

Laser-Induced Fluorescence Measurement of the Anomalous Collision Frequency in a 9-kW Magnetically-Shielded Hall Thruster

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Abstract: We present a simple fluid model relating the electron anomalous collision frequency to plasma properties measurable with laser-induced fluorescence (LIF). Assuming that an anomalous drift measured in azimuthal ion velocity is caused by a friction force against the electrons and ions, the electron anomalous collision frequency that must exist to explain that drift can be calculated. The technique is demonstrated to estimate the anomalous collision frequency profile along the channel centerline of the H9 Hall thruster. The LIF-based results are in agreement with previous simulations and probe-based measurements of the anomalous collision frequency profile. The LIF-based anomalous collision frequency profile was used in a Hall2De simulation, and the resulting simulated ion velocity profile is reasonably close to the experimentally measured velocity profile, lending further support to the new method. The importance of this method is twofold. First, this method provides a non-intrusive way to validate the collision frequency profiles found in simulations. Second, it provides strong evidence that instabilities caused by the relative drift between electrons and ions can dominate anomalous collisionality since it only detects collisions related to a friction force between electrons and ions.

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Nomenclature

$\vec{B}$	= magnetic field
B_x	= radial component of magnetic field
$\vec{E}$	= electric field
E_z	= axial component of electric field
$g_z(v_z)$	= LIF profile of the axial component of velocity
I_{SP}	= specific impulse
IVDF	= ion velocity distribution function
LIF	= laser-induced fluorescence
m_i, m_e	= ion mass, electron mass
n_i, n_e	= ion density, electron density
P_i, P_e	= ion pressure, electron pressure
q	= electric charge
r_m	= mean discharge channel radius
t	= time
$\vec{V}_e$	= bulk electron velocity
$\vec{V}_i$	= bulk ion velocity
V_{iy}	= azimuthal component of bulk ion velocity
V_{iz}	= axial component of bulk ion velocity
V_{ey}	= azimuthal component of bulk electron velocity
v_z	= dummy variable for integrating over axial velocity
w_k	= ratio of the k^{th} to the $(k-1)^{th}$ raw moments of the axial IVDF
x, y, z	= radial, azimuthal, axial dimension
ν_{in}	= ion-neutral momentum loss collision frequency
ν_e	= total electron collision frequency
ν_{in}	= ion-neutral collision frequency
$\nu_{an(i)}$	= ion anomalous collision frequency
$\nu_{an(e)}$	= electron anomalous collision frequency
ω_{ce}	= electron cyclotron frequency
Ω_e	= Hall parameter
$\Omega_{an(e)}$	= anomalous Hall parameter

I. Introduction

A. Background and Motivation

ANOMALOUS electron transport across the magnetic field is one of the most important unsolved problems in Hall thruster research. It has been known for decades¹ that the electron mobility across the magnetic field in a Hall thruster, often discussed in terms of an effective electron collision frequency, is orders of magnitude higher than classical estimates based on collisions with neutrals or ions. One main problem stemming from anomalous transport is that enhanced current represents a power loss. Secondly, since the mechanisms causing enhanced collisionality are not well known and the effective collision rate cannot be predicted from first principles, Hall thruster simulations are limited to using an input collision rate profile instead of self-consistently solving for it along with other plasma properties. Better understanding anomalous electron transport might allow better and more efficient thrusters to be designed.

Possible mechanisms that may lead to the enhanced electron mobility have been investigated such as interactions with the discharge channel wall and instabilities such as the electron drift instability. No proposed mechanisms have yet been fully developed with the capability to accurately predict the anomalous collision frequency. Part of the difficulty in solving the anomalous electron mobility problem may be that there is no single mechanism but many different ones that may be active in different parts of the thruster and at different operating conditions. There may be a complicated interaction between the different mechanisms as well.

The current state-of-the-art in Hall thruster simulation (e.g. Hall2De²) requires inputs of experimental measurements of thrust and discharge current, and the code iterates over the anomalous collision frequency profile to converge on a simulation that agrees with the experimental results.

Accurate measurements of the anomalous collision frequency profile would be a boon to evaluating theories and validating simulations. In addition, it would be helpful to have a method to distinguish between different types of mechanisms rather than simply a measurement of the total effective collision rate. Some measurements of the anomalous collision frequency have been made with probe-based techniques.³ This method measures the total rate without regard to mechanisms, and the accuracy of probe measurements near the Hall thruster discharge can be suspect due to perturbing operation.

Laser-induced fluorescence (LIF) measurements use a laser to excite fluorescence from a particular energy level scheme. The spectrum of collected fluorescence is related to the ion velocity distribution function (IVDF) by the Doppler shift. It can have submillimeter spatial resolution inside or outside the discharge channel (limited by optical access to the injected lasers and collection optics) with a bandwidth up to about 1 megahertz.⁴⁻⁶ It is generally considered nonintrusive because it does not measurably change thruster operation as probes can, though it does shift energy level populations along the laser path, potentially leading to a slight optogalvanic effect in the thruster.

Electron confinement can be characterized by the dimensionless Hall parameter, the ratio of the electron cyclotron frequency to the electron collision frequency:

$$\Omega_e = \frac{\omega_{ce}}{\nu_e}. \quad (1)$$

The aim of this work is to demonstrate for the first time that nonintrusive LIF measurements can provide estimates of the Hall parameter or electron anomalous collision frequency in a Hall thruster. The method can help verify whether the anomalous collision frequency profiles obtained in modeling are actually realized in the laboratory and therefore aid efforts to understand and predict Hall thruster operation.

B. Organization

The remainder of the paper is organized as follows.

Section II describes the experimental setup used to carry out these tests. Laser-induced fluorescence is a nonintrusive technique to measure the ion velocity distribution function (IVDF). Two lasers, along axial and azimuthal dimensions of the thruster, excite fluorescence from xenon ions. The IVDF is inferred from the Doppler shift of the observed fluorescence spectrum.

Section III summarizes the theories and analysis procedure underpinning the work. First, Subsection III A details a simple fluid theory linking properties of the IVDF to Hall parameter. The theory requires an electric field profile, which can also be estimated from LIF data. Subsection III B summarizes the methods

used to estimate electric field. These methods require accurate measurements of some statistical properties of the IVDF, particularly the first three raw moments of the distribution. The method used to obtain measurements of the moments are detailed in Subsection III C.

The results are presented in Section IV. Subsection IV A summarizes the raw IVDF measurements obtained at the 300 V, 15 A operating condition of the H9. Subsection IV B presents the Electric field and ionization frequency profiles obtained by the models of Subsection III B and examines the difference between the results of the different models. Finally, Subsection IV C briefly presents the anomalous collision frequency profiles estimated from the LIF measurements at the 300 V, 15 A operating condition of the H9.

Section V discusses the results in further detail and in context with the literature. The first two subsections build a case that the LIF-based anomalous collision frequency is reasonably accurate. First by comparing with simulations and probe-based measurements previously published (Subsection V A) and then by comparing with a Hall2De simulation using the LIF-based anomalous collision frequency profile as its input (Subsection V B). The possible influence of systematic error is briefly discussed in Subsection V C. Finally, a preliminary uncertainty analysis is given in Subsection V D before concluding the paper.

II. Experimental Setup

A. H9 Hall Thruster

This experiment used the H9, a 9-kW magnetically-shielded Hall thruster with a nominal discharge voltage of 800 V. It was recently developed as part of a longstanding collaboration between the NASA Jet Propulsion Laboratory (JPL), the Air Force Research Laboratory (AFRL), and the University of Michigan. Three copies were made to use as a common research platform at the three facilities. Its main heritage is the H6 Hall thruster, also jointly developed by the three institutions, and the H6 MS, a modified, magnetically-shielded version created by JPL.^{7–9} Papers on the design¹⁰ and performance¹¹ of the H9 were presented at this conference.

The thruster was operated at the 300 V, 15 A operating condition for this experiment. The H9 provides convenient optical access for an azimuthal laser since most of the ionization and acceleration occurs downstream of the exit plane. The technique presented here, however, is not specific to any particular features of the H9, and should in principle be applicable to any Hall thruster.

B. Facility and Experimental Configuration

The experiment was carried out in the Large Vacuum Test Facility at the Plasmadynamics and Electric Propulsion Laboratory at the University of Michigan. It is a 6 m by 9 m chamber pumped by 7 cryo pumps with a total pumping speed of 250,000 l/s on xenon.

A typical time-averaged laser-induced fluorescence (LIF) setup was used. Many other publications have described the concept of LIF experiments in detail, e.g. Manzella¹² and Mazouffre.¹³ A common LIF level scheme for Xe II was used with laser injection near 834.724 nm (in air) and collection near 541.9 nm (in air). Figure 1 shows the experimental setup schematically. Axial and azimuthal lasers, chopped at different frequencies for signal multiplexing, were aligned to a single interrogation volume near the thruster with a diameter of about 1 mm. Lasers were aligned at the 12 o'clock position on the thruster centerline so that a horizontal beam corresponds to azimuthal measurements along the channel centerline (see Fig. 1(b)). The optics were fixed whereas the thruster was on motion stages to allow mapping the IVDF at a number of spatial locations along the discharge channel centerline.

Fluorescence from the interrogation volume was collected and sent out of the chamber by optical fiber. The collected light was filtered by a monochromator with a 1-nm pass band, and the signal from the photomultiplier was recovered by lock-in amplifiers with a time constant of 30 to 100 ms in most cases.

The convention for this sign of ion velocity of this paper is that positive velocity is flow counter to the laser beam. Thus, positive axial velocity is downstream (out of the thruster). The negative measured azimuthal velocity of Fig. 5, with sign determined by the direction of the laser, is in the $\vec{E} \times \vec{B}$ direction (and also the direction of the expected ion swirl).

