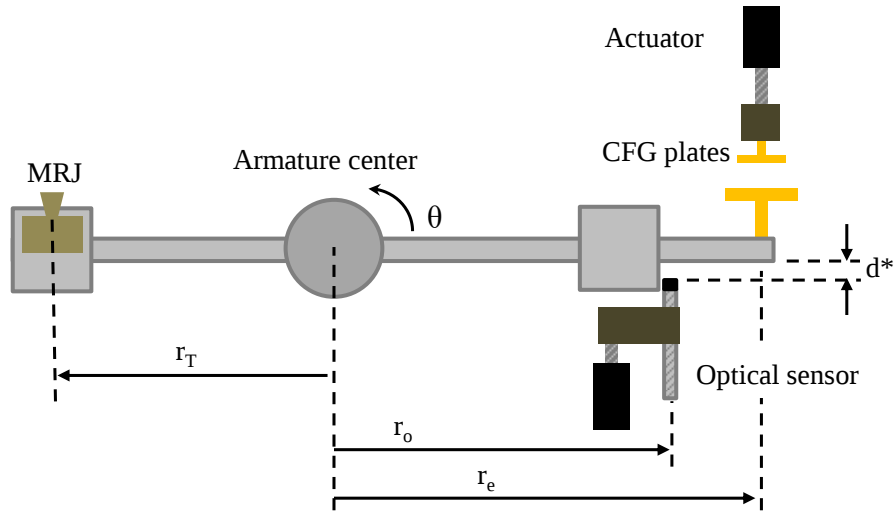


Figure 2. Conceptual diagram of torsional thrust stand arm. Radial positions of interest are: the resistojet at r_T , the ODS at r_o , and the CFG plates at r_e .

The torsional stiffness, k_θ , relates the torque applied to the thrust stand arm, τ , and the angular displacement the arm undergoes, $\Delta\theta$.

$$\tau = -k_\theta \Delta\theta \quad (1)$$

The angular displacement can be approximated using the small angle approximation and the change in position as observed by the ODS, Δd^* .

$$\Delta\theta \approx \tan \Delta\theta = \frac{\Delta d^*}{r_o} \quad (2)$$

The observed angular displacements are all less than 0.03° , so this approximation incurs an additional uncertainty of, at most, 9.14×10^{-8} , which is negligible.

During calibration, the source of the torque is the force created by the CFG plates at the radial position of the electrodes. Substituting Equation (2) and the CFG force into Equation (1) yields a relation for the torsional stiffness.

$$k_\theta = \frac{r_e r_o F_{CFG}}{\Delta d^*} \quad (3)$$

Once the exact value of the torsional stiffness has been empirically determined through the calibration process, the thrust of the test article can be calculated from the observed arm displacement during operation.⁵

$$F_T = k_\theta \frac{\Delta d^*}{r_o r_T} \quad (4)$$

B. Optical Displacement

The displacement of the balance arm is measured using a Philtec muDMS-D63 optical displacement sensor. A dual fiber optic cable connects the electronics outside the vacuum chamber to a sensor tip located next to the balance arm. The sensor tip contains both transmitting and receiving fibers; the transmission fibers emit a small amount of light that reflects off the balance arm and is measured by the reception fibers. The reflectance ratio of this incident light yields a gap distance between the sensor tip and the balance arm within a certain range. The D63 operates in either a near (0-0.25 mm) or far (0.5-1.5 mm) sensitivity range. The D63 has a resolution of 25 nm and a maximum range of 3 mm.⁶ A high reflectance target is used for the ODS to maximize the signal-to-noise ratio.

C. Vibration Isolation

In order to limit the impact of facility vibrational noise on the accuracy of the measurements,² the thrust balance system is mounted atop a Minus-K model BM-1 vibration isolator. The BM-1 series uses a passive system of springs, flexures, and a critically buckled beam to create a compound stiffness that is very low, enabling the isolator to have a natural frequency close to 1 Hz when properly loaded. The isolation system has been observed to decrease the background vibrational noise by at least a factor of four. The Minus-K system is mounted on an optical table for stability, which in turn is mounted to the chamber. At each mechanical junction, a 0.25 in. thick layer of Sorbothane is added to increase the overall system damping to reduce the impact of mechanical shocks caused by general activity near the chamber. Figure 3 shows a picture of the thrust stand mounted on the Minus-K system.



