

Radial-azimuthal particle-in-cell simulation of a Hall effect thruster

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Abstract: Plasma waves in the azimuthal direction are considered to be a potential source of the anomalous mobility observed in Hall thruster discharge plasma. In this paper, a two-dimensional (2D) particle-in-cell (PIC) simulation is developed to model the azimuthal waves induced by the ExB drift in the cross-field configuration. The radial-azimuthal simulation is an extension of the one-dimensional (1D) azimuthal simulation, in which the axial electric field and radial magnetic field are prefixed. Multidimensional effect due to the plasma sheaths are accounted for, but the coherent ion acoustic wave survives for over a few μs . The effects of axial boundary conditions and the importance of the ion density fluctuation on the electron transport are discussed.

I. Introduction

Hall effect thruster (HET) is a cross-field device where the ions flow electrostatically and electrons are magnetized. Plasma instabilities and oscillations in the cross-field configuration have been first studied in the 1970s¹ and predicted that the ExB drift can induce collisionless fluctuations that can contribute to electron transport depending on the ion-to-electron temperature². In a purely one-dimensional setup, the free energy from the ExB drift is transferred into the plasma wave, which in turn heats up both the ions and electrons.

It has been shown by numerous simulations that the electron resistivity must be higher than classical theory, i.e., when only intermolecular collisions are accounted for, to obtain the time-averaged plasma properties and thruster performance that are experimentally observed. For instance, the electric field, the potential drop, is largest near the channel exit where the magnetic field is strongest and the potential drop in the plume is small. In order to achieve such a potential profile, the electron conductivity in the plume must be much larger than that near the channel exit, assuming Ohm's law for electrons. Because the electron collision frequency decreases due to the decrease in the neutral atom density downstream the thruster exit, anomalous transport is therefore needed mainly in the plume region of a HET. Revealing the electron transport mechanism in the HETs is the key to understanding how to improve the thruster performance and how to design new thrusters.

Three types of modeling techniques have been developed in the HET community³. (i) One is the fluid approach in which macroscopic quantities, such as density, mean velocity, and temperature, are obtained by solving the

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conservation fluid equations. In modeling electron dynamics, it is common to use a quasineutral assumption and Ohm's law, which eliminates the need to resolve the Debye length and electron plasma frequency. Instead of solving the Poisson's equation, the potential can be obtained from the Ohm's law and charge balance equation. Anomalous transport is typically taken into account by prescribing an empirical profile^{4,5} or self-consistently solving a set of azimuthal wave equations based on the linear dispersion relation of the ExB driven instability⁶. (ii) Kinetic model in which the kinetic equations, such as the Vlasov equation, are solved. The most common is particle-based kinetic approaches, including particle-in-cell (PIC) and Monte Carlo collision (MCC) models^{7,8}. The other is grid-based kinetic approaches, such as the direct kinetic (DK) method⁹ or discrete velocity method (DVM), in which the kinetic equations are directly solved on discrete phase space. (iii) Hybrid model employs both the fluid and kinetic approaches depending on the species^{10,11}. The most common for the HET discharge plasmas is to use a kinetic model for heavy species while a fluid model for electrons. Using a fluid and hybrid model, the electrons are modeled as a fluid, and therefore cannot investigate the anomalous transport due to kinetic effects.

Recent simulations and experiments have shown that ion acoustic waves induced by the ExB drift are the main source of anomalous transport. Experiments by Tsikata¹² showed for the first time that the plasma oscillations measured in the magnetron discharge, which is another type of cross-field plasma device, agree with the dispersion relation of the ion acoustic wave. Azimuthal dynamics in HETs have been studied by Adam and Heron¹³ and Hirakawa¹⁴. Lafleur et al. recently developed a simplified test problem where fixed electric and magnetic fields are applied and the particles experience an ExB drift using a particle-in-cell (PIC) simulation^{2,15}. It was shown that classical mobility can be obtained when the Poisson solver in the azimuthal direction is turned off and anomalous mobility can be observed in the presence of the azimuthal plasma wave. The reported mobility showed the values when the plasma instability fully saturates, which occurs in the time scale on the order of μs . It was also reported that the electron energy grows indefinitely without a boundary condition that removes the high-energy particles from the system; an axial boundary condition was applied in the paper, serving as the only damping mechanism of the plasma wave growth.

