High-Energy Density Electromechanical Thruster Based on Stabilized Liner Compression of Plasma
IEPC-2017-316

Presented at the 35th International Electric Propulsion Conference
Georgia Institute of Technology • Atlanta, Georgia • USA
October 8 – 12, 2017

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Abstract: We have previously discussed the conceptual design of an advanced propulsion system for crewed, high-energy, deep-space missions based on controlled thermonuclear fusion and stabilized liquid metal liner implosion. The basic dynamics of such implosion technology was demonstrated many years ago at the Naval Research Laboratory and, in principle, permits stable, repetitive exchange of the kinetic energy of a liquid metal shell with a low mass-density payload of plasma and magnetic flux during high radius-ratio compression and expansion. Operation at peak payload fields of a megagauss offers the opportunity for lower cost fusion power. In the course of discussion, the notion was introduced that the basic pneumatically-driven, stabilized implosion system could be used as a novel method to obtain high speed plasma as a new kind of electric thruster, even without energy from controlled fusion. The liner implosion system receives energy from a thermal source (e.g., nuclear fission reactor) and adiabatically compresses a plasmoid to achieve high temperature plasma. The plasmoid is ejected via a magnetic nozzle to obtain thrust at high specific impulse. The liner returns to its original position for the next implosion and plasma compression event. The efficiency of this space-based, reciprocating engine is, of course, limited by the ratio of source to heat-rejection temperatures. The principal interest here is that the limitations on energy, power density and lifetime associated with pulsed electric thrusters based on energy storage in capacitors and repetitive, high-current switching can be avoided for the major portion of the power flow by use of pneumatic and inductive energy storage and manipulation. We discuss several details of a high energy-density, electromechanical thruster based on stabilized liner compression of plasma.

Nomenclature

a	=	radius ratio of inner surface implosion
a_m	=	maximum spacecraft acceleration
B_i	=	initial magnetic field at entrance to nozzle
B_o	=	magnetic field compressed by FRC at nozzle entrance
E	=	FRC elongation, length divided by diameter
f_b	=	ratio of annular area between nozzle coil and FRC at nozzle entrance to coil area
g_o	=	gravity at earth's surface
h_n	=	thickness of nozzle conductor
L	=	inductance [henries]
L_n	=	inductance of nozzle (as single-turn coil)
m_{Li}	=	mass of lithium ion [kg]
m_n	=	mass of nozzle
P_n	=	power needed by nozzle
r	=	radius of inner surface of liner
r_c	=	inside radius of nozzle coil
r_{co}	=	inside radius of nozzle coil at entrance
r_f	=	minimum radius of inner surface of liner
r_n	=	(separatrix) radius of FRC at nozzle exit

r_o = (separatrix) radius of FRC at nozzle entrance
r_p = (separatrix) radius of FRC
r_s = inner radius of single-turn coil in pulsed MHD section
R = resistance [ohms]
S = operating stress in coil
S* = ratio of separatrix radius of FRC to ion gyro-radius at maximum field value
t_e = characteristic time for energy loss from initial FRC (prior to compression)
t_ϕ = characteristic time for flux loss from initial FRC (prior to compression)
t_o = characteristic time for liner implosion through compression ratio a.
T = plasma temperature
u = exhaust speed
u_o = optimum exhaust speed for maximum delivered mass fraction
u_2 = radial speed of inner surface of liner at twice r_f
v = azimuthal speed of inner surface of liner
V = charging-voltage for capacitor
V_o = rated voltage for energy-storage capacitor
w_m = energy per unit mass
W_n = energy stored in magnetic nozzle at initial field
z_s = length of coils in pulsed MHD section
Z = atomic number of propellant; also, number of free electrons per ion
α = specific power of supply [W/kg]
Δv = speed change of spacecraft
ε = conversion efficiency of supply power to thrust power
ϕ = magnetic flux
γ_e = ratio of conductivity to that of Lorentz gas
κ = factor between 1 and 2 for optimum exhaust speed
η = electrical resistivity
$\ln \Lambda$ = coulomb logarithm
ρ_n = mass density of nozzle
θ = half-angle of FRC (separatrix) path in nozzle
τ = thrust duration
τ_i = time interval between implosions

I. Introduction

AS electric rocket propulsion has developed over more than half a century, several mechanisms for creating high specific impulse have been explored. Many of these have been largely discarded due to system issues, such as electrode erosion that limits applicability to long-duration missions, or matters outside the thruster itself, e.g., capacitor lifetime. We have followed the usual "S-curve" of technology (Fig. 1) in which early experiences with poorly understood schemes gradually gave way to an exciting time of rapid development of a few concepts with which we may now be heading toward a mature plateau for continued operational use (e.g., ion engines, Hall thrusters, kilowatt arcjets, and a few specialized devices). It is appropriate, therefore, to extend our considerations to new S-curves, addressing longer term needs for higher powers and exhaust speeds appropriate for crewed-missions to the outer planets, planetary defense, continual operations in the asteroid belt, and perhaps advanced techniques for lunar and sub-lunar maneuvers. Some of these considerations may include the use of controlled thermonuclear fusion, while others may use related technologies for nearer term applications.

The present paper explores the use of a special electromechanical system, based on stabilized liner implosion, to generate high temperature plasma by adiabatic compression at high energy-density. Such plasma would be used to create a directed exhaust at high specific impulse (5000 – 500,000 sec). Previously[1-3], we have considered the operation of a stabilized liner-driven propulsion system in the context of controlled fusion. Conceptual calculations suggest that such a system could enable crewed voyages to Jupiter or Saturn requiring only a few months, and transporting the very large masses needed for radiation safety. In that consideration, attainment of the necessary high specific power was sought by reducing heat production by fusion neutrons (eschewing the more accessible D-T fusion reaction in favor of D-He^3) and operating in the open thermodynamic cycle of a conventional rocket engine. These features were enabled by magnetic confinement of the fusion plasma in a Field-Reversed Configuration

(FRC)[4,5] that could be compressed adiabatically to temperatures > 100 keV, and then expanded out a magnetic nozzle, converting the plasma and magnetic energy of the FRC into directed kinetic energy. In this concept, the plasma energy is substantially enhanced by deposition of high-energy charged particles from the D-He^3 reactions, with the open-cycle avoiding the burden of radiators for more than a few percent of the thrust power (Fig. 2.) As an intermediate technology stage on the way to such an advanced fusion propulsion system, we explore here the utility of retaining the features of liner-driven adiabatic compression and magnetic nozzle expansion, while accepting the radiator burden of a closed cycle to power the liner implosions.