Figure 3. Torsional thrust balance mounted on vibration isolator.

D. Magnetic Damping

To further supplement the Minus-K system, the balance arm is modified to include an additional magnetic eddy-current damper. The magnetic damper consists of two 1 x 1 x 0.375 inch nickel-plated neodymium magnets mounted perpendicular to the balance arm. A 0.25 inch thick aluminum 90° angle bracket is mounted to the balance arm with one face parallel to the magnets. The degree of damping is coarsely set by adjusting the gap between the aluminum bracket and the magnets. This damper is the primary means of controlling whether the thrust stand system operates in an over-damped or an under-damped regime. It has been observed that the best overall performance of the thrust stand is achieved when the magnetic damping is as high as possible while still maintaining an under-damped regime. A critically damped regime is ideal, but given the imprecise nature of setting the damper gap distance, this is difficult to readily achieve. As of this writing, the system is being upgraded with an actuated damper that offers more precise control of the damper gap. Due to the desire to avoid an over-damped regime, the required gap is also dependent on the mass of the test article. As an example, the current gap distance used for resistojet testing is approximately 2.4 mm; while a smaller gap is readily achievable, doing so has been found to decrease thrust stand response time.

E. Facilities

The vacuum system used is NRL's South Vacuum Chamber, which is 2 m diameter by 2.3 m tall. The chamber is pumped with a 48" NRC diffusion pump with a nominal pumping speed of 100,000 l/s for air. The diffusion pump is backed by a Leybold WH2500 blower and a SV-630B roughing pump with a combined pumping speed of 1530 CFM. Base pressure at high vacuum operation is 5.5×10^{-6} Torr. Pressure in the chamber is measured using a Bayard-Alpert type ion gauge in high vacuum, and rough vacuum pressure is measured using a Granville Phillips 275 convectron gauge.

III. Operational Methodology

A. Calibration

While the flexural pivots set the baseline for the balance arm torsional stiffness, any wires or propellant lines required by the test article will also contribute. In order to capture this effect, the overall stiffness needs to be empirically determined.

Thrust stand calibration is performed using CFG plates, which utilizes the electric field between two charged discs to apply a non-contact force on the thrust stand arm. The force generated can be calculated from measurements of the capacitance of the two plates and the voltage drop applied. The process consists of successive steps of: capacitance measurement across a range of electrode gaps, measurement of thrust stand arm deflection across a range of CFG plate voltages, and calculation of the effective thrust stand arm torsional stiffness. Thrust can then be found by comparing the thrust stand deflection during operation of the thruster to the determined torsional stiffness.

Capacitance Measurement

For two parallel plates the capacitance, C , is a function of the area of the plates, A , the distance between them, d , and the permittivity of free space, ϵ_0 .

$$C = \frac{A\epsilon_0}{d} \quad (5)$$

However, for the specific case of the thrust stand CFG plates there are two deviances from this ideal: the plates have unequal areas and as the arm rotates the plates are no longer strictly parallel. To correct for these effects, the CFG plates can be thought of having an effective area, A_{eff} , which accounts for the unequal plate areas and edge effects, and a zero-shift to the plate gap, Z_s , which accounts for the variation of parallelism with gap distance. These quantities are determined by taking the measured capacitance as a function of gap spacing and curve-fitting to the equation below.

$$C = \frac{A_{eff}\epsilon_0}{d + Z_s} \quad (6)$$

The gap spacing is first zeroed out by driving the actuator forward until the two plates achieve electrical contact. The actuator is then incrementally retracted while measuring the capacitance between the CFG plates. The capacitance is measured using a BK Precision 889B LCR meter. An example plot of the measured capacitance and the corresponding curve-fit are shown below in Figure 4. Note that while a relatively wide range of values are measured, the distances of interest are those that the CFG plates are expected to experience during calibration. A separation gap that is less than 0.6 mm makes the plates prone to contacting and micro-welding during calibration. Conversely, gap distances above 1.5 mm result in calibration forces that are too small for this application. To improve the accuracy of the two fit parameters, the measured data is split into near and far regions each with its own curve fit. The far region is chosen to contain the CFG gap range of interest, so further analysis utilizes the values of A_{eff} and Z_s from the far fit.