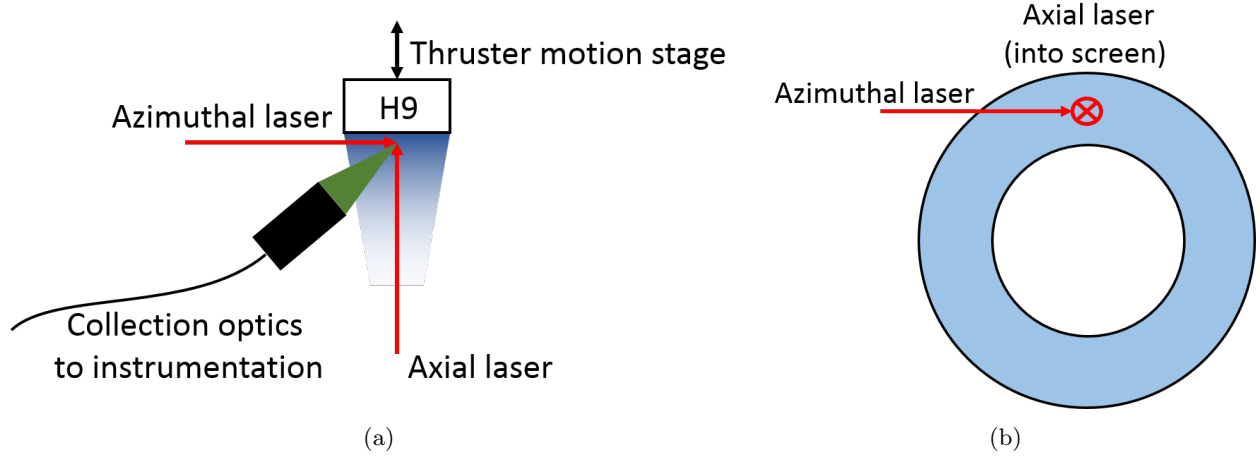


Figure 1: (a) Top-down schematic view of the experimental setup showing thruster, optics, and the axial motion stage. The thruster moves axially to allow mapping the IVDF along the discharge channel centerline. (b) Head-on view of the thruster showing the laser alignment at the 12 o'clock position. The “Azimuthal laser” measures the azimuthal IVDF for interrogation volumes along the channel centerline.

III. Methods

A. Model to Calculate Hall Parameter from LIF data

One of the key results of Subsection IV A is that the ions accelerate in azimuthal speed much faster than would be expected by the Lorentz force on the ions moving through the magnetic field. The central idea of the model is that this extra azimuthal ion acceleration comes from a drag force against electrons in the Hall current. The electrons in turn experience an equal and opposite change in momentum due to the drag force. Based on the drag force measured on the ions via LIF, the drag force on the electrons is known and can be related to an effective momentum loss collision frequency.

Using a simple fluid model, we formulate a theory linking the properties of the axial and azimuthal IVDF profiles that are measurable with LIF to the Hall parameter at the same measurement locations. First, consider the ion momentum conservation equation:

$$m_i n_i (\vec{V}_i \cdot \nabla) \vec{V}_i + \nabla P_i - q n_i (\vec{E} + \vec{V}_i \times \vec{B}) = -n_i m_i \nu_{an(i)} (\vec{V}_i - \vec{V}_e) - n_i m_i \nu_{in} \vec{V}_i, \quad (2)$$

where m_i is ion mass, n_i is ion density, $\vec{V}_i$ is ion velocity, P_i is ion Pressure, q is ion charge (the electron charge for the Xe II ions of interest), $\vec{E}$ is electric field, $\vec{B}$ is magnetic field, $\nu_{an(i)}$ is the ion anomalous collision frequency for momentum loss, $\vec{V}_e$ is electron velocity, and ν_{in} is the ion-neutral collision frequency. The first term on the right hand side represents the drag force on the ions from the electrons $\vec{V}_e$ and the second term represents ion-neutral collisions.

A number of simplifying assumptions are made. The annular geometry of the thruster is ignored and we consider a simplified coordinate system where the z axis is axial, y is azimuthal, and x is radial. We assume azimuthal symmetry of ion temperature and pressure to neglect the ion pressure term. $\vec{E}$ is considered purely axial and $\vec{B}$ is considered purely radial. The ion velocity (order 10^4 m/s) is negligible compared with electron drift velocity (order 10^7 m/s), so we can assume the drag force is purely azimuthal. Finally, the ion-neutral collision frequency will also be neglected since it is expected to be small compared with the anomalous collision frequency.

Using these simplifications, we can solve the azimuthal component of Equation 2 for the ion anomalous collision frequency:

$$\nu_{an(i)} = \frac{V_{iz}}{V_{ey}} \left(\frac{\partial V_{iy}}{\partial z} - \frac{q}{m_i} B_x \right). \quad (3)$$

The Bulk ion velocities are known via LIF measurements. The only variable for which we need a further estimate is V_{ey} , the electron azimuthal drift velocity. We expect it to be near the single particle $\vec{E} \times \vec{B}$ drift

velocity but can estimate it more accurately by considering the electron Ohm's law:

$$\nabla P_e - qn_e(\vec{E} + \vec{V}_e \times \vec{B}) = -n_e m_e \nu_{an(e)}(\vec{V}_e - \vec{V}_i). \quad (4)$$

Similarly, we neglect the pressure term on the left and ion velocity on the right and solve for the electron azimuthal drift velocity:

$$V_{ey} = \frac{E_z}{B_x} \left(\frac{\Omega_{an(e)}^2}{\Omega_{an(e)}^2 + 1} \right), \quad (5)$$

where an anomalous hall parameter is defined as the ratio of the electron cyclotron frequency to the anomalous electron collision frequency from the friction force:

$$\Omega_{an(e)} = \frac{\omega_{ce}}{\nu_{an(e)}}. \quad (6)$$

The anomalous Hall parameter becomes exact in the limit that $\nu_{an(e)}$ dominates all other sources of electron collisions (as we have assumed in this model). The electron drift speed is nearly the single-particle $\vec{E} \times \vec{B}$ speed except for a factor involving the anomalous electron Hall parameter $\Omega_{an(e)}$. The dependence on this factor intuitively makes sense since at large Hall parameter the electrons are well confined and the limiting drift speed is the single-particle $\vec{E} \times \vec{B}$ speed, whereas collisions dominate at low Hall parameter and the limiting drift speed is zero.

Now, we link the electron anomalous collision frequency to the ion anomalous collision frequency by assuming the ions gain equal and opposite momentum as the electrons lose, as they must unless there is some other momentum transfer:

$$m_e \nu_{an(e)} = m_i \nu_{an(i)}. \quad (7)$$

Then the electron anomalous collision frequency can be written as:

$$\nu_{an(e)} = \frac{m_i}{m_e} \frac{B_x V_{iz}}{E_z \frac{\Omega_{an(e)}^2}{\Omega_{an(e)}^2 + 1}} \left(\frac{\partial V_{iy}}{\partial z} - \frac{q B_x}{m_i} \right). \quad (8)$$

Or, writing entirely in terms of the anomalous Hall parameter:

$$\frac{\Omega_{an(e)}}{\Omega_{an(e)}^2 + 1} = \frac{m_i}{q} \frac{V_{iz}}{E_z} \left(\frac{\partial V_{iy}}{\partial z} - \frac{q B_x}{m_i} \right). \quad (9)$$

This implicit equation for the anomalous Hall parameter can be conveniently solved since all variables on the right hand side are known or can be measured with LIF. Alternatively, since the Hall parameter is large everywhere, Eq. 8 can be approximated as an explicit formula for the anomalous collision frequency in terms of measured variables:

$$\nu_{an(e)} = \frac{m_i}{m_e} \frac{B_x V_{iz}}{E_z} \left(\frac{\partial V_{iy}}{\partial z} - \frac{q B_x}{m_i} \right). \quad (10)$$

Equation 9 provides a formula to calculate the anomalous Hall parameter. We can regard the right hand side as a single measurable quantity f :

$$\frac{\Omega_{an(e)}}{\Omega_{an(e)}^2 + 1} = \frac{m_i}{q} \frac{V_{iz}}{E_z} \left(\frac{\partial V_{iy}}{\partial z} - \frac{q B_x}{m_i} \right) = f. \quad (11)$$

Equation 11 can be written with the anomalous Hall parameter as the roots of a quadratic function and solved as:

$$\Omega_{an(e)} = \frac{1 + \sqrt{1 - 4f^2}}{2f}, \quad (12)$$

where we have taken larger root because physically the Hall parameter is larger than 1. We can then solve for the anomalous collision frequency:

$$\nu_{an(e)} = \frac{2f\omega_{ce}}{1 + \sqrt{1 - 4f^2}}. \quad (13)$$

Mathematically, there are two distinct solutions of anomalous Hall parameter $\Omega_{an(e)}$ for every value of f (except for $\Omega_{an(e)}(f = 0.5) = 1$); one solution greater than 1 and one solution less than 1. This ambiguity is not generally a problem because physically we know the Hall parameter is large near the thruster. This subtlety might become problematic if the method were used farther downstream of the thruster, where the Hall parameter is expected to be near 1.

B. Models to Calculate Electric Field From LIF data

To apply the model of the previous section for estimating Hall parameter from LIF data, we solve Equation 9. We require the electric field profile, which can also be estimated from LIF data.