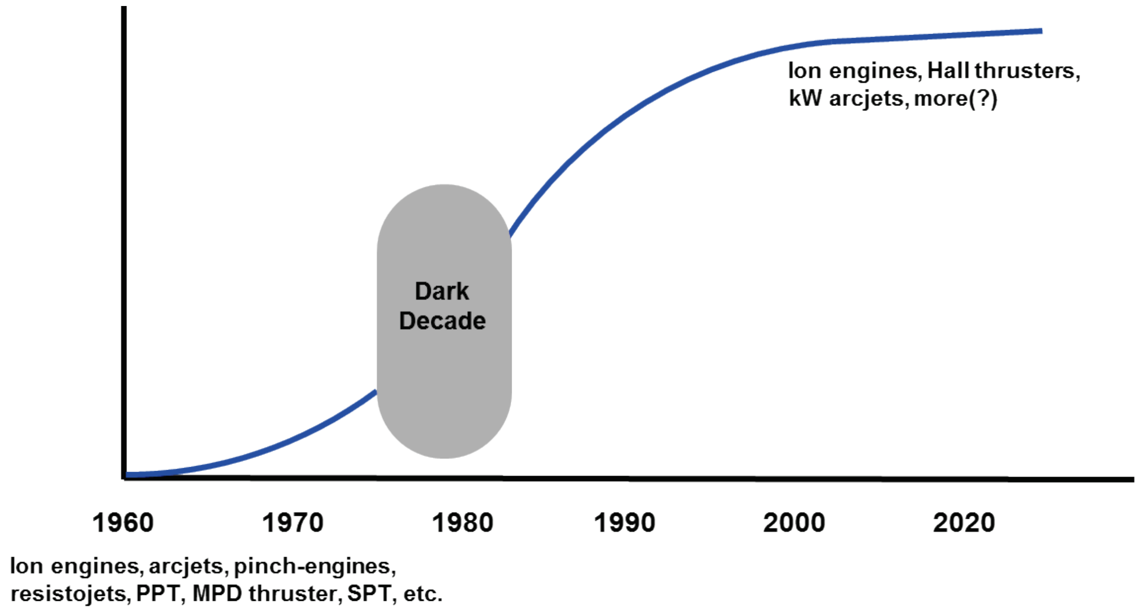

Figure 1. Electric propulsion has followed the traditional "S-curve" for technology, with gaps for the post-Apollo "Dark Decade" and the 'false dawn' of the Space Exploration Initiative, c. 1991.

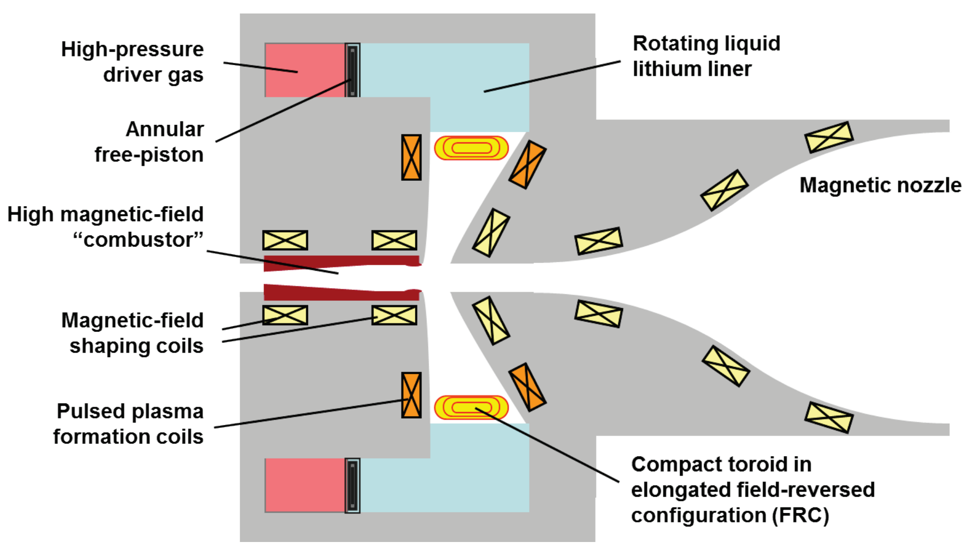

Figure 2. Basic arrangement of stabilized liner compressor for fusion rocket (Ref. 3) indicating use of elongated FRC, a high magnetic-field "combustor" section for D-He^3 operation, and a magnetic nozzle. (Pulsed MHD generator section at end of nozzle not shown here.)

II. Space Propulsion Considerations

The principal consequence of development of the means to compress an FRC to high temperature using liner implosion is that very high exhaust speeds would be possible at extremely high total powers. Electric thrusters can already achieve speeds up to 10^5 m/s at powers greater than 10 kW. Such capability is near the top of the "S-curve" for today's electric propulsion. The interest here is to push to a new S-curve with speeds >> 100 km/s and system powers >> 10 MW. These values are beyond the needs of any presently planned missions, but represent a start toward the next generation of advanced space propulsion. Previous consideration[1] of crewed voyages to Jupiter, for example, indicated that short trip-times with heavily-shielded spacecraft and significant acceleration might be possible, but only with advanced fusion fuels (e.g., D-He^3) that would provide system specific powers in the limit of values for the thermal radiators. Basically, with thrust power P scaling as the product of thrust and exhaust speed u, and with acceleration given by thrust divided by mass, the maximum acceleration a_m is constrained by the specific power α (divided by exhaust speed):

$$a_m = 2\,\alpha/u \tag{1}$$

For exhaust speed comparable to the necessary Δv (> 100 km/s) and an acceleration of 1% g_o to assist crew health, the minimum specific power is 5 kW/kg, a value far above solar and present nuclear-fission sources.

Without advanced fusion capability, we might still explore propulsion values at lower specific powers. The associated lower acceleration would not help much for astronauts experiencing extended time at zero gravity, so other means to avoid such crew issues would be needed. Robotic operations near the outer planets and even the asteroid belt may benefit from accelerations comparable to the local solar gravity. At 4 a.u., for example, this is only $3.9\ 10^{-5}\ g_o$ At five times this value and an exhaust speed of 100 km/s, the specific power would be about α = 100 W/kg, a value once sought for space-nuclear power[6], and perhaps available with future fission systems.

The optimum exhaust speed u_o depends on the specific power and the time τ over which thrust is provided[7]:

$$u_o = (\kappa\alpha\varepsilon\tau)^{1/2} \tag{2}$$

where ε is the total efficiency of converting power from the supply into the jet power and κ depends on the desired (accumulated) speed change Δv relative to $(2\alpha\varepsilon\tau)^{1/2}$. From Stuhlinger[7], we note that the maximum for delivered mass fraction can be rather broad, if $\Delta v/\ (2\alpha\varepsilon\tau)^{1/2} \ll 1$. Also, the optimum speed shifts from κ = 1 to 2 as the desired relative speed decreases.