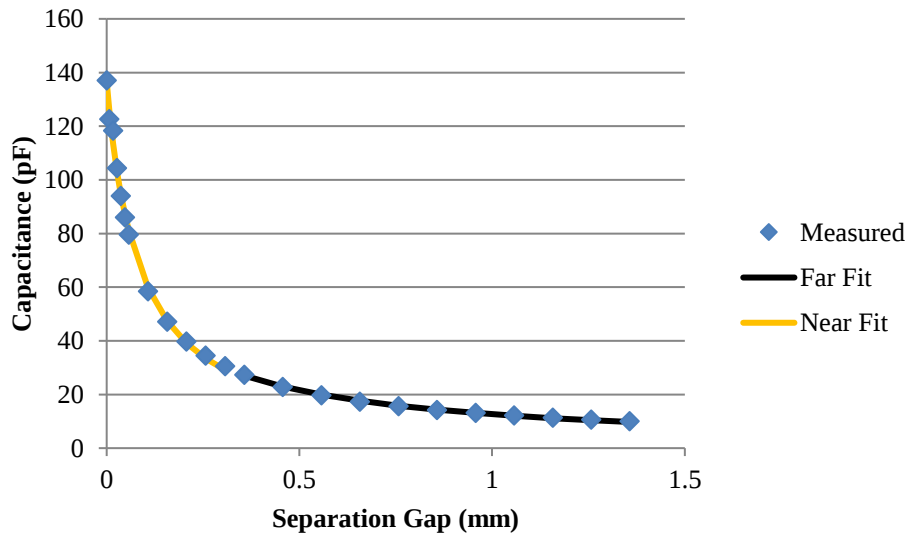


Figure 4. Example near and far capacitance curve fits. Curve fit values for Far: $A_{eff} = 0.001725 \text{ m}^2$, $Z_s = 0.208213 \text{ mm}$; for Near: $A_{eff} = 0.001285 \text{ m}^2$, $Z_s = 0.083274 \text{ mm}$.

Capacitive Force Generation

The idealized force between two parallel plates in a capacitor is given by

$$F = \frac{1}{2} \frac{A \epsilon_0 V^2}{d^2} \quad (7)$$

where V is the voltage drop across the two plates. Just like in the previous section, due to the non-ideal geometry used the effective area and zero-shift must be used, resulting in the below equation.

$$F_{CFG} = \frac{1}{2} \frac{A_{eff} \epsilon_0 V^2}{(d + Z_s)^2} \quad (8)$$

With the effective area and zero-shift determined by the capacitance measurement curve-fit, the force generated by the CFG on the thrust stand arm can be known given the voltage drop applied and the gap distance. The former can be easily measured using a volt meter across the output leads of the power supply providing the voltage on the plates, while the latter is measured using an optical displacement sensor. However, because the ODS and CFG plates are at different positions of the arm, the linear change in position induced by a rotation at one is not equal to the linear change in position of the other. Instead, the actual change in position at the CFG plates is related to the ratio of their radial positions of the arm. Thus, the gap distance is given in terms of the change in the optical sensor position, Δd^* , as shown in the equation below.

$$d = d_0 - \frac{r_e}{r_o} \Delta d^* \quad (9)$$

Substituting Equation (9) into Equation (8) yields the CFG calibration equation.

$$F_{CFG} = \frac{1}{2} \frac{A_{eff} \epsilon_0 V^2}{\left(d_0 - \frac{r_e}{r_o} \Delta d^* + Z_s \right)^2} \quad (10)$$

With the relation between calibration force, voltage, and gap distance, the CFG initial gap distance is set using the actuator, generally 0.8 mm. The voltage across the CFG plates is gradually increased, which generates a force on the arm and decreases the gap between the plates. At each voltage the position of the arm is measured by the optical displacement sensor, resulting in a “staircase” plot that increases with the incremental calibration force. Figure 5 shows an example calibration plot that highlights the key characteristics. It should be noted that the spikes that occur at each step transition are indicative of the under-damped balance arm. While this means that the system requires some finite time to stabilize and let the oscillations dampen out, the system is more responsive and will reach steady state conditions more quickly than an over-damped system.