In this paper, we develop a 1D azimuthal and 2D radial-azimuthal PIC simulation to further investigate the ion acoustic wave generated in the cross-field configuration. A radial-azimuthal simulation is employed to investigate the multidimensional effects and coupling of the ion acoustic wave with plasma-wave interaction, i.e., plasma sheaths.

II. Particle-in-cell simulation

Schematic of the cross-field configuration is shown in Figure 1. A static electric field (uniform) is imposed between the anode and cathode surfaces and a static magnetic field is imposed in the perpendicular direction. In this paper, we present two types of simulation results and models. (i) 1D azimuthal simulation and (ii) 2D radial-azimuthal simulation. In both simulations, the electrons are injected from the cathode side and ions are injected from the anode side once they move out of the computational domain. In the direction of ExB drift, we apply a periodic boundary condition.

A. 1D Azimuthal PIC simulation

The basic model is identical to Lafleur's 1D azimuthal PIC simulation in Ref. 2. Ion and electron densities are obtained only in the azimuthal direction to solve the Poisson's equation. In other words, the azimuthal electric field is distributed to particles in any z or r coordinates. The Boris method is used for advancing the particle velocity, and the leap frog method is used to update particle positions. The particle weights are linearly distributed to the cell centers. The Poisson's equation with a periodic boundary condition in the azimuthal direction is solved using a fast Fourier transform (FFT) solver.

One important assumption is that there is an axial boundary condition imposed as anode and cathode surfaces. The reason why this is critical is that the heated electrons and ions via ion acoustic wave will be depleted by these two boundaries to saturate the growth of the wave.

B. 2D planar PIC simulation with dielectric walls

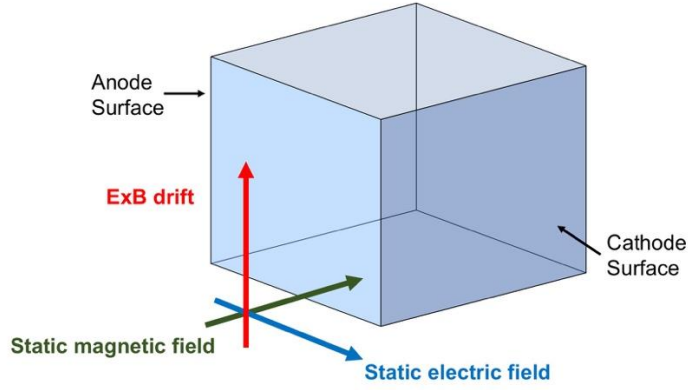


Figure 1. Schematic of the cross-field configuration (planar approximation)

In the 2D case, the ion and electron densities are stored on cell centers and hence the Poisson's equation is solved for the potential on cell centers. Charges are accumulated on the dielectric wall surface. This is the main difference with the recent 2D PIC simulation¹⁶. As shown in Fig. 2, those charges are stored on cell interfaces along the walls. The electric field on the walls are determined by taking the Gauss's law on the walls

$$\epsilon_0 E_0 - \epsilon_d E_d = e\sigma \quad (1)$$

where E and ϵ are electric field and permittivity, subscript 0 and d denote the plasma and material, respectively, e is the elementary charge, and σ is the surface charge. Typically, $E_d \approx 0$ inside the dielectric material. The Poisson's equation with Neumann boundary conditions in one direction and periodic boundary conditions in the other, the discretized system of equation cannot be solved. Hence, we employ the compatibility condition, i.e., the total space charge in the cells is equal to the total surface charge along the walls, so that a reference potential can be set at an arbitrary cell interface along the walls. Here, a zero potential is set at the northwest corner of the computational domain.