The simplest method to estimate electric field from LIF data is to neglect all effects such as ionization and other collisions and assume that the change of bulk ion velocity is only due to acceleration through the electric field. The bulk velocity can be calculated as the peak or mean velocity of the measured IVDF. The force on a single ion in the axial electric field is:

$$qE_z = m_i \frac{dV_{iz}}{dt} = m_i \frac{dV_{iz}}{dz} \frac{dz}{dt}, \quad (14)$$

and therefore the electric field is estimated as:

$$E_z = \frac{m_i}{q} V_{iz} \frac{\partial}{\partial z} V_{iz}. \quad (15)$$

This method will be referred to as the simple E-field estimate.

A potential problem with this method is that we want to estimate the Hall parameter throughout the entire spatial range of LIF data. This includes up to the exit plane, which includes at least some of the ionization region, where neglecting ionization is likely a bad approximation.

Alternatively, we can use a more sophisticated model to estimate electric field and ionization profiles from measured LIF data. Perez-Luna¹⁴ describes the method in detail, but we summarize the pertinent results here. The theory makes several key approximations. First, temporal variation is neglected, effectively assuming the plasma source operates in steady state without oscillations, or at least that variation in time is small compared with variation in space and velocity space. Second, ionization is assumed to dominate charge exchange. Third, the neutral gas distribution from which ions are born is considered to be a Dirac delta function. This is a reasonable approximation if the IVDF is much broader than the neutral gas distribution, which is typical of a Hall thruster.

A simplified Boltzmann equation is obtained from these assumptions. Moments of the simplified Boltzmann equation provide a system of equations that is solved for analytical expressions for electric field and ionization profiles in terms of moments of the distribution. The results, written in the conventions of this paper and with a typographical error corrected, are:

$$E = \frac{m_i}{q} \frac{w_2 w_1}{2w_1 - w_3} \frac{\partial w_3}{\partial z} \quad (16)$$

and

$$\nu_i = \frac{a_z}{w_2} - \frac{w_1}{w_2} \frac{\partial w_2}{\partial x}. \quad (17)$$

Here $a_z = qE_z/m_i$ is the axial component of acceleration from the electric field on the ions. The w_k variables are the ratio of the k^{th} raw moment to the $(k-1)^{th}$ raw moment of the IVDF:

$$w_k = \frac{\int_{-\infty}^{\infty} v_z^k g_z(v_z) dv_z}{\int_{-\infty}^{\infty} v_z^{k-1} g_z(v_z) dv_z}. \quad (18)$$

C. Distribution Moments from Fitting to Sums of Skew Gaussian Functions

The Perez-Luna model of the previous section requires highly accurate measurements of the moments of the IVDF because it relies on derivatives of ratios of moments up to the third raw moment. The k^{th} raw moment of the IVDF, the numerator of Equation 18, is the weighted average velocity of the IVDF with a weight factor of v_z^k . This makes the calculation extremely susceptible to even small-amplitude noise at the high-velocity region of the measured VDF profile, even after substantial filtering to remove noise or defining tighter limits of integration than the extent of measured data. We were not able to easily develop a reliable method to numerically calculate the moments of the IVDF using raw or filtered data.

Instead, we turn to fitting a function to the data because moments can be accurately calculated for the noiseless fitting functions. This technique is also challenging because there is no general function that will fit to arbitrary LIF profiles. As we can see in Appendix A, there are many instances of highly skewed IVDFs for which Gaussian and even single skew Gaussian functions would be a poor fit. The skew Gaussian function

is described briefly in Appendix B. It is a generalized Gaussian function that has adjustable skewness. The measured IVDFs of the 300 V, 15 A operating condition of the H9 happen to be relatively amenable to fitting to a single skew Gaussian. In other operating conditions (not presented here), there are also more complicated IVDFs that are bimodal, asymmetric, extremely broad, and often have a uniform region between the two main peaks. We found, however, that sums of a few skew Gaussian functions provide enough freedom to very well fit arbitrary IVDF profiles without overfitting.

To achieve the most accurate fits possible, the results presented here were calculated using a sum of two skew Gaussians as the fitting function. Wherever statistical properties of the IVDF or calculations based on them are reported in this paper, those moments used are numerically calculated from the fit to a sum of skew Gaussians as shown in Appendix A.

IV. Results

The following results are reported with normalized thruster dimensions. The axial position is normalized by the mean channel radius. The origin at $z/r_m = 0$ is taken to be the axial position at the outermost edge of the boron nitride.

A. Ion Velocity Distribution Measurements

Figure 2 displays the measured IVDF traces as a function of axial position along the channel centerline for the 300 V, 15 A operating condition of the H9. Each profile is individually normalized to its own peak. Note that the step size of axial position is not uniform; it tends to be smaller where the gradient is larger to better capture the change in the IVDF. Figure 3 shows the mean axial velocity obtained by the mean of the IVDFs (measured by the first moment of the fitting function). These mean values are later used in Section IV C to calculate the anomalous collision frequency from Equation 9.

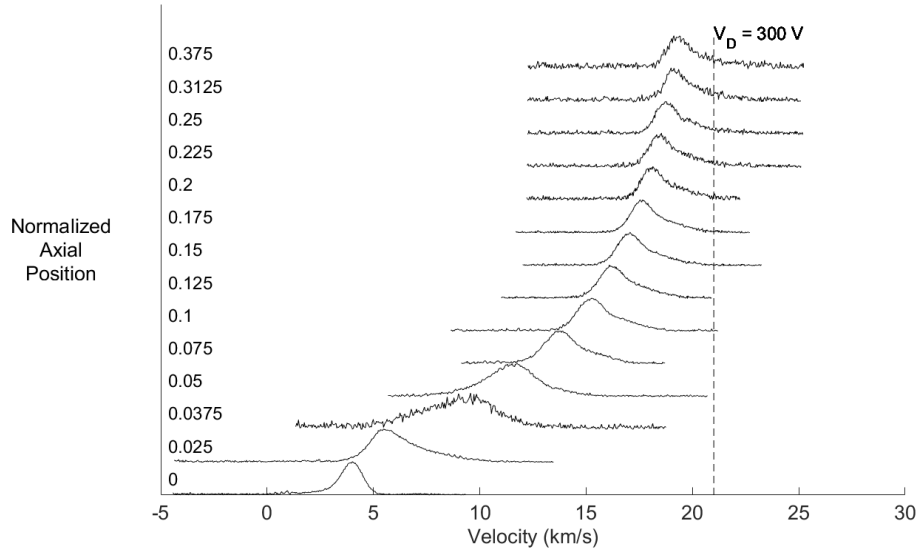


Figure 2: Axial IVDF at the 300 V, 15 A operating condition of the H9. Most of the acceleration occurs shortly downstream of the exit plane.

Most of the acceleration occurs outside the discharge channel. The final mean velocity of 19.96 km/s measured at $z/r_m = 0.375$ implies a voltage utilization of at least 90% and an I_{SP} of more than 2000 s.

The spread of the IVDF is quantified by the standard deviation of the function fit using the method of Subsection III C and plotted in Fig. 6. The spread of the axial IVDF reaches a maximum shortly downstream of the exit plane. The broad spread of the distribution is possibly due to overlapping ionization and acceleration regions, a common effect in Hall thruster IVDF measurements.¹³ The spread declines thereafter possibly due to kinematic compression.¹⁵

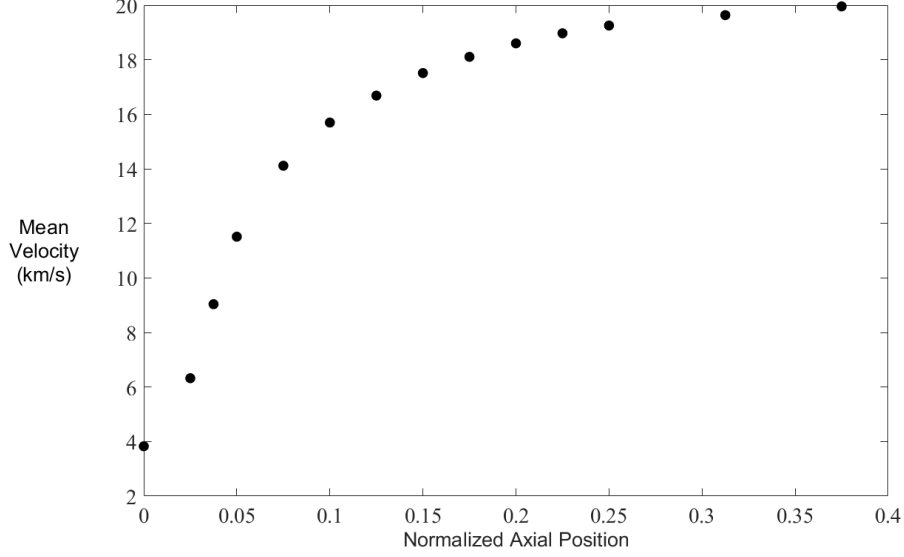


Figure 3: The mean axial velocity for the 300 V, 15 A operating condition of the H9 obtained from fitting. Most of the acceleration occurs between the exit plane and about 0.4 discharge channel radii downstream.

The skewness of the distribution increases toward the end of the acceleration region, resulting in a significant high-energy tail. The increasing skewness is also generally consistent with kinematic compression. There may be a slight population of ions in the high-energy tail at energies above the discharge potential, the corresponding to the vertical dashed line in Fig. 2.