To inform calculations of the thruster, it is useful to suggest a particular mission. Suppose, for example, a transit from earth to the asteroid belt, followed by several years of "prospecting" maneuvers. The first part of the mission would involve a Δv of about 15 km/s for a quasi-circular spiral over a few years from earth (at an orbital speed of 30 km/s) to, say, 4 a.u. (at 15 km/s). The maneuvers for the next several years could require an accumulated Δv of about 30 km/s (equivalent to full reversal of orbital velocity). With a total thrusting time of ten years, and an efficiency of 50%, Eqn. 2 provides u_o = 177 km/s. The ratio of $\Delta v/u_o$ = 0.25, so acceptance of a 20% final delivered-mass fraction would allow an exhaust speed of about 530 km/s. The desire for such a high exhaust speed arises in the present discussion from the need to have adequate plasma temperature for successful plasma compression by the stabilized liner implosion. This requires adequate energy per unit mass $u^2/2$, which equals 1.4 x10^5 MJ/kg in the present example. Such high energy-density with advanced space propulsion, based on stabilized liner compression of FRCs using nuclear fission sources, would then contribute later to the more interesting fusion-powered propulsion at similar energy-densities for crewed-missions to the outer planets.

III. Background on Stabilized Liner Implosions

Efficient compression of a low mass density plasma target by implosion of a high mass density shell (aka, liner) is subject to Rayleigh-Taylor instability[8] that would cause small perturbations on the inner surface of the liner to grow rapidly, penetrating the plasma target and resulting in radiation that prevents adiabatic compression of plasma to high temperatures. Rotation of a cylindrical, liquid liner can provide a centripetal term ($-v^2/r$) that counters the deceleration of the inner surface of the liner, reversing the direction of the effective "gravity" there in favor of stability[9]. This technique does not apply to the outer surface of the cylindrical shell during acceleration of this surface by a low mass density "fluid" in the form of a pulsed magnetic field or high-pressure gas. By eliminating the free outer surface of the liner, driving it instead with a solid free-piston, the problem of Rayleigh-Taylor instability is avoided, and energy can be exchanged reversibly between the driver-fluid and the liner kinetic energy. The mechanical behavior of a liner implosion stabilized on its inner surface by rotation and on its outer portion by free-piston drive was demonstrated[10,11] in the late 1970's at the Naval Research Laboratory (NRL), Washington, DC. Figure 3 provides images from an experimental movie of the stabilized liner implosion technique with a rotating liner of water compressing trapped air.

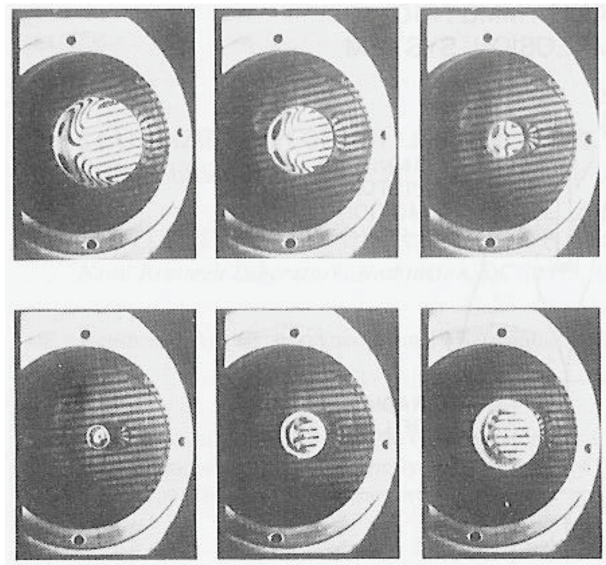

Figure 3. Frames from high-speed movie of rotating liquid liner implosion driven by annular free-piston display stable, reversible motion of inner surface. Liner is water compressing trapped air. View at slight angle to axis shows line-pattern on bottom plate reflected off inner surface of liner (Refs. 10, 11).

Recently, ARPA-E initiated the ALPHA program for exploring low-cost fusion technologies enabled by magnetic flux compression to megagauss field levels where system costs may be minimum[11]. The Stabilized Liner Compressor (SLC) project is sponsored by this program and seeks to extend the NRL experiments to much high energy-densities and speeds. At drive pressures > 12 kpsi, implosion speeds with alkali metals (e.g., Na, Li, NaK) in excess of 1000 m/s should be possible. We use MACH2, a 2-1/2 dimensional MHD code, to develop the apparatus for such implosions. Figure 4 displays frames from a numerical simulation of the implosion of a NaK liner on to trapped magnetic flux, increasing the magnetic field from 0.2 T to 91 T. Such an implosion operating on an FRC plasma target with initial diameter of 10 cm could enable repetitive laboratory experiments to develop the proper plasma[11].

IV. Background on Plasma Targets

The notion of strong adiabatic compression of a magnetized plasma target by liner implosion depends, of course, on the possibility of developing a plasma target of adequate stability and lifetime. Studies[4,5] of the Field-Reversed Configuration (FRC) indicate that stable arrangements require a ratio of $S^*/E < 3$ to 7, where S^* is essentially the number of ion gyro-radii within the maximum radius of the magnetic field line separating closed and open field-lines, and E is the elongation, the ratio of FRC length to diameter. Low values of S^*/E indicate more opportunity to counter the adverse curvature of the magnetic field line at the ends of the FRC (i.e., center of curvature inside the plasma) by the finite gyro-radius of the ions, and also by reducing the average curvature with longer, straighter portions of the field lines. With only poloidal magnetic flux (i.e., field components only in the rz-plane) surrounding the plasma, the particle density peaks in a region of zero-field, allowing a high value (> 0.3) for the average ratio of plasma to magnetic pressure. For large values of E, the ratio of plasma energy to magnetic energy is about unity[2]. An additional feature of the ideal FRC is that it contracts axially during radial compression, with a length varying as $r_p^{0.4}$. This increases the temperature rise for a given radial compression ratio. For a 10:1 implosion of radius, the temperature would increase by a factor of about 40 for ideal adiabatic compression.