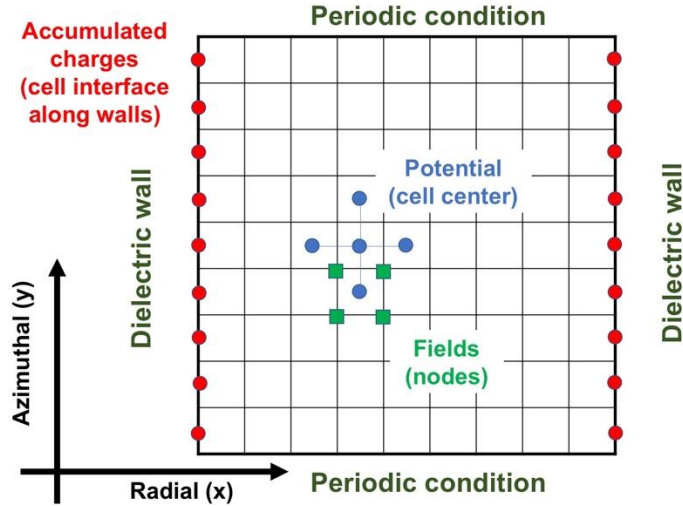


Figure 2. 2D planar grid and boundary condition

Ions and electrons are injected the same way as the 1D azimuthal simulation, in that the ions are injected from the anode side ($z=0$) when they move out of the system and electrons are injected from the cathode side when moving out of the system. When the electron or ion collides with the wall, the charge is recorded at discrete cell interfaces (red circle in Fig. 2). In order to keep the spatially averaged plasma density constant, ions and electrons are reinserted into the domain only when the ions collide with the wall. With this method, the total number of ions is kept constant and

the compatibility condition can be satisfied. The computational domain and numerical parameters assumed in this paper is shown in Table 1. The ion is assumed to be xenon (131 amu).

Table 1. Numerical parameters used in the PIC simulations

Axial domain length (z)	1 cm
Azimuthal domain length (y)	5 mm
Radial domain length (x)	1 cm
Number of grid points (y)	256
Number of grid points (x)	512 (for 2D)
Time step	2 ps
Total time	4 μ s
Axial electric field	20,000 V/m
Radial magnetic field	200 G
Ion density	10^{17} m^{-3}
Number of particles per cell	20

C. Parallelization

The 2D PIC simulation employs parallel computing using message passing interface (MPI) method, which is a distributed memory system. Here, each processor has the global domain information, i.e., not using domain decomposition, and macroparticles are distributed in the global domain. The ion and electron densities are first obtained locally in each processor, and then collected globally using MPI_Allgather.

The Poisson's equation is solved using a linear algebra software called HYPRE¹⁷. HYPRE submodule has its own distributed grids from the global domain, which then solves for a linear system such as $Ax = b$, where A is a matrix, x is the vector to solve, and b is the right-hand side. There are multiple methods available in HYPRE to solve such a system, including generalized minimal residual (GMRES), multigrid (MG), and conjugate gradient (CG) methods with preconditioners. We selected the SMG (semi-coarsening multigrid) method using a GMRES preconditioner. Here, A verification test case using method of manufactured solution is described in Ref. [IEPC¹⁸]. The computational wall time increases when the HYPRE grid is distributed in different computational nodes. Using the ada cluster at Texas A&M University has 20 processor cores per node. Thus, the number of HYPRE grid is set to maximum 20 cores. After the potential profile is obtained using the HYPRE software, the potential is gathered in the root processor and broadcasted to all processors, after which the electric field on the global domain is calculated in all processors. In this calculation, the 2D simulation employs 20 particles per cell and 40 processors in total. The total computational wall time is approximately 3 days.