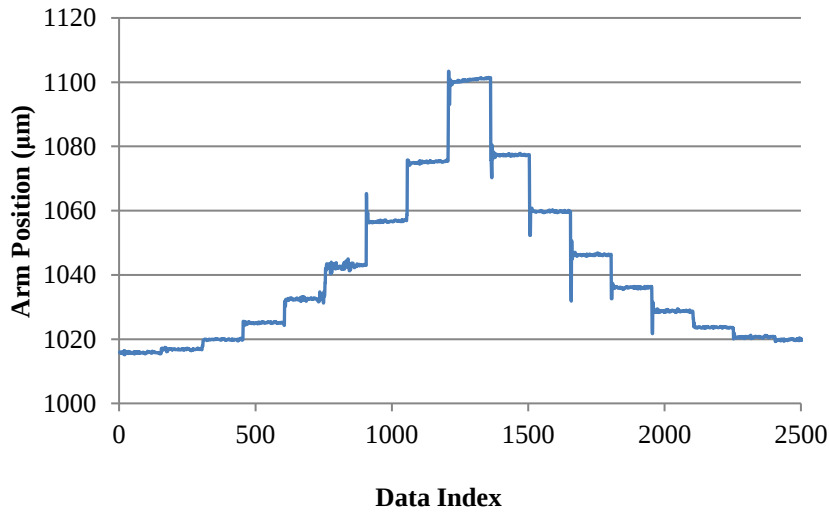


Figure 5. Example calibration plot of arm position.

From this data, the arm position at each voltage is averaged and compared to the initial null position to create a the change in position, Δd^* , at each voltage. Using Equation (10), this data can be converted into a line of generated force as a function of observed displacement. An example of one such plot is shown below in Figure 6. Note that this linear relation is characteristic of the system and is easily observed elsewhere in other systems of the same design.⁴ It should be noted that calibration should only be performed either at atmospheric or at high vacuum conditions to prevent breakdown and arcing, as low vacuum pressures coincide with a minima in the Paschen curve.

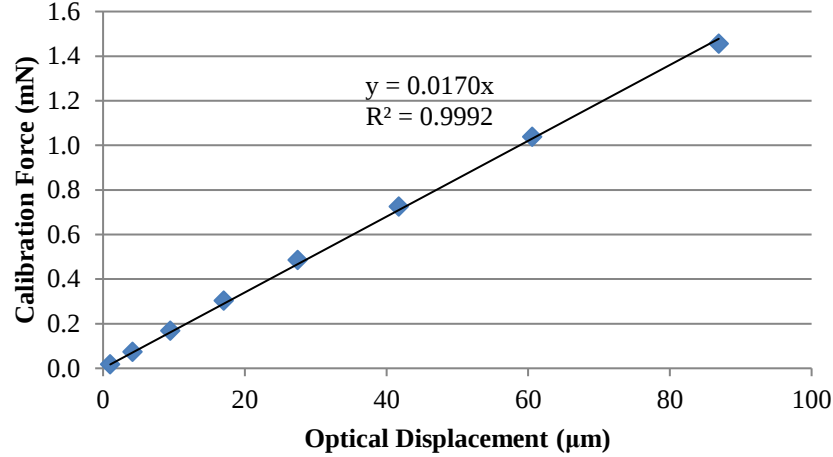


Figure 6. Example capacitive force as a function of the observed displacement as seen by the optical sensor.

Torsional Stiffness Calculation

Keeping Equation (4) in mind, the goal of calibration is to empirically determine the torsional stiffness of the balance arm. The slope of the line in Figure 6 is an effective linear stiffness, k_x , but it is not a true torsional stiffness. Comparing k_x and Equation (3), the torsional stiffness can be found as follows.

$$k_\theta = r_e r_o k_x \quad (11)$$

Since the flexures used in the thrust stand, as well as any wires or propellant lines, have a linear response across the small deflections observed, the torsional stiffness of the thrust stand must also behave linearly. This is reason for the linear nature of Figure 6. Once the torsional stiffness is calculated, the thrust produced by a test device can be determined from displacement of the arm using Equation (4).