Figure 4 shows the azimuthal velocity distribution mapped along the channel centerline starting with the exit plane at $z/r_m = 0$ (the furthest upstream location possible for a radial laser beam without modifying the thruster). The mean of the azimuthal IVDF is plotted in Fig. 5, showing a nearly linear acceleration in bulk azimuthal velocity. The slope of the linear fit (in units of meters not normalized units) is used to approximate $\frac{\partial V_{iy}}{\partial z}$ in Eq. 9 to calculate the Hall parameter and anomalous collision frequency. This measured azimuthal acceleration in red implies an ion swirl that is more than twice what would be expected for the magnetic field of this thruster. The blue data show the hypothetically expected acceleration in azimuthal velocity given the Lorentz force on the ions moving through the radial magnetic field of the thruster.

It is clear from Fig. 4 that the spread of the distribution monotonically increases. This is quantified in Fig. 6 with a plot of the standard deviation of the measured azimuthal IVDF. The measurement comes from the square root of the variance, the central second moment of the distribution, calculated from the fit to a sum of skew Gaussian functions. Note that the azimuthal spread begins far below the axial spread at the exit plane, but they converge to approximately equal values by about 0.2 channel radii downstream. Following that point, the axial spread appears to break its decreasing trend and rises along with the azimuthal velocity. More points are necessary to determine whether that trend continues, but it appears that the ions may be thermalized downstream of about 0.3 channel radii. More importantly, ions are clearly not thermally isotropic upstream of that location, and therefore assuming thermalized ions may be a poor assumption for models to make in the ionization and acceleration zones of Hall thrusters.

B. Electric Field and Ionization Profiles

Figure 7 shows the electric field profiles from the models described in Subsection III B evaluated on data from the 300 V, 15 A operating condition of the H9 of Subsection IV A. The two electric field models tend to agree very closely where there is little ionization, but the electric field from the Boltzmann model tends to be slightly higher than the field from the simple model where there is significant ionization. This result provides some evidence that the field estimates are reasonable and accurate. Despite the calculations by completely different methods (compare Eq. 15 and Eq. 16), we would expect the electric fields calculated to be equivalent where there is no ionization because that is the main assumption of the simple electric field

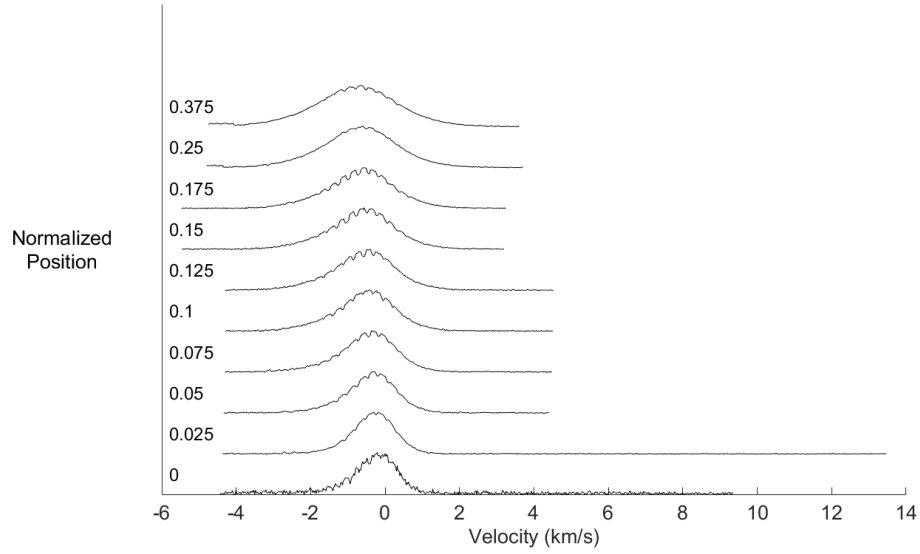


Figure 4: The mean speed and spread of the measured azimuthal IVDF monotonically increase with axial position.

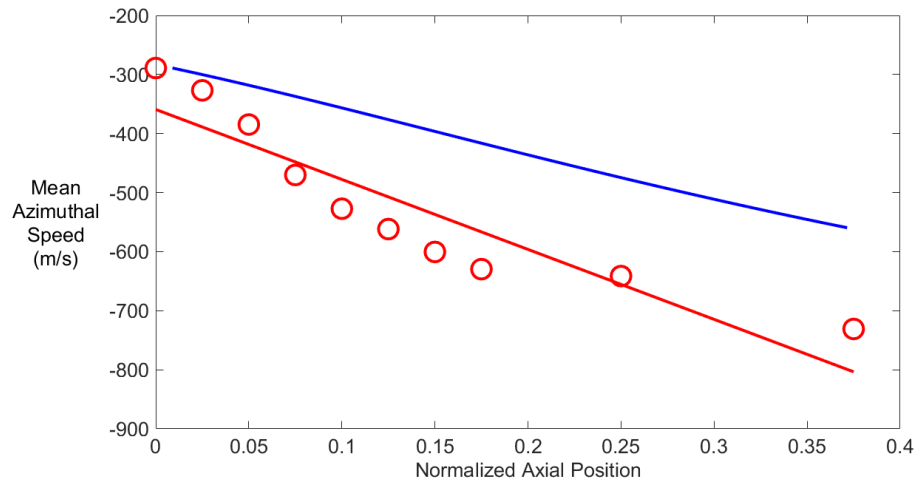


Figure 5: The measured azimuthal mean velocity (red) accelerates more than two times faster than expected by the Lorentz force alone on the ions in the magnetic field of the thruster (blue). A linear Fit is included as a guide to the eye and used in the model calculation for the derivative of azimuthal bulk velocity.

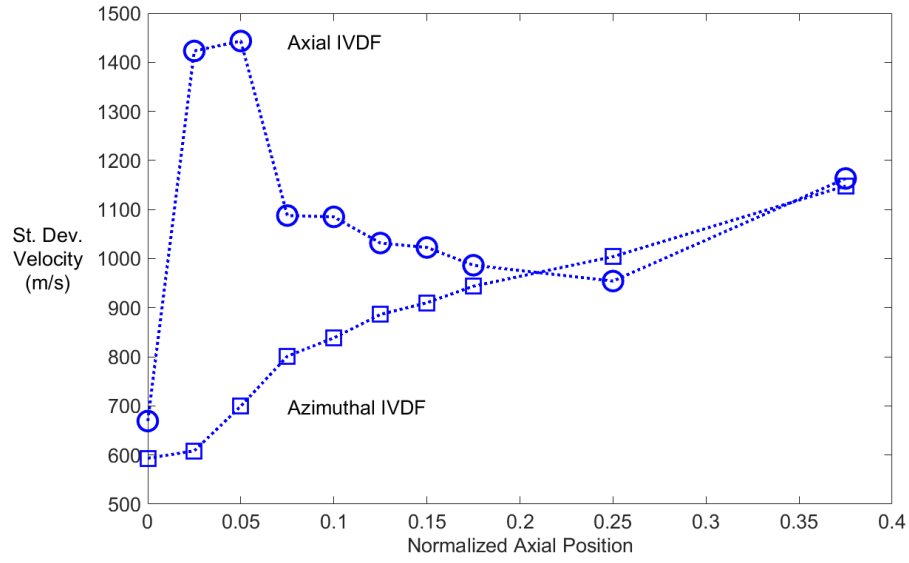


Figure 6: The axial spread is much larger than the azimuthal IVDF spread in the region where acceleration is highest. The azimuthal IVDF spread monotonically increases with axial position. The steepest slope is shortly outside the discharge channel.

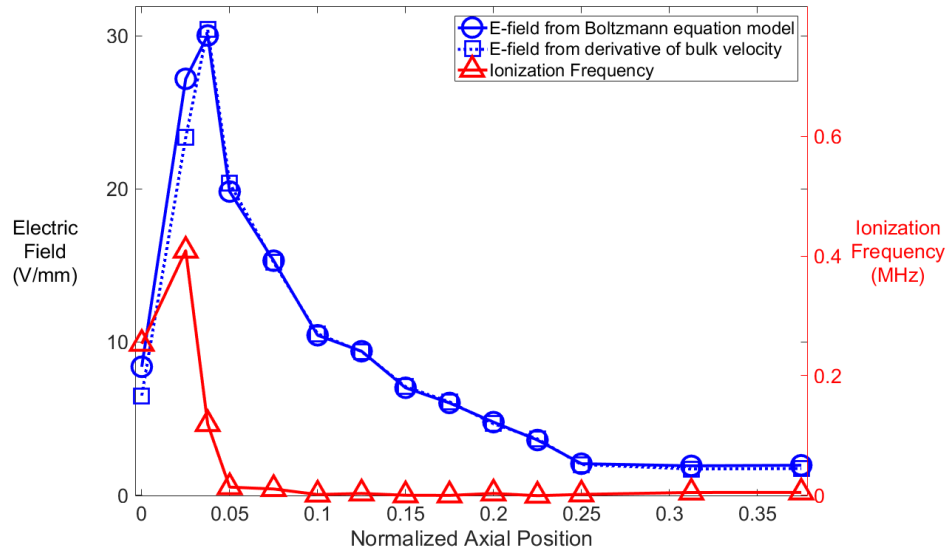


Figure 7: The electric field from the Boltzmann model is slightly higher than the field estimated by the simple model where at positions where there is ionization, and the two are equivalent where there is no ionization. The ionization profile overlaps the acceleration zone and peaks shortly upstream of the peak in electric field.

model. We also expect the electric field from the Boltzmann model to have a larger magnitude where there is ionization because ionization generates low-velocity ions that tend to depress the mean velocity and its derivative, leading to a smaller magnitude of estimated electric field by Eq. 15.

Over 240 V of the voltage drop occurs downstream of the exit plane, and most of that within 0.1 channel radius. The peak in electric field occurs near 0.0375 radii downstream, whereas the ionization profile peaks slightly upstream near 0.025 radii downstream. The acceleration and ionization zones clearly overlap significantly. Lacking data further upstream, however, we are unable to determine how much or whether there is any part of the ionization zone at low electric field.