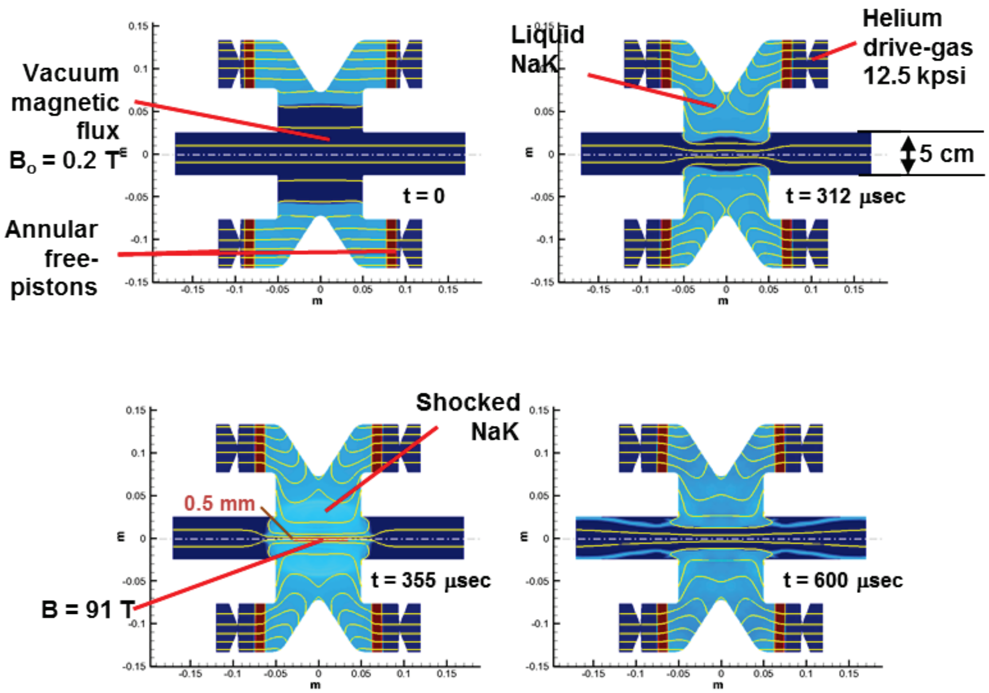

Figure 4. Frames from MACH2 simulation of Stabilized Liner Compressor imploding NaK liner onto axial magnetic flux. The driver-gas is helium, initially at 12.5 kpsi. The initial magnetic field is 0.2 T and increases to 91 T by magnetic flux compression.

Such ideal compression, of course, depends on limiting the loss of particles, magnetic flux and energy from the FRC, and represents a separate condition on the plasma target, apart from simple stability. Basically, the liner implosion must occur in a time short compared to that for loss of energy, particles or magnetic flux. As a diffusional process, the loss times scale as the square of a characteristic dimension, while the implosion time for a liner of a given mass density, driven by a given pressure, will increase only linearly with dimension. This suggests that there is always a solution for a large enough system (in common with many aerospace systems). The actual size at which success is possible will depend on the appropriate diffusion coefficients, which may vary greatly with plasma conditions. This has been explored briefly in the context of fusion power development[11] and applies equally for the present consideration of space propulsion. Considerable differences are observed for situations of Bohm diffusion vs enhanced classical diffusion, (as indicated in Fig. 14 and Table I, Ref. 11). From the associated formulation, we may derive useful relationships among the several parameters affecting liner compression of an FRC.

For example, the flux loss time t_ϕ is about twice the energy loss time t_e, which had been set to twice the characteristic implosion time t_o. To accomplish this condition, we need:

$$t_\phi = t_o = a^2 r_f / 4u_2 \quad (3)$$

where a is the radial compression ratio of the FRC to a minimum radius r_f and u_2 is the radial speed of the inner surface of the liner at twice this radius. For an implosion by a factor a = 10, to a radius r_f = 1 cm, with a speed u_2 = 2 km/s (corresponding to a MACH2 simulation with lithium and an initial drive-pressure of 25 kpsi), the initial loss time for magnetic flux would be 125 µsec For an elongated FRC (E >> 1), the time for flux loss is about:

$$t_\phi = (\mu/2\eta)\, a^2 r_f^2 \quad (4)$$

with the electrical resistivity η given classically by[12]:

$$\eta = 3 \times 10^{-5}\, (Z/\gamma_e) \ln \Lambda / T^{3/2} \quad (5)$$

where γ_e is a function of Z and the temperature T is given in eV (vs K) to obtain the resistivity in Ω-m. For Z = 3, γ_e = 0.7, and ln Λ = 10, the classical resistivity at T = 100 eV is $\eta = 1.3 \times 10^{-6}$ Ω-m. The enhancement factor used[11], corresponding to neoclassical diffusion[13], is a hundred in the loss rate, so $\eta = 1.3 \times 10^{-4}$ Ω-m. To obtain the desired flux loss time for the present example, we need to increase the initial temperature of the FRC to 187 eV or increase the minimum radius to r_f = 1.6 cm; to allow some margin, we shall use r_f = 2.6 cm.

V. Electromechanical Generation of High Speed Plasma

The basic mechanism here for electromechanical generation of a high speed plasma (depicted in Fig. 5) comprises four elements used in a sequence of stages. The first element is creation of the stabilized liquid metal liner implosion. This uses the high pressure of a drive-gas, energized by a separate power supply (e.g., fission reactor). Expansion of the drive-gas adiabatically by the axial displacement of the annular free-piston accelerates the inner surface of the rotating liquid liner to high enough speed for plasma compression. In addition to the kinetic energy associated with radial motion of the liquid, a significant amount of energy becomes stored in rotational energy of the liner needed for stabilization of the inner surface in the final compression. Calculations[14] suggest that the peak rotational energy is about 50% of the peak energy of the compressed FRC.

The second element of our mechanism is creation of the plasma target in the form of an FRC. This requires the rapid manipulation of magnetic flux on timescales much shorter than the liner implosion (tens vs hundreds of μsec) into which the FRC is injected when the liner speed has become adequate. Injection of the FRC displaces open-field lines outward between the closed-field of the FRC and the liner surface to achieve much higher magnetic field values than supplied statically by a coil surrounding the liner.

Compression of the FRC by the imploding liner leads to the third element of the system, namely a magnetic nozzle by which the internal energy of the FRC in both thermal and magnetic forms is converted into the directed kinetic energy of the exhaust. Note that the FRC is *borne* by the open field lines of the magnetic nozzle, not *born* on them, so it does not have to lose energy to resistive diffusion in extracting itself from the nozzle field. The magnetic field lines of the nozzle meld with the open field-lines separating the FRC from the inner surface of the liner.