B. Uncertainty Analysis

While CFG calibration techniques are useful in that they provide a non-contact means to apply a force to the torsional arm, the primary drawback is relatively large number of sources of uncertainty. This is a result of using a curve-fit to determine the zero-shift and the effective area, as well as the many parameters needed to calculate the electrostatic force, each requiring accurate measurement. Therefore, in order to ensure a highly accurate thrust measurement, each source of uncertainty must be identified and minimized.

Capacitance Curve Fit Uncertainty

Proceeding in order from the calibration procedure to the thrust measurement, the first source of uncertainty is the capacitance curve-fit used to determine the zero-shift and effective area. In statistical analysis, the standard error, σ , of a parameter from a least-squared curve-fit is a function of the residual sum of the squares, R_{ss} , the number of data points, N , the number of parameters to fit, n_p , and the covariance matrix, as seen in the equation below.

$$\sigma(p_i) = \sqrt{\frac{R_{ss}}{N - n_p} \text{cov}_{ii}}$$

The above equation is difficult to calculate analytically, due to the complexity of the covariance matrix. For convenience, a Matlab script is used to fit the capacitance data to Equation (6) using a non-linear least-squares function, *fit*.

This internal Matlab function returns a 95% confidence interval for the two parameters, which can be used to determine the standard error of each parameter using the equation below.

$$\sigma = \frac{CI_{Upper} - CI_{Lower}}{3.92} \quad (12)$$

Note that the denominator varies with the confidence interval; for a 99% confidence interval it is 5.15, and for a 90% confidence interval it is 3.29. Added to this is the uncertainty of the device used to measure the capacitance, U_C , resulting in the following.

$$U_{Zs} = \sqrt{U_C^2 + \left(\frac{\sigma_{Zs}}{Zs}\right)^2} \quad (13)$$

$$U_{Aeff} = \sqrt{U_C^2 + \left(\frac{\sigma_{Aeff}}{A_{eff}}\right)^2} \quad (14)$$

In order to minimize error, the curve fit should be restricted to within the bounds of gap distances that are expected to be used. Furthermore, collecting as many data points as possible increases the number of degrees of freedom of the curve-fit which decreases the standard error.

CFG Calibration Uncertainty

The next source of uncertainty is the CFG force calculation and the multiple diagnostics required for the various parameters in Equation (10). For parameters that can be directly measured, the uncertainty is dictated by the accuracy of the diagnostic used, summarized in Table 1. For other parameters that are calculated from these basic values the uncertainty is determined through standard error propagation methods.

Table 1. Summary of parameter diagnostics and uncertainties.		
Parameter	Diagnostic	Uncertainty
$r_e, r_o, r_a, r_T,$	Direct measure	$\pm 1/64$ in
V	HP 974A	$\pm 0.05\%$
d_o, d^*	Philtec model D63	± 50 nm
$\dot{m}$	Alicat M series	$\pm(0.4\% \text{ RD} + 0.2\% \text{ FS})$

The change in arm position as measured by the optical sensor, Δd^* , is calculated from the difference of two arm positions at different calibration force or thrust levels. As such, the relative uncertainty can be found as

$$U_{\Delta d^*} = \frac{\sqrt{(\delta d_1^*)^2 + (\delta d_2^*)^2}}{\Delta d^*} \quad (15)$$

where δd^* is the absolute uncertainty of the arm position as measured by the optical displacement sensor at operating condition 1 and 2 from which Δd^* is calculated. Due to the likely presence of minute vibrations of the arm, each d^* measurement is an average of multiple measurements, the population of which will have a mean and a standard deviation (or standard error) about the mean, σ_{d^*} . The absolute uncertainty of an averaged measurement is

$$\delta d^* = \frac{\sigma_{d^*}}{\sqrt{N}} \quad (16)$$

where again N is the number of data points in the measurement population. As an aside, the uncertainty caused by vibration or noise in the arm position can thus be mitigated by increasing the measurement sample size. The compound uncertainty of the CFG force is calculated by conducting uncertainty propagation through Equation (10).

$$U_{F_{CFG}} = \sqrt{U_{Aeff}^2 + 4U_v^2 + 4 \frac{\sigma_{d_o}^2 + \sigma_{Z_s}^2 + \left(\frac{r_e}{r_o} \Delta d^*\right)^2 (2U_r^2 + U_{\Delta d^*}^2)}{d_0 - \frac{r_e}{r_o} \Delta d^* + Z_s}} \quad (17)$$