C. Electron Anomalous Collision Frequency

With the results of the previous subsections, we are in a position to apply the model of Subsection III A to estimate the electron anomalous collision frequency profile along the thruster channel centerline. Figure 8 shows the collision frequency profile calculated using Eq. 9 plotted together with the normalized magnetic field profile. The collision profile appears to reach a minimum shortly before the peak in the magnetic field profile and rises monotonically thereafter in the measured range. The slope of the rise in anomalous collision frequency after the minimum becomes more gradual at locations further downstream, but it is unclear due to noise and a low density of points.

The collision frequency profiles from the two electric field models agree within 3% of each other except where ionization was detected in Fig. 7. Despite electric field profiles that are deceptively close in Fig. 7, the largest relative difference between the collision frequencies from the two electric field models is up to 25% in the ionization region near the exit plane. The Perez-Luna collision frequency is lower than the simple collision frequency since the Perez-Luna electric field is higher.

There is also a significant difference between the two collision rate estimates at the last two points. The Perez-Luna model detected a small ionization rate there, though it may only be due to some error such as inaccuracies in the least squares fits from which moments are calculated.

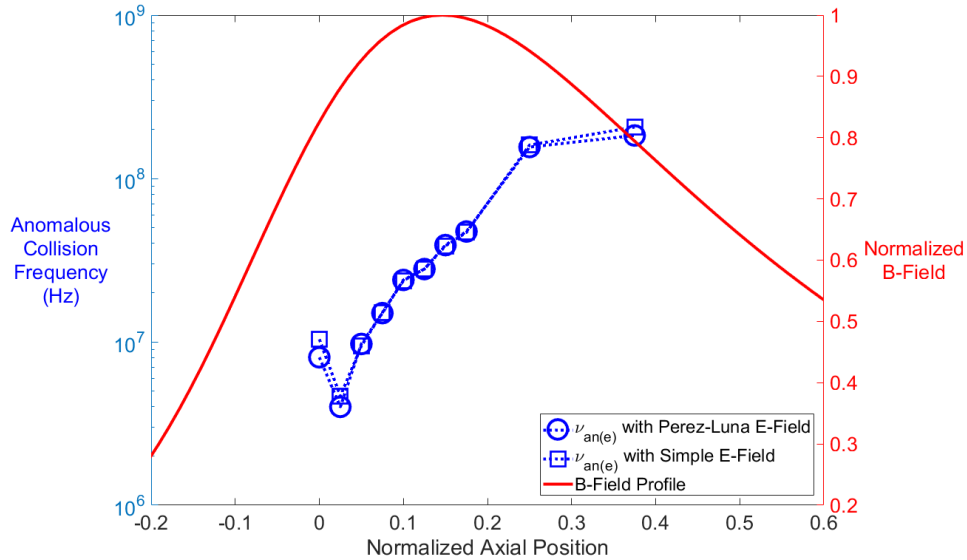


Figure 8: The electron anomalous collision frequency calculated from the two electric field profiles are similar qualitatively. Where there is ionization, the rate from the Perez-Luna electric field can be about 25% lower than the rate from the simple electric field.

V. Discussion

A. LIF-Based Anomalous Collision Rate Versus Simulation and Probe-Based Experiment

The LIF-based collision frequency profile of Fig. 8 strongly resembles the results found in simulations and probe-based experiments.

Figure 9 shows an example of the collision frequency profile from a Hall2De simulation of the H6. The z/L axis is axial position normalized by the discharge channel length from the anode to the exit plane. Hence $z/L = 1$ is akin to the normalized axial position 0 at the exit plane in the figures of this paper.

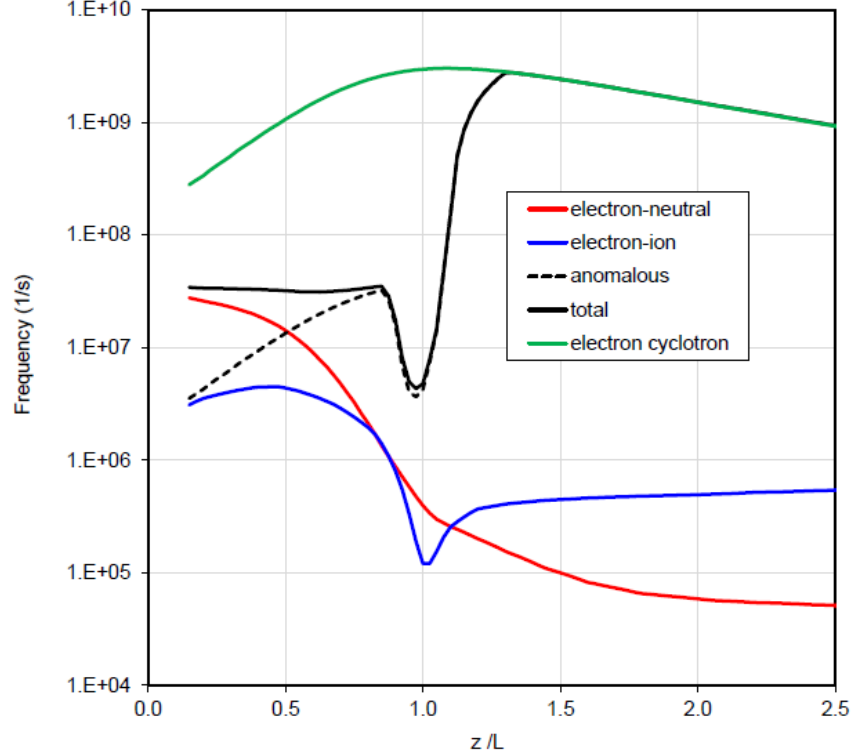


Figure 9: An anomalous collision frequency profile (black dashed line) from a Hall2De simulation of the H6 shows similar trends and order of magnitude as the laser-based collision profile measurement. Reproduced from Mikellides.¹⁶

Like the LIF-based result, there is a minimum just upstream of the maximum in the magnetic field profile. The green electron-cyclotron frequency curve is essentially a proxy for the magnetic field profile since it is proportional to B-field magnitude. In addition, the slope of the anomalous collision frequency downstream of the exit plane reduces somewhat as it approaches the electron-cyclotron frequency, similar to the 'kink' found in Fig. 8 near the normalized axial position of 0.1.

Note that the laser-based measurement spans only the range of magnetic field above about 80% of the peak, covering much less range than the simulation of Fig. 9. Therefore, that simulation contains features not resolved by our measurement, including the local maximum in anomalous collision frequency upstream of the minimum and the limit at the electron-cyclotron frequency.

Figure 10 shows an example of probe-based measurements of anomalous collision frequency at two different operating conditions of the NASA-173Mv1. Like our LIF-based measurements on the H9, there is a minimum in the anomalous collision frequency of about 1 MHz just upstream of the peak in magnetic field. Note the "Bohm" curves are a proxy for magnetic field because the Bohm diffusion frequency is proportional to electron-cyclotron frequency.

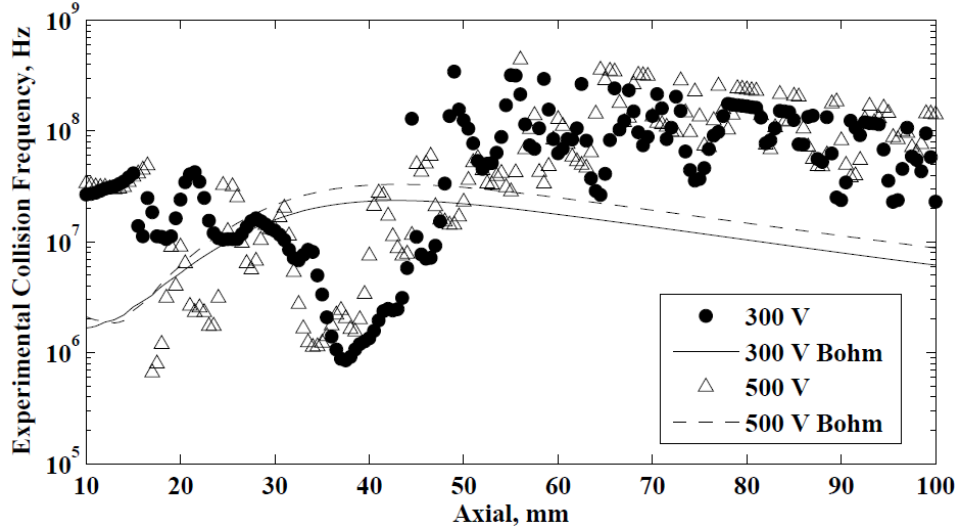


Figure 10: Probe-based measurements of the anomalous collision frequency profiles of the NASA-173Mv1 show similar trends and order of magnitude compared with the LIF-based results. Reproduced from Linnell.³

B. Hall2De Simulation Using the LIF-Based Anomalous Collision Profile

Hall2De is a two dimensional (axial and radial) multi-fluid solver including multiple ion charge states, neutrals, and electrons. It was developed at the NASA Jet Propulsion Laboratory² and has successfully supported investigations of magnetic shielding^{7,9} and flight development programs. Like all state-of-the-art Hall thruster fluid code, it requires an anomalous collision frequency profile that currently cannot be predicted from first principles. Hall2De can converge on a reasonable anomalous collision frequency profile by iterating over the profile to match experimentally measured discharge current and thrust, but it is not clear that the resulting profile is necessarily physically correct.