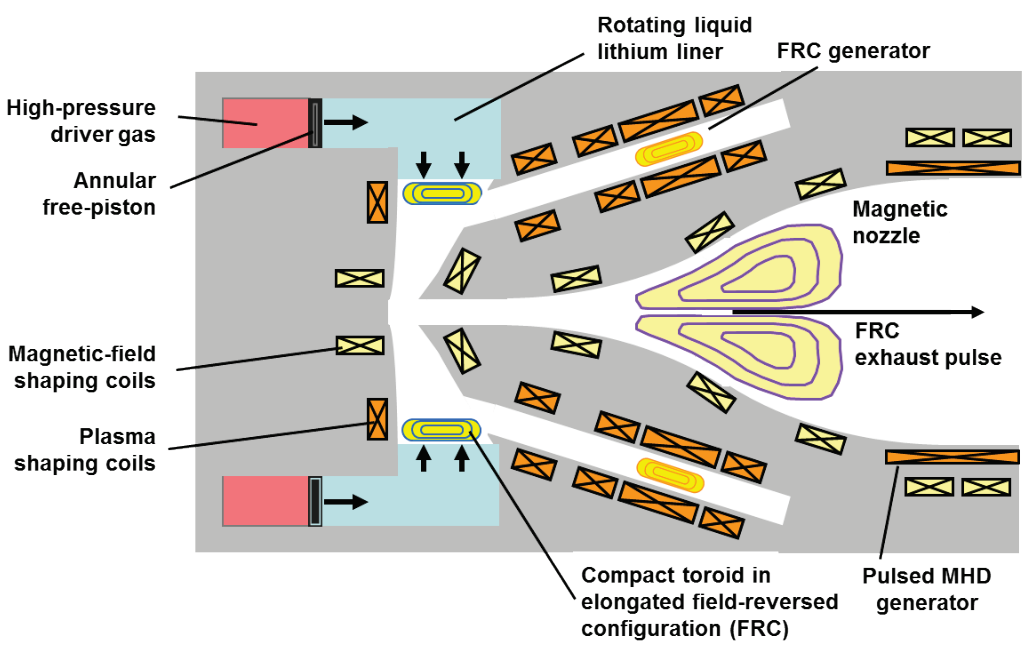

Figure 5. Basic arrangement of stabilized liner compressor as electromechanical rocket. The pulsed MHD generator at the nozzle exit is used with inductive energy storage to provide the electrical pulse for the FRC generator. Not shown: the external power supply drives a pressure booster for restoring the drive-gas pressure and initial volumes after the liquid liner returns from peak compression.

The fourth element is a magnet and inductive storage circuit that converts a portion of the exhaust kinetic energy into electromagnetic energy needed to create the next FRC. The use of the pulsed motion of the exiting FRC in a magnetic field along with inductive energy storage and switching can avoid the need for capacitive energy storage for the full requirements of FRC generation. This offers the much higher energy density available with inductive vs capacitive energy storage, (especially for extended thrust times).

After the rapid expulsion of the FRC following its compression, the liquid liner returns toward its original position by a radial expansion that initially re-compresses the drive-gas adiabatically. With energy transferred to the FRC (and some lost to friction and resistive diffusion), the volume of the driver-gas cannot be reduced directly to its original value by the remaining liner energy, which resides mostly in rotation. Instead, a separate pressure booster receives energy from the external power supply in order to accumulate the drive-gas to its initial volume at high pressure. This allows the annular free-piston to achieve its original position. Notionally, we might consider operation at a frequency based on the time for implosion and re-expansion of the liner. This would offer easier connection of the pulsed MHD system in the exhaust to the initial plasma generator. While the "water model" at NRL (Fig. 3) did indeed cycle freely, the power level in the present application may be too high relative to the power source and thermal management systems, so inductive energy storage is required between shots.

VI. Propellant Considerations

The basic operation of our electromechanical thruster can, in principle, allow many choices for the propellant. There are, however, various constraints that enter this decision. These range from the lifetime and stability of the FRC to the usual concern for frozen-flow losses. Ideally, by conversion of both the magnetic and thermal energy of the adiabatically compressed FRC to the directed-kinetic energy of the ions, we would need a peak plasma energy per unit mass that is about half of that represented by the exhaust. Based on the optimum speed from Eq. 2, this would be $w_m = \kappa\alpha\varepsilon\tau/4$ for any propellant. The relation of this energy per unit mass to the peak plasma temperature, however, depends on factors such as the degree of ionization. If, for example, we used lithium (6.9 amu) as the propellant (for its excellent storability), an exhaust speed of 530 km/s (i.e., three times the optimum speed for our previous mission example) corresponds to a kinetic energy per ion of about 10 keV. With w_m at peak compression of about 70,000 MJ/kg, the peak plasma temperature would be[15] about 800 eV. We would expect that the lithium would be fully ionized at this temperature, so there would be three electrons (Z = 3) for every lithium ion. The thermal energy per unit mass in kinetic motion would then be $3kT(Z + 1)/2m_{Li}$, which accounts for essentially all the plasma energy because the cost of ionization is less than 3000 MJ/kg. Thus, almost all of the thermal energy would be available for ion kinetic energy; ideally, less than 5% would be lost to frozen-flow in the form of ionization.

For higher atomic number propellant, say iron (Z = 26), mined *in situ* perhaps, losses could be much greater. The energy per unit mass for the same exhaust operation would be the same, but the temperature decreases[16] to 620 eV. Roughly, the ionization cost can be about ten times the plasma temperature; (e.g., argon with a first-ion cost of 15.7 eV is typically singly-ionized in low-density plasma at only 1.5 eV.) For a total ionization cost of 6200 eV, the iron would be ionized to about 24 free electrons. The kinetic portion of the energy per unit mass is then about 40 MJ/kg, so upwards of 43% of the peak plasma energy could be lost to frozen-flow (and radiation). Better estimates of plasma chemistry during compression and expansion, and system considerations for propellant availability might still offer interest in such higher Z materials. The possibility of *in situ* propellant, of course, alters the discussion[7] of optimum exhaust speeds substantially.

For a 10:1 radial implosion, ideal adiabatic compression of an FRC of lithium would indicate an initial plasma temperature of only about 22 eV in the present example. Diffusional loss of energy, particles and magnetic flux[11] would increase the required temperature of the initial plasma to 57 eV or 26 eV, based on Bohm or neoclassical diffusion, respectively. Both these values are still very low compared to present experience with deuterium FRCs at temperatures upwards of hundreds of eV. More exploration of non-hydrogenic plasma in FRCs is certainly needed in order to pursue the present advanced propulsion notions.