Equation (17) only gives the uncertainty of a single force measurement based on a transition between two force conditions (i.e. you need two data points to calculate Δd^*). However, the end goal of the calibration is to determine the torsional stiffness, k_θ . From the trend line in Figure 6 we can extract the slope and the accompanying standard error. The uncertainty of k_θ itself is a somewhat difficult question, as we are essentially trying to determine the uncertainty of the slope of a trend line of calculated points with their own individual uncertainty. As a conservative approach, the uncertainty of k_θ is calculated as the propagated uncertainty of: the average uncertainty of the CFG force, the uncertainties of the torque moment arms, and the standard error of the slope of the trend line in Figure 6.

$$U_{k_\theta} = \sqrt{\bar{U}_{F_{CFG}}^2 + 2U_r^2 + \left(\frac{\sigma_{k_x}}{k_x}\right)^2} \quad (18)$$

Thrust Measurement Uncertainty

The next step is to calculate the uncertainty of any measured thrust based on the calibration. Using Equation (4), we can propagate the additional quantities into the total uncertainty of the thrust measurement.

$$U_{F_T} = \sqrt{U_{k_\theta}^2 + 2U_r^2 + U_{\Delta d^*}^2} \quad (19)$$

It should be noted that the uncertainty of the arm displacement is a function of both the vibrational noise occurring at the time of the measurement and the magnitude of the displacement itself. Thus, the thrust of a test article has a unique uncertainty at each operating condition, rather than a flat accuracy across a range of conditions. The reported uncertainty later in this paper is an average of a set of uncertainties and is meant for illustrative purposes of the system's capability.

Specific impulse is determined using the thrust measurement,

$$I_{sp} = \frac{F_T}{\dot{m}g} \quad (20)$$

where $\dot{m}$ is the mass flow rate of the propellant through the nozzle and g is the acceleration due to gravity at sea level. The mass flow can be calculated from the volumetric flow rate by the equation below,

$$\dot{m} = \frac{q[\text{sccm}]}{1344} MW[g / \text{mol}]$$

where q is the volumetric flow rate, and MW is the molecular weight of the propellant used, each with the units denoted. The uncertainty of the specific impulse calculation is

$$U_{I_{sp}} = \sqrt{U_{F_T}^2 + U_m^2} \quad (21)$$

where U_m is the uncertainty from the flow meter, found in Table 1.

Beyond the uncertainty of a measurement, the thrust stand performance can be considered in terms of a resolution. The resolution of a device is the smallest measure that can be distinguished by the instrument. For direct measurements, this usually takes the form of the smallest increment, be it the 1/64 inch mark on a measuring tape, or the last digit of a digital display. In this case, thrust is derived from a statistical average of a set of direct measurements of another parameter. The resolution is then the smallest change of the direct measurement, in this case arm position, that can be clearly registered by the system. To be conservative, resolution is defined at the standard deviation of a position measurement converted to the equivalent thrust. While arm displacements smaller than the standard deviation can be analyzed to derive a thrust, the margin of error is usually large enough to make comparative analysis between operating conditions difficult.⁷

Drift vs. Hysteresis

The keen observer will have noticed that the calibration curve in Figure 5 demonstrated an oddity – the position of the arm at the beginning and the end of the calibration process is not the same. There are two possible explanations for the change in arm position: either the null arm position drifted through the calibration process, or there is hysteresis in the motion of the arm. The former could be caused by a change in thermal or mechanical strain of some element within the system, while the latter would be likely caused by friction during motion. Thermal or mechanical strain can be remedied by allowing sufficient time for the system to stabilize to a steady state, whereas hysteresis can only be remedied by adjusting the setup or by accounting for it in the uncertainty calculations. In order to identify the cause for null position drift, the thrust stand is repeatedly perturbed using the CFG plates to observe the change in arm position between null cases, as shown in Figure 7.