We carried out a simulation with Hall2De using the LIF-based anomalous collision profile. Because the measurements were sparse, we fit them to a piecewise function to use as input for the simulation (see Fig. 11 (a)). The region upstream of the LIF data was set to a low value from previous simulations of the H6MS. The simulation should not be very sensitive to the anomalous collision frequency here because electron-neutral and electron-ion collisions begin to dominate in this region (see Fig. 9). Jorns¹⁷ came to a similar conclusion in a parametric study. Downstream of the LIF data, the collision frequency is extrapolated out to the bounding value of the electron-cyclotron frequency (i.e. $\Omega_e = 1$). (b)

The resulting simulated ion velocity profile is shown together with the experimental profile in Fig. 11 (b). The simulation nearly correctly captures the acceleration region. There are some discrepancies; the acceleration of the simulation seems to begin slightly upstream of the experiment and the simulation has a lower final velocity. These discrepancies likely result from errors made in the collision frequency profile that was fit to the experimental data and input to the simulation. The exact shape of the profile is uncertain due to the limited range and density of points. Nonetheless, this result demonstrates that the experimental collision profile is reliable enough to yield reasonable results from the simulation.

C. Potential Sources of Systematic Error

Though the model has been applied to estimate the electron anomalous collision frequency, the accuracy of those profiles is not well known. In fact, there are some causes for concern.

The Perez-Luna model assumes a negligible time derivative, but Hall thrusters often have major oscillations in the ion velocity distribution.^{6,18} Applying the model to the time-average of an oscillatory VDF may not necessarily capture accurate estimates of time-averaged electric field and ionization profiles. The 300 V, 15 A condition of the H9 is relatively quiescent, thus there is not a major concern here.

Applying the Perez-Luna model at the 500 V condition, however, led to erroneous results. We numerically integrated the estimated electric field profile to obtain the estimated voltage drop across the interrogated

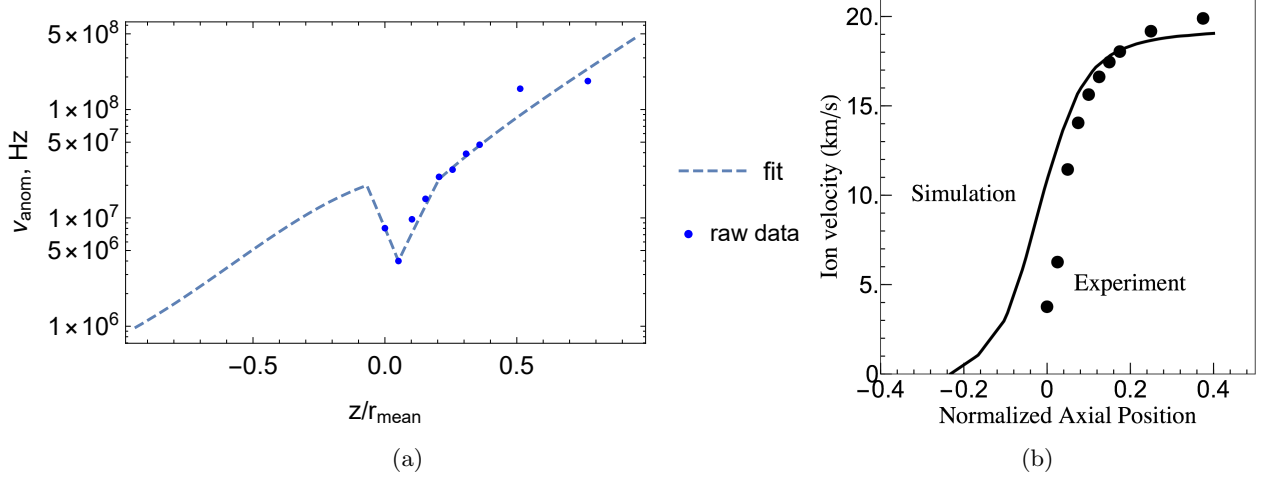


Figure 11: (a) A continuous piecewise function was fit to the LIF-based anomalous collision frequency for use with Hall2De. (b) The Hall2De simulation using the LIF-based anomalous collision profile is similar to the experimentally measured velocity profile.

region. The result is a plausible voltage drop of about 240 V over the dataset of the 300-V operating condition but an obvious overestimate of about 550 V for the 500-V operating condition. The reason for the overestimate in one case rather than the other may be that the 500-V condition is highly oscillatory while the 300-V condition is relatively quiescent, though the mechanism that may be causing the overestimate is not known.

The simple model of electric field may also not be appropriate to apply on time-averaged IVDFs measured in a strongly oscillatory condition. In addition, as shown in Fig. 7, the simple model can be inaccurate in the ionization region.

Another factor potentially contributing significant systematic error in this implementation is that the derivative of bulk azimuthal velocity is taken to be constant due to the lack of points and noise in numerically calculating the derivative. Instead of numerically differentiating, the drift of bulk azimuthal velocity is taken as the slope of a linear fit shown in Fig. 5. The drift may not be constant in reality. Without further data to better resolve the profile, it may be reasonable to fit a piecewise linear function to a few regions of the data or to interpolate a smooth curve between the points and differentiate that curve. In either case, the results at some points may change by as much as about a factor of two, but the qualitative features of the anomalous collision profile would likely remain.

D. Preliminary Uncertainty Analysis

Consider a quantity $x(u_1, u_2, \dots)$ calculated as a function of some set of measurements $\{u_1, u_2, \dots\}$, all with uncertainty that can be described as the standard deviation σ of some probability density function. A standard formula¹⁹ estimating the propagation of uncertainty for uncorrelated and small relative uncertainties is given by:

$$\sigma_x^2 = \sum_{i=1}^N \left(\frac{\partial x}{\partial u_i} \right)^2 \sigma_i^2, \quad (19)$$

where σ_x^2 is the variance of the calculated x and the set of σ_i^2 are the variances of the measured variables u_i . The uncertainty is considered to be the standard deviation of the distribution (square root of variance). In words, measurement error amounts to a small change in the argument u_i used to calculate $x(u_1, u_2, \dots)$. For a given uncertainty σ_i , the uncertainty of the calculated x is larger when the partial derivative of x with respect to u_i is larger because the function changes more for a small change in the argument u_i .

A quantitative analysis of uncertainty has not already been completed, but we can point out the overar-

ching trends based on the functional form of $\nu_{an(e)}$. Applying Eq. 19 gives:

$$\sigma_{\nu_{an(e)}}^2 = \left(\frac{\partial \nu_{an(e)}}{\partial f} \right)^2 \left(\left(\frac{\partial f}{\partial V_{iz}} \right)^2 \sigma_{V_{iz}}^2 + \left(\frac{\partial f}{\partial E_z} \right)^2 \sigma_{E_z}^2 + \left(\frac{\partial f}{\partial V_{iy}} \right)^2 \sigma_{V_{iy}}^2 \right), \quad (20)$$

where each σ^2 with a subscript of a variable name is the variance associated with the random distribution of that variable. Equation 13 is a nonlinear function of f , and f itself is a nonlinear function of the measured quantities, raising the question of whether the derivatives (and therefore the uncertainty in $\nu_{an(e)}$) are significantly larger in some conditions compared with others.

Though Eq. 13 is a nonlinear function of f , it has a large and approximately linear region including all f values from data presented in this paper. Figure 12 shows the derivative of anomalous collision frequency with respect to f to be nearly constant in the range of about $0 < f < 0.2$, corresponding to $4.79 < \Omega_{an(e)} < \infty$. This makes sense because in the limit of large Hall parameter, the anomalous Hall parameter is inversely proportional to f , whereas the anomalous collision frequency is proportional to f . In the range of f of interest, therefore, the function $\nu_{an(e)}(f)$ does not contribute significantly higher uncertainty at some f values than others.

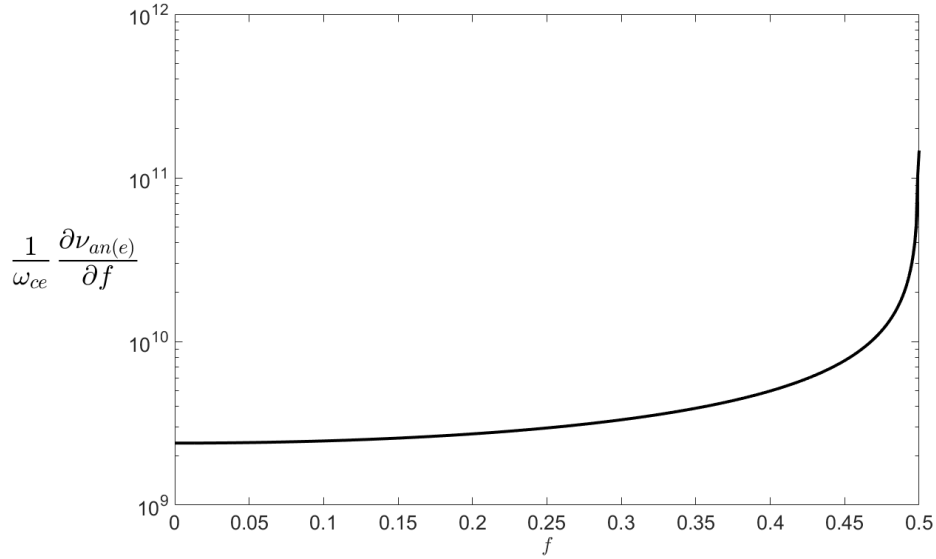


Figure 12: The derivative of anomalous collision frequency with respect to f is nearly constant at low f values (large Hall parameter).