VII. Magnetic Nozzle

The magnetic nozzle could be accomplished as a conical coil (Fig. 6) whose field matched reasonably to the flux lines surrounding the compressed FRC. Initially, the flux of this coil would fill the entire interior of the nozzle, but expulsion of the FRC would force the flux outward, toward the coil, to provide a continuing buffer field with a pressure matching that of the expanding FRC. As in the adiabatic compression of the FRC, azimuthal currents in the plasma offer the electromagnetic force needed for axial containment, with the length ideally varying as $r_p^{0.4}$. By allowing the FRC to expand to larger radius in the nozzle, the pressure will decrease as r_p^{-4}. The coil radius r_c needed to match this pressure is then given by:

$$r_c/r_{co} = r_p/r_o \tag{6}$$

where 'o' indicates values at the entry to the nozzle for the coil and the plasma. The magnetic energy in the nozzle associated with its initial flux distribution is then:

$$W_n = (B_o^2/2\mu)\ f_b^2\ \pi r_{co}^2\ r_o[(r_n/r_o) - 1]/(r_n/r_o)\ \tan\theta \tag{7}$$

where $f_b^2 = (r_{co}^2 - r_o^2)/r_{co}^2$, r_n is the FRC radius at the nozzle exit and the path of the plasma edge follows a half-angle θ (assumed constant here). We might take θ as 5 degrees to allow a gentle guide for the FRC and a nozzle length that is much larger than the plasma length. The modest profile loss could be corrected in the subsequent pulsed MHD generator stage near the end of the nozzle.

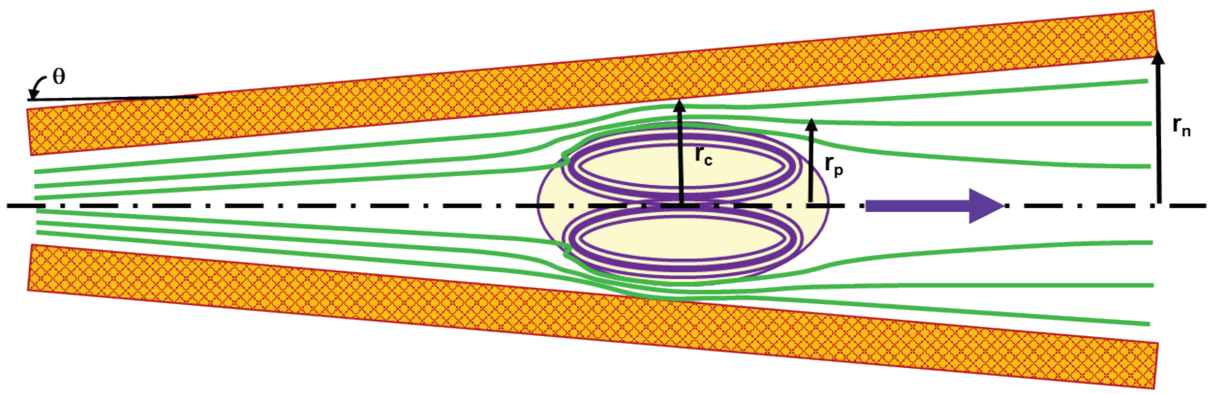

Figure 6. The magnetic nozzle in the form of a conical coil has a gentle angle to allow for the increase of axial length of the FRC as it expands. For conserved magnetic flux, the diameter of the coil increases in proportion to that of the FRC in accord with Eq. 6.

With a magnetic flux $\phi = B_o\, f_b\, \pi r_{co}^2$, the magnetic energy is associated with an inductance:

$$L_b = \phi^2 / 2W_n = \mu\pi r_{co}^2\ (r_n/r_o)\ \tan\theta/r_o\ [(r_n/r_o) - 1] \tag{8}$$

Note that this is the circuit inductance for a single-turn coil, but could be larger by use of multiple turns in order to match other circuitry. The corresponding mass of the conical coil, with slant length $l_n = r_{co}[(r_n/r_{co}) - 1]/\sin\theta$, thickness h_n and mass density ρ_n is:

$$m_n = \rho_n \pi h_n\ r_{co}\ [(r_n/r_{co}) - 1](r_n + r_{co})/\sin\theta \tag{9}$$

For normal conductors, the necessary thickness involves the electrical power via the resistance; use of superconductors would lie along other design axes that depend on factors associated with cryogenic systems.

The power required by a resistive coil is based on the so-called L/R-time. For slender cones (θ << 1), this characteristic time is approximately $(\mu/4\eta_n)(r_{co} + r_n)h_n$, where η_n is the resistivity of the magnetic nozzle conductor. We may set the time between pulses τ_i equal to half the L/R-time, and solve for the necessary coil thickness; the factor of two accounts for energy scaling as the square of the current, which decays faster than the current itself. The average power required by the magnetic nozzle is:

$$P_n = W_n/\tau_i \tag{10}$$

The mass of the nozzle coil is then:

$$m_n = \rho_n\pi(W_n/P_n)(8\eta/\mu)\ (r_o/r_{co})\ [(r_n/r_{co}) - 1]/\sin\theta$$
$$= \rho_n\pi\ (B_o^2/2\mu)f_b^2\pi r_{co}^3\ r_o[(r_n/r_o) - 1]\ [(r_n/r_{co}) - 1]\ (8\eta/\mu)/(r_n/r_o)P_n\ \sin\theta\tan\theta \tag{11}$$

which indicates that we can reduce the nozzle mass if we are willing to supply more power. This power could come from the external supply, increasing the mass of that subsystem; a trade-off is indicated to minimize the net mass. Alternatively, we might be able to extract the necessary power usefully from the high speed plasma exhaust via a pulsed MHD generator.

VIII. Pulsed MHD Power

Generation of the initial FRC requires manipulation of magnetic flux in times of several microseconds. Typically, this is accomplished using energy-storage capacitors, charged initially to tens of kilovolts, and delivering multi-megampere currents, when discharged through fast-closing switches. These currents rise quickly in single-turn coils that both create the plasma, wrap it in magnetic flux, and accelerate it to high speeds (> 10's km/sec). The electrical load appears as an initial low inductance that increases rapidly due to plasma motion inside the FRC coil(s). Such a load is amenable to inductive energy storage for which the required dissipation in opening-switches is typically the energy stored multiplied by the ratio of the small initial inductance to the storage inductance. The importance of inductive vs capacitive energy storage in the present space propulsion application is the higher energy-storage density the former allows and the limited shot-life of energy-storage capacitors. Magnetic energy storage in a simple coil is limited by the strength of materials to about $S/2\rho$, where S is the working stress in the coil material and ρ is its mass density. For S = 60 kpsi, and $\rho = 2.7\ t/m^3$ (Al), the energy storage per unit mass is 76 kJ/kg. Capacitors, on the other hand, store perhaps 250 J/kg. A capacitor bank would thus require 4 t to store a megajoule, while a high strength inductor could store this energy in only 13 kg.