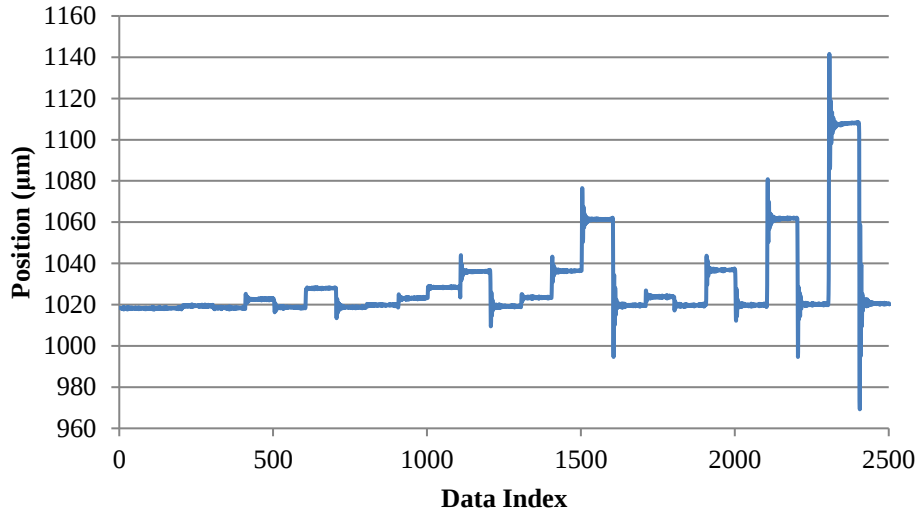


Figure 7. Perturbation of the thrust stand arm through the CFG plates.

The change in the null points is then compared to the drift of the thrust stand arm throughout the process. The arm dynamics occur in two primary timescales: the shorter timescale of mechanical motion of the arm in response to a change in force, and the longer timescale of the arm responding to mechanical and thermal strain relaxation or generation. The arm response to a change in force is relatively rapid, as a new steady state position is reached within six seconds. Once this motion has subsided, there is a small drift at this otherwise stable position. The shift in the null points is then compared to the average of this drift, as shown in Figure 8.

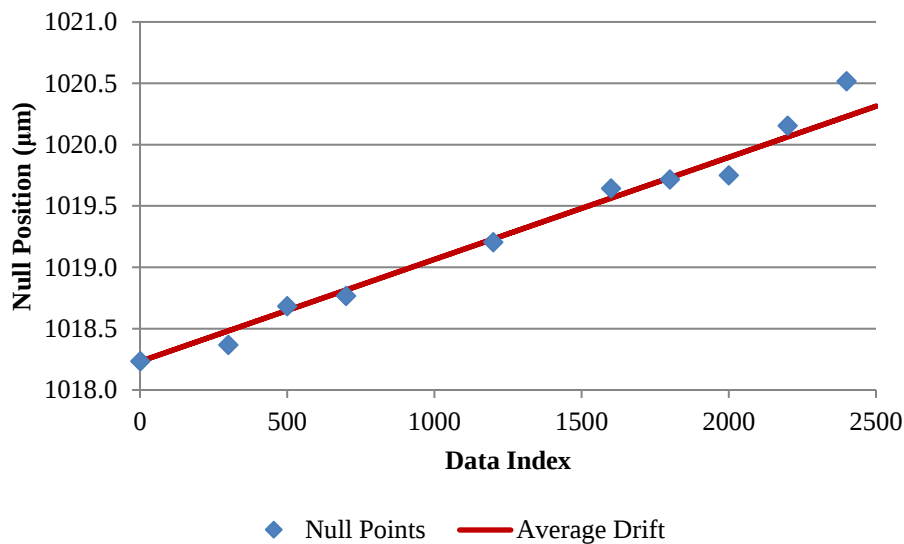


Figure 8. Change in thrust stand arm null position compared to average drift.