We further consider the partial derivatives of f with respect to the uncertainties in the measurements of V_{iz} , E_z , and $V'_{iy} = \frac{\partial V_{iy}}{\partial z}$:

$$\frac{\partial f}{\partial V_{iz}} = \frac{m_i}{q} \frac{1}{E_z} \left(\frac{\partial V_{iy}}{\partial z} - \frac{q B_x}{m_i} \right), \quad (21)$$

$$\frac{\partial f}{\partial E_z} = -\frac{m_i}{q} \frac{V_{iz}}{E_z^2} \left(\frac{\partial V_{iy}}{\partial z} - \frac{q B_x}{m_i} \right), \text{ and} \quad (22)$$

$$\frac{\partial f}{\partial V'_{iy}} = \frac{m_i}{q} \frac{V_{iz}}{E_z}. \quad (23)$$

The partial derivative of f with respect to V_{iz} will clearly be largest in the near-field plume downstream of the acceleration region since the derivative is proportional to the inverse electric field and the derivative of azimuthal velocity is nearly constant. Similarly, the partial derivative of f with respect to electric field will be largest in the downstream region since it is proportional to the inverse square of electric field and the bulk axial ion velocity V_{iz} is maximal there. The same is true of the partial derivative of f with respect to the derivative of azimuthal velocity. All of these contribute to the same conclusion that the calculated

anomalous electron collision frequency will have a larger absolute uncertainty downstream where the electric field is low and the axial velocity is large.

The anomalous collision frequency also rises precipitously in the downstream region. Then although the absolute uncertainty may rise orders of magnitude there, the relative uncertainty may not be strongly affected. Note from Eq. 20 that the uncertainty in $\nu_{an(e)}$ is a weighted sum in quadrature of the uncertainties of the measured quantities with the derivatives as weight factors. The relative uncertainty has the same terms added in quadrature except divided by a factor of $\nu_{an(e)}$:

$$\frac{\sigma_x}{\nu_{an(e)}} = \sqrt{\left(\frac{1}{\nu_{an(e)}} \frac{\partial \nu_{an(e)}}{\partial f} \frac{\partial f}{\partial V_{iz}}\right)^2 \sigma_{V_{iz}}^2 + \left(\frac{1}{\nu_{an(e)}} \frac{\partial \nu_{an(e)}}{\partial f} \frac{\partial f}{\partial E_z}\right)^2 \sigma_{E_z}^2 + \left(\frac{1}{\nu_{an(e)}} \frac{\partial \nu_{an(e)}}{\partial f} \frac{\partial f}{\partial V_{iy}'}\right)^2 \sigma_{V_{iy}'}^2}. \quad (24)$$

Given the above analysis, the weight factors of the relative uncertainty of Eq. 24 can be computed as expressions of the measured properties of the IVDF and electric field:

$$\left| \frac{1}{\nu_{an(e)}} \frac{d\nu_{an(e)}}{df} \frac{\partial f}{\partial V_{iz}} \right| = \sqrt{\frac{q^2 E_z^2}{-4V_{iz}^2 (V_{iy}' m_i - qB_x)^2 + q^2 E_z^2}} \frac{1}{V_{iz}}, \quad (25)$$

$$\left| \frac{1}{\nu_{an(e)}} \frac{d\nu_{an(e)}}{df} \frac{\partial f}{\partial E_z} \right| = \sqrt{\frac{q^2 E_z^2}{-4V_{iz}^2 (V_{iy}' m_i - qB_x)^2 + q^2 E_z^2}} \frac{1}{E_z}, \text{ and} \quad (26)$$

$$\left| \frac{1}{\nu_{an(e)}} \frac{d\nu_{an(e)}}{df} \frac{\partial f}{\partial V_{iy}'} \right| = \sqrt{\frac{q^2 E_z^2}{-4V_{iz}^2 (V_{iy}' m_i - qB_x)^2 + q^2 E_z^2}} \frac{m_i}{(V_{iy}' m_i - qB_x)}. \quad (27)$$

The weight factors of relative uncertainty do not diverge to infinity in the limit of low electric field. This potentially enables the model to accurately capture the anomalous collision frequency farther outside of the acceleration region than the current dataset.

All of the weight factors are calculated and shown in Figure 13 for the results of our dataset. There is a difference of up to about a factor of 10 between the maximum and minimum of the weight factors. The weight factor associated with electric field uncertainty tends to rise downstream, whereas the weight factor associated with bulk velocity uncertainty falls, and the factor associated with the derivative of azimuthal bulk velocity changes little. The behavior of the uncertainty of $\nu_{an(e)}$ will depend on the relative magnitudes of the uncertainties of the three measured quantities. The worst case would be if the uncertainty in electric field dominates, in which case the uncertainty of $\nu_{an(e)}$ varies by about the same factor of 10.

VI. Conclusion

A new method was developed to measure the electron anomalous collision frequency in a Hall thruster using only LIF measurements of the axial and azimuthal components of the IVDF along the channel centerline. The method detects the enhanced electron collision frequency from mechanisms associated with a friction force between electron and ions, i.e. instabilities driven by the relative drift between the two species.

Initial results for the 300 V, 15 A operating condition of the H9 demonstrate that the method yields a reasonable estimate of the profile shape and order of magnitude. The LIF-based profile for the H9 has many key features previously found in simulations and probe-based experiments, including a minimum in anomalous collision frequency of order megahertz just upstream of the peak in magnetic field.

Two models for the electric field were used in the calculation of anomalous collision frequency: a simple derivative neglecting ionization and the Perez-Luna model taking ionization into account. The estimated collision rate varies as much as 25% in regions with ionization when comparing the estimate made with the Perez-Luna electric field vs the simple electric field. This implies that a more sophisticated model than the simple model for electric field may be necessary to accurately estimate the anomalous collision rate profile for a Hall thruster, though both models capture the same trends.

The main significance of the method to measure the electron anomalous collision frequency profile is that it can help validate the profiles found in simulations such as Hall2De, which cannot be predicted from first principles. In addition, since it specifically detects enhanced collisionality related to a friction force between electrons and ions, the method can help determine what types of potential mechanisms may be active. In the case of the 300 V, 15 A operating condition, we find that the LIF-measured anomalous collision frequency

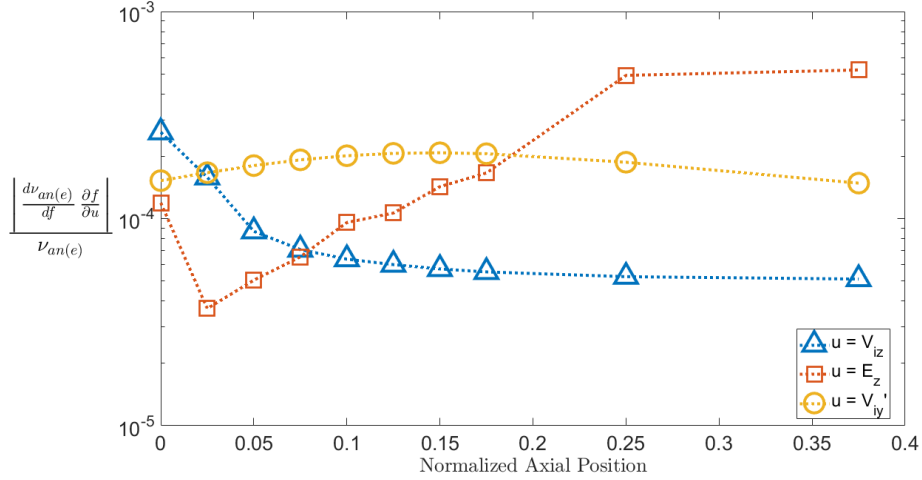


Figure 13: The relative uncertainty of the calculated $\nu_{an(e)}$ is a weighted sum in quadrature of the uncertainties of the measured quantities from the IVDF. The weight factors plotted do not change by more than about a factor of 10 in the range of data collected, indicating the uncertainty of $\nu_{an(e)}$ is constrained to change by at most that factor.

is commensurate with estimates of the total anomalous collision frequency from simulations (Fig. 9) and experiment (Fig. 10). Therefore, the results provide strong evidence that mechanisms related to instabilities driven by the relative drift between electrons and ions can be a dominant source of the anomalous electron collision frequency in a Hall thruster.

Appendix

A. Raw Data and Fits

For reference, this section contains plots of the raw data and the fits to sums of skew Gaussian functions used in the analyses of Section IV. Figure 14 contains the axial IVDF data and Figure 15 contains the Azimuthal IVDF data for the 300 V, 15 A operating condition. The blue line in the subplot is the difference between the raw data and fit, visually showing that the difference is only random noise and there is little if any systematic error of the fit from the raw data.