Apart from energy storage per unit mass, there is the issue of storage time. For capacitors, this could be indefinitely long, but the number of discharges in the lifetime of a capacitor is not infinite. A well-engineered high-voltage capacitor for laboratory use might have a life (after "infant mortality" losses) of 10^5 shots. At 1 Hz, this would be much less than a year. Reduction of the charge-voltage V compared to the rated voltage V_o can increase this time substantially, e.g., by $(V/V_o)^7$, but at the cost of reduced energy storage per unit mass, as $(V/V_o)^2$. Inductive storage, on the other hand, is a transient affair (except for superconducting systems, where the limitation may be cryogenics). As with the magnetic nozzle, the energy decay time is scaled by half the so-called L/R-time of the coil, which is proportional to the radius and thickness of the coil. The coil length would be at least a few times the coil radius to allow adequate flux efficiency. The storage time for a coil of 0.1 second, using aluminum (at a resistivity of 2.7 $\mu\Omega$-cm), with a length four times the radius and a radial thickness of a third the radius would require a mass of about 100 kg. For a stress in the coil of 30 kpsi, the energy per unit mass is 38 kJ/kg, so the coil could store about 3.8 MJ. This is much more than would be necessary, and indicates that comparison of capacitive and inductive storage must consider more than the intrinsic energy per unit mass. For 1 MJ stored, the effective energy density of the inductive store, based on storage time, is reduced to 10 kJ/kg, which is still much higher than a capacitor bank (especially with voltage de-rated to increase shot lifetime).

We have an opportunity within the present consideration of using the magnetic nozzle as part of the inductive store. Its inductance can be much higher than the initial inductance of the FRC generator. The stored energy in the initial nozzle field, however, may be much less than needed by the plasma, which makes the nozzle itself inadequate for inductive storage to drive the FRC generator.. The principal power requirement for the nozzle is due to resistive loss between shots. This can be replaced either by the external power supply or by the pulse of energy from the exhaust flow.

Figure 7 suggests an arrangement for the pulsed MHD generator as a final stage of the magnetic nozzle. A multi-turn coil supplies the magnetic field at this stage with a value required for the FRC generator. The energy provided by the pulsed MHD action is approximately the product of the volume of the exiting FRC and the initial magnetic energy density. The magnetic field may, therefore, be much higher here than initially elsewhere in the nozzle. Magnetic flux is displaced by the FRC past the single-turn, conducting cylinder notionally through a "slot" of switching that is initially open to allow flux transfer. This switching is closed as the FRC exits in order to trap the magnetic flux and energy in the circuit formed by the single-turn coil in series with the surrounding multi-turn coil.

To generate the next FRC, this circuit is interrupted to divert energy to the several pulsed coils that create the plasma and magnetic field of the FRC and deliver it to the implosion chamber.

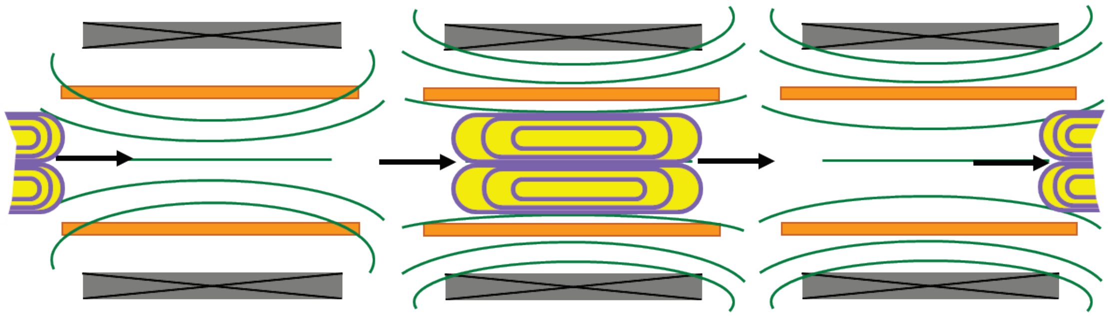

Figure 7. Sequence of operation of the pulsed MHD generator at the nozzle exit. The interior single-turn coil (orange) has a switch that is initially open (as perhaps a set of switches along an axial "slot" in this coil. This allows the FRC to displace magnetic flux toward the surrounding multi-turn coil. The switches are closed before the FRC exits, capturing flux and energy in the volume between the two coils for use by the FRC generation circuit.

IX. A Sample Design

The consequences of some choices for propellant and operating conditions can be considered in a brief sample design. Based on the discussion in Section VI, let us use lithium at an initial temperature for the FRC of 100 eV. The peak temperature after a 10:1 radial implosion, including neoclassical losses would be 3.1 keV. The associated energy density of the plasma is then 2.6 $\times 10^5$ MJ/kg. The exhaust kinetic energy density could be twice this, corresponding to a maximum exhaust speed of 508 km/sec. This is close to the previously calculated speed based on the assumed mission.

At a peak magnetic field of 50 T, the ion density at peak compression at the field-zero point of the FRC is 5 $\times 10^{17}$ /cm^3. For a minimum radius of 2.6 cm and an elongation of E = 10, the FRC volume is 1100 cm^3, so, if the average density is a third of the maximum density, the plasma mass is 2.1 $\times 10^{-6}$ kg. The total plasma energy at peak compression is then 0.55 MJ. With an equal amount of magnetic energy, the total peak energy of the compressed FRC is 1.1 MJ. The rotational energy needed to avoid Rayleigh-Taylor instability at the inner surface of the liner during final compression (and re-expansion) is about half of the total FRC energy or 0.55 MJ, for a combined liner kinetic energy of 1.65 MJ. With an additional 20% of this energy allowed for losses due viscous and magnetic diffusion, the work done on the liner by the pneumatic drive during implosion is 1.98 MJ. Expulsion of the FRC and the various losses would allow perhaps just the rotational energy for the return of the liner and re-compression of the drive-gas. This means that the external power supply, acting through a pressure booster, would need about 1.6 MJ to accumulate and restore the drive-gas to its initial pressure and volume for the next implosion event.

At a repetition rate of 10 Hz, the power supply would operate at about 16 MW(e). For the assumed specific power of 100 W/kg, the power supply mass is then 160 t. The initial SLC device[11] under development for ARPA-E has an outer diameter of 45 cm, a length of 64 cm and, based on titanium, a core mass of 450 kg. We are aiming for a minimum radius of the inner surface of the liner of 2.5 mm. The present sample design, with a minimum radius of 2.6 cm, could have a diameter of about 135 cm and a length (with only one annular free-piston) of 130 cm. The core mass would then scale to about 8.2 t, or about 5% of the mass of the power and propulsion system (not including the nozzle and auxiliary systems).