The data suggests that the observed change in null position of the thrust stand arm before and after calibration is due to aggregate drift of the null position over time, rather than friction-induced hysteresis. Therefore, the effect can be averaged across the number of steps within the calibration and accounted for within the uncertainty analysis. For a given total drift of H across a calibration with N steps, the uncertainty of the CFG force is given by a modification of Equation (17).

$$U_{F_{CFG}} = \sqrt{U_{Aeff}^2 + 4U_V^2 + 4 \frac{\sigma_{d_0}^2 + \sigma_{Z_s}^2 + \left(\frac{r_e}{r_0} \Delta d^*\right)^2 (2U_r^2 + U_{\Delta d^*}^2)}{d_0 - \frac{r_e}{r_0} \Delta d^* + Z_s} + \left(\frac{H/N}{\Delta d^*}\right)^2} \quad (22)$$

IV. Results

To demonstrate thrust stand performance and operational characteristics, a cold gas thruster is used as a test article. The thruster uses unheated nitrogen gas that is metered using an Alicat M series flow controller. The gas is expanded through a micro-nozzle with a throat diameter of 150 microns, an angle of 38.9°, and an area ratio of 96.6. A maximum operating pressure of 8.85×10^{-5} Torr is reached with a nitrogen flow rate of 80 sccm. At each flow rate the thruster is given time to settle (approximately 10 minutes required) and then run for 3 minutes at steady state. Data collection of all diagnostics is automated at 4 Hz through LabView. The data is then averaged for each steady state condition and the resulting uncertainties calculated. Figure 9 shows the performance of the thruster as measured by the thrust stand. The thrust stand demonstrated an average resolution of 4.8 μ N based on the RMS vibrational noise characteristics and an average thrust uncertainty of 14.6 μ N, or 2.9% of the measured quantity.

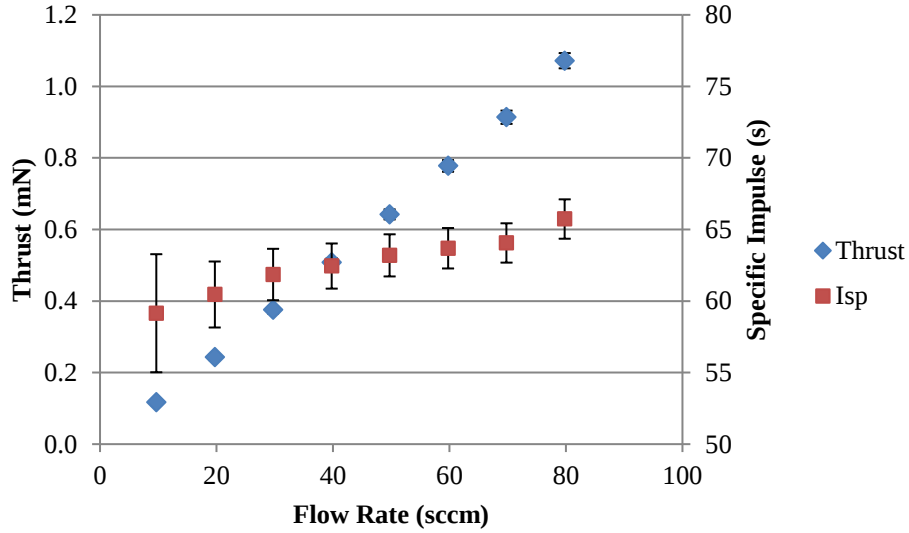


Figure 9. Thrust and specific impulse of a cold gas micro-thruster.

Despite the large number of parameters in the uncertainty propagation, most can be controlled through careful selection of diagnostics to ensure maximum accuracy. The primary drivers of uncertainty were found to be not the diagnostics, but rather the aggregate statistical parameters Z_s , Δd^* , and H . Uncertainty in the zero shift term can be addressed by increasing the number of capacitance measurements used in the curve fit, which reduces the standard error σ_{Z_s} . Uncertainty in the ODS measurements is driven primarily through vibrational noise, so additional damping and extended data sampling can improve accuracy. Reducing total drift through calibration is currently being investigated through better temperature control of the ODS electronics.

V. Conclusion

A torsional balance thrust stand system has been developed and tested with a cold gas thruster. High resolution measurements, reaching as low as 4.8 μ N, have been achieved with an average uncertainty of 14.6 μ N for a thrust range of 117-1070 μ N. Environmental noise is mitigated through the use of a low-stiffness vibrational isolation system and adjustable magnetic damping of the balance arm. A relatively fast response time of approximately 5 seconds can be achieved by

operating the system close to a critically-damped regime, although other settling times can be achieved by varying the damping according to the dynamics of the test article.

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