B. The Skew Gaussian Distribution Function

The skew Gaussian function is a generalized version of the Gaussian function with an additional parameter α that creates an asymmetric bell-shaped curve with a longer tail on one side depending on the sign of α . The probability density function (PDF) of the standard normal distribution is $\phi(x)$:

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \quad (28)$$

The cumulative distribution function (CDF) of the standard normal distribution is $\Phi(x)$:

$$\Phi(x) = \int_{-\infty}^x \phi(t) dt = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{x}{\sqrt{2}} \right) \right] \quad (29)$$

The distribution function $\phi(x)$ describes a Gaussian function of mean μ and standard deviation σ with x transformed as:

$$x \rightarrow \frac{x - \mu}{\sigma} \quad (30)$$

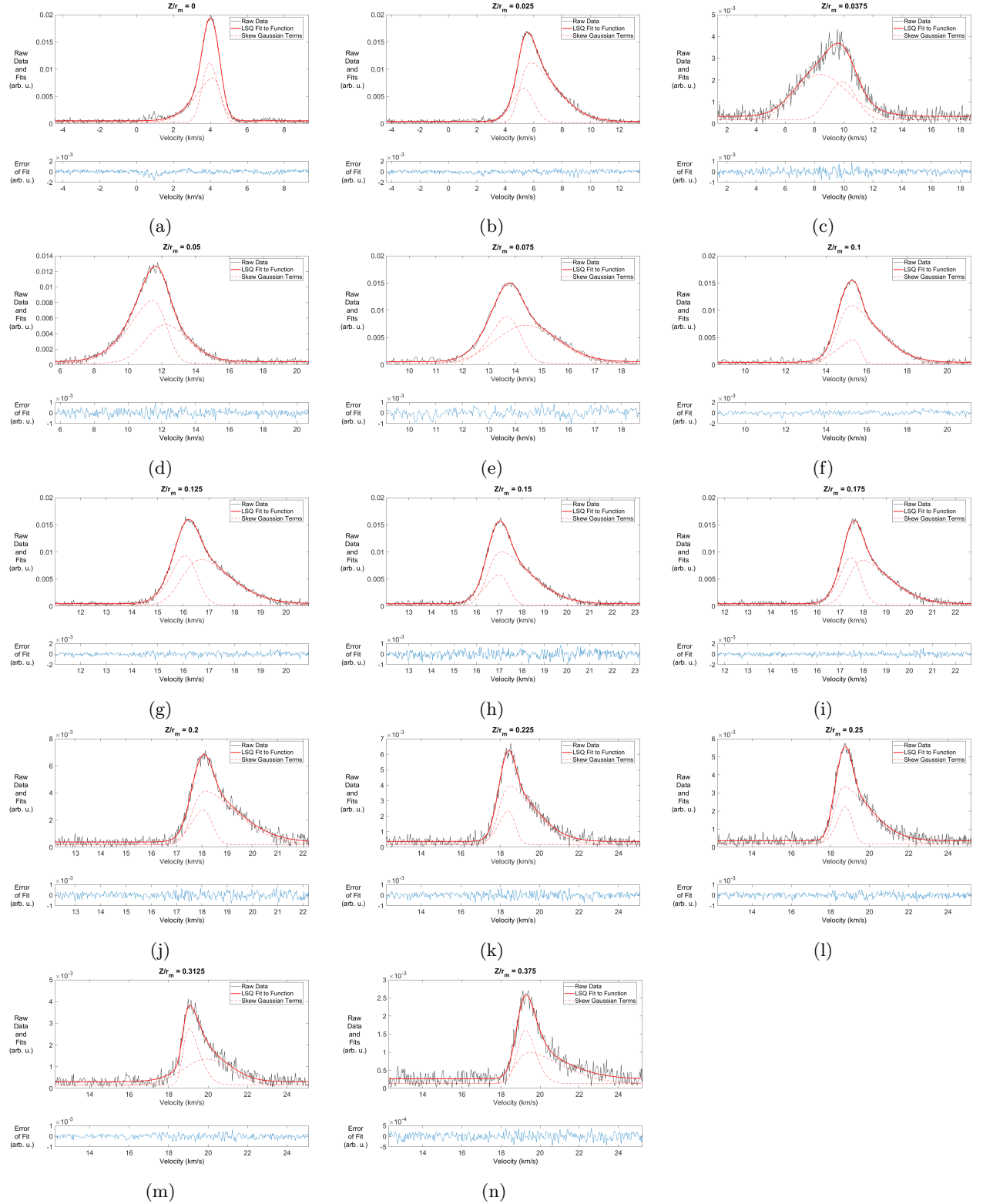


Figure 14: Plots of axial IVDF for the 300 V, 15 A operating condition at the same spatial points from 2. Each plot includes the raw data (black), the fit sum of skew Gaussian function (thick red curve), and the individual skew Gaussians in the sum (dashed red curves).

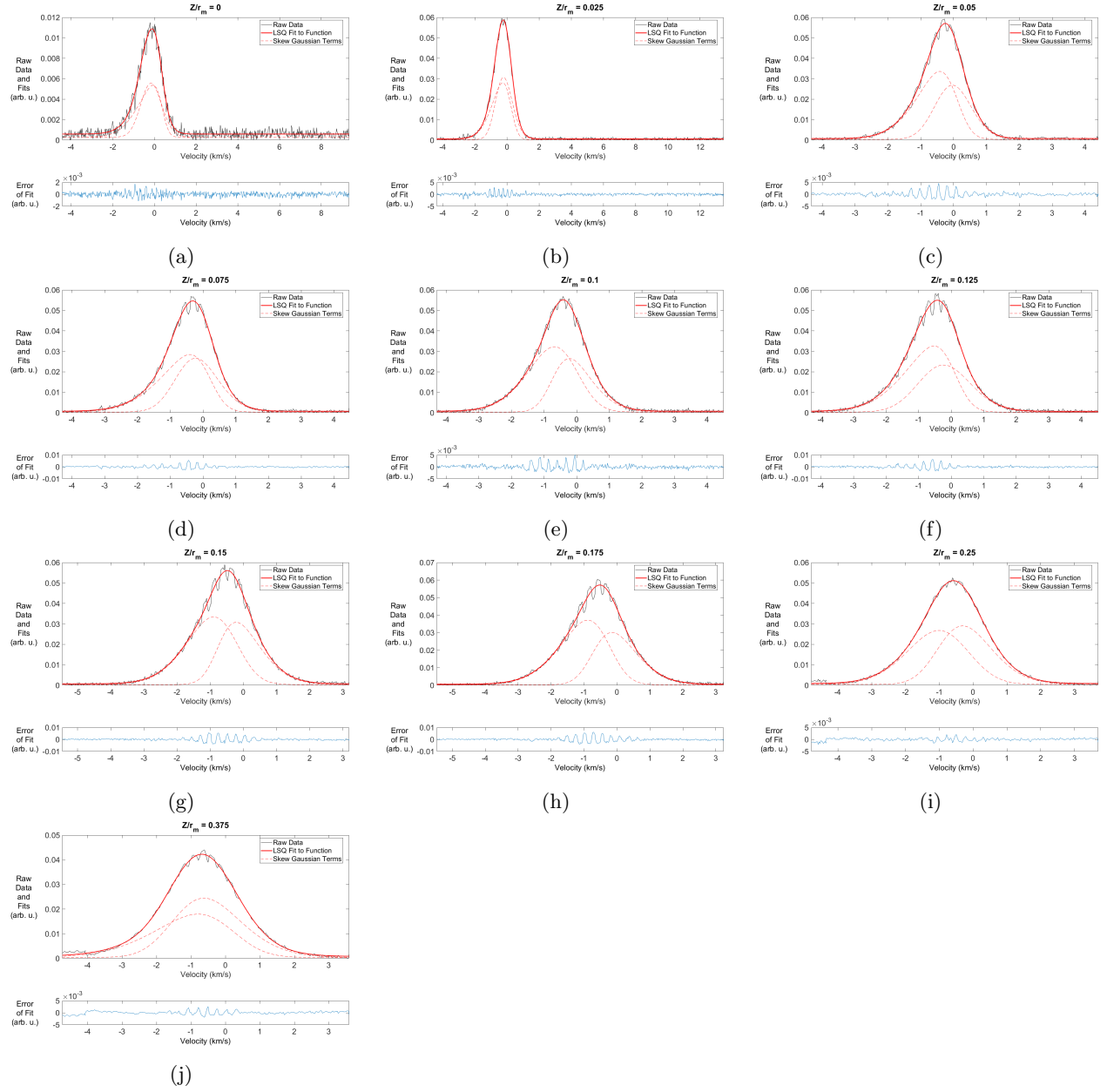


Figure 15: Azimuthal IVDF plots for the 300 V, 15 A operating condition at the same spatial points from 4. Each plot includes the raw data, the fit sum of skew Gaussian function, and the individual skew Gaussians in the sum.

A skew Gaussian distribution is the product of the PDF multiplied with the CDF where the argument of the CDF is scaled by α :

$$f(x) = 2\phi\left(\frac{x-\mu}{\sigma}\right)\Phi\left(\alpha\frac{x-\mu}{\sigma}\right). \quad (31)$$

Increasing skewness is obtained by increasing the parameter α . Negative α leads to symmetric curves with negative skewness. Examples of representative curves of this family are shown in Fig. 16.

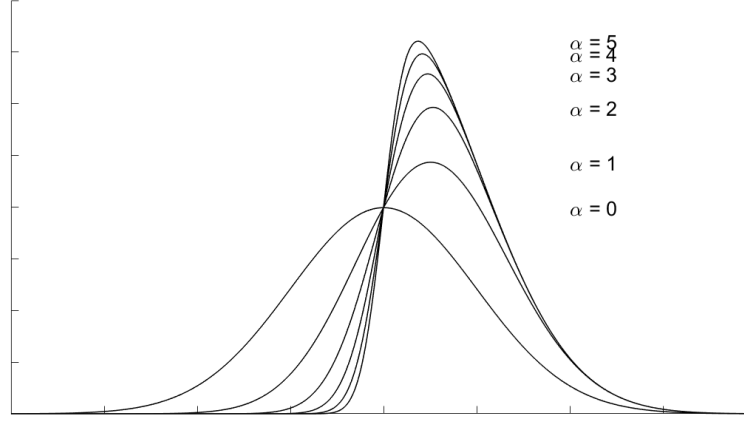


Figure 16: The skew Gaussian distribution is a probability density function that can be fit to LIF data. In this paper, we fit to sums of skew Gaussians to obtain an extremely good fit without overfitting to calculate moments of the measured IVDF.

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