The energy of the initial FRC, based on neoclassical losses, is about 50kJ, including a factor of two for the magnetic energy. Typically, electromagnetic acceleration of plasmoids can be accomplished with an efficiency of 25% from stored energy, with some energy retained by the driving circuit, allowing a net efficiency of upwards of 40%. If we provide the energy for creation and acceleration of the FRC by means of a pulsed MHD generator action on the exhaust flow, we might anticipate a conversion efficiency of 25% to electricity and an equal fraction to heat in the exhaust flow. The directed exhaust flow would then lose an energy of about (50 kJ)/(0.4)(0.5) = 250 kJ. The resulting exhaust speed therefore decreases to 447 km/s, which is closer to the optimum speed for the assumed overall efficiency ($\varepsilon = 0.5$) and specific power of the supply; the delivered mass fraction[7] could increase from 0.2 to 0.3. Opportunities for optimization in the various assumed values would await better understanding of the several processes, including losses from the FRC during compression, and the efficacy of generating the initial plasma using pulsed energy from the exhaust.

For these sample conditions, Eq. 7 indicates that the magnetic nozzle would contain an energy of about 18 kJ, for a half-angle of 5 degrees from an entry radius r_{co} = 4.8 cm to an exit value r_n = 48 cm. The magnetic field at the entrance is limited by static mechanical strength to about 20 T vs the 50 T peak field attained by liner compression, so the plasma radius at the entry, with f_b = 0.2, is 4.1 cm. Expansion of the plasma by an order of magnitude in radius would increase its length from the value at peak compression of 52 cm to 130 cm at the exit. The choice of half-angle was informed by this extended length, which is allowed to occur over a nozzle length of 500 cm.

The energy stored by the (quasi-)static magnetic field of the magnetic nozzle is much less than that needed for the pulsed MHD generator to supply energy for creating the initial FRC. The nozzle coil could be segmented to allow a higher magnetic field in the last meter or so of the nozzle, without increasing the field near the nozzle entrance. The last segment can serve as the inductive store and thereby use part of the mass required by the magnetic nozzle.

The magnetic flux displaced by the high speed FRC as it enters this segment is associated with a reduction in the inductance in the manner of a magnetic-flux compression generator. Ideally, for constant magnetic flux (at the very high magnetic Reynolds number in the present situation), the work W_g done by the FRC motion is the initial energy within the multi-turn coil multiplied by change in inductance divided by the final inductance, given approximately here by the ratio of areas:

$$W_g = (B_i^2/2\mu)\, \pi r_c^2\, z_s\, [\, r_p^2/(r_c^2 - r_p^2)] \tag{12}$$

where z_s is the length of the FRC, which roughly equals the coil lengths. Closure of the switching for the single-turn coil captures the magnetic flux and energy between this coil at radius r_s and the surrounding coil at r_c, but misses the energy from r_p to r_s, so the energy available for the FRC generator is reduced by $(r_c^2 - r_s^2)/(r_c^2 - r_p^2)$.

For an FRC with r_p = 50 cm and z_s = 130 cm, r_s = 55 cm, and r_c = 65 cm, the previously estimated magnetic energy of 50 kJ/0.4 = 125 kJ for the FRC generator circuit requires an initial magnetic field in the MHD segment of about 0.42 T. (This is much greater than the initial magnetic field in the portion of the nozzle immediately preceding the MHD stage of $B_o f_b/(r_n/r_{co})^2 \approx 0.04$ T.) The peak field due to FRC compression becomes about 1 T, which is quite tolerable mechanically. We would retain the energy in the annular inductive store (between r_c and r_s) until the pulse is delivered to the FRC generator. The remaining flux and energy can then be released to the full area of the coil. We could restore some energy to the coil from the external power supply. Alternatively, we could extract more energy from the exhaust flow. Some optimization can be performed here with improved performance in terms of delivered mass fraction at somewhat lower exhaust speed, but fundamentally the cost of resistive losses must be covered by the power supply. If we let the L/R-time of the single-turn coil equal five times the specified interval time of 0.1 sec, an aluminum coil (η = 2.7 $\mu\Omega$-cm) would need a thickness of about 4 cm. The surrounding multi-turn coil could then have an inner radius of 65 cm, and would need a similar thickness. The masses of these coils are 560 kg and 660 kg, respectively; (the earlier portion of the magnetic nozzle would have a mass of about 760 kg.) For coil lengths of 150 cm, the inductance of the intervening annular region is 290 nH, which is well above the tens of nH for the initial inductances of the FRC generator; note that this inductance is for a single-turn outer coil and can be increased with multiple turns to accommodate the FRC circuitry. Sample design values are summarized in Table I.

X. Concluding Remarks

The notion of yet another way to achieve electric rocket propulsion, in the present case by a unique electromechanical system, is offered here as a technological step toward the much greater capability of space propulsion based on controlled fusion. Such fusion propulsion, employing ultrahigh magnetic fields provided by stabilized liner compression, and using D-He^3 to access much higher values of specific power, would enable crewed-missions to our nearest "solar systems": Jupiter and Saturn. These missions would have trip-times of only several months[1] and deliver relatively large masses to protect astronauts from the radiation dose during transit and the intense radiation fields near objects of great interest (e.g., Europa and Enceladus). Experience with the SLC-driven propulsion systems described here would inform the design of such advanced propulsion and could also provide useful capabilities for extended robotic missions beyond Mars. In the mean time, our project within the ARPA-E ALPHA program will perform the detailed engineering design, fabrication and operation of a prototype SLC. Further work in the near future may then extend to creation and liner compression of relevant FRC plasma targets. With better understanding of issues of SLC design and FRC behavior (especially for non-hydrogenic plasma, e.g., lithium), we can examine optimizations, such as the use of pulsed MHD generators vs longer-life capacitors, or increased power from the external supply.

Table I
Summary of Sample Design Values

Mission parameters	
Specific power of external supply: α = 100 W(e)/kg	Total thrust time: τ = 10 years
Thrust efficiency: ε = 0.5	Final mass fraction: > 0.2
Design choices and results	
Liquid liner material: Lithium	Propellant: Lithium
Plasma radial compression ratio: a = 10	Initial FRC temperature: 100 eV
Minimum plasma radius: r_f = 2.6 cm	Peak magnetic field: 50 T
FRC elongation at minimum radius: E = 10	Peak FRC energy: 1.1 MJ
Magnetic nozzle length: z_n = 500 cm	Exhaust speed: 447 km/sec
Power input: 16 MW(e)	FRC generator power: 1.25 MW(e)
Power supply mass: 160 t	Rocket mass (core, nozzle, MHD): 13 t

Acknowledgments

Work supported by ARPA-E ALPHA Program to enable low-cost controlled fusion

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