III. 1D tests

1D azimuthal PIC simulation proposed by Lafleur et al.² was further studied before running the 2D simulations. Several collisionless 1D studies are shown in this section. The ion acoustic wave is formed in the direction of the ExB drift. Here in the simulations, the ExB drift propagates in the $-y$ direction. It is to be noted that the 1D results shown in this section are benchmarked with the results obtained using the collision PIC simulation built in Thermophysics Universal Research Framework (TURF)¹⁹.

A. Number of macroparticles (numerical convergence)

The number of macroparticles per cell (N_p) is varied to investigate the numerical convergence. Here, $N_p = 2, 5, 15, 100, 500$, and 2000. Figure 3 shows the time evolution of the electron temperature and the ion azimuthal mean velocity as a function of time. The overall trend agrees with Lampe's PIC simulations¹, in which an electron drift instability (EDI) is excited at the first few 10-100's nanosecond and an ion acoustic wave initiates. What is remarkable here is that the ion acoustic oscillations are captured even when using only 5 macroparticles per cell. However, it can be seen that the case with $N_p = 5$ approaches the $N_p = 2$ case, hence it is recommended that $N_p > 15$ is used. Below this, numerical heating occurs due to the statistical noise from the under-resolved distribution function, which numerically increases the electron temperature within the first 10 ns.

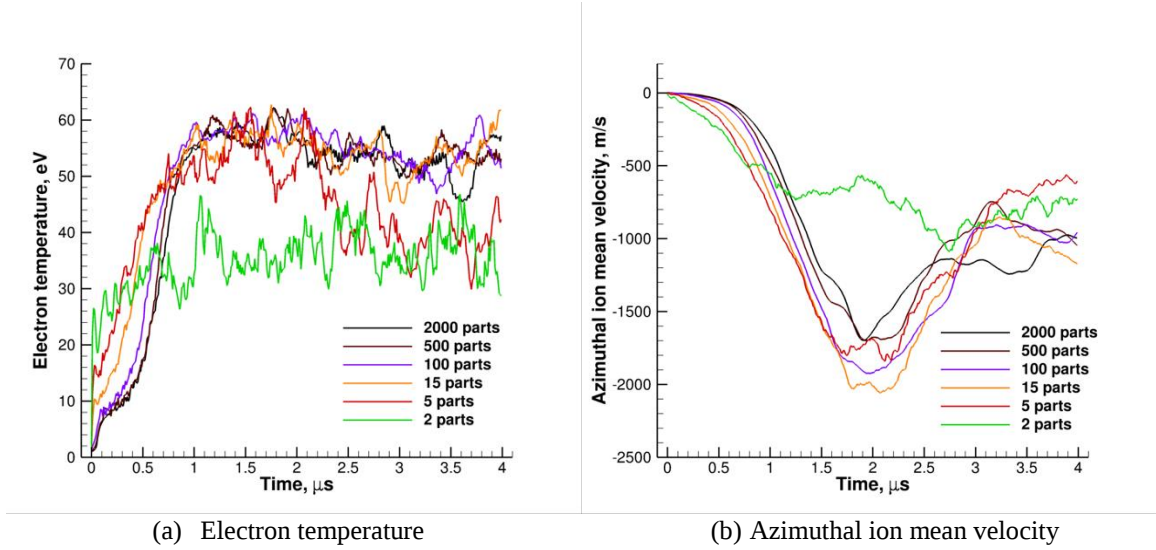


Figure 3. Numerical convergence test varying number of macroparticles per cell in the 1D azimuthal simulation.

Figure 4 shows the ion phase space for $N_p = 15$ and 500. Both cases show a feature of ion acoustic wave. The phase velocity is approximately 6,500 m/s from the particle distribution function. The ion acoustic speed can be calculated as $c_s = \sqrt{k_B T_e / m_i} \approx 6,300$ m/s assuming $T_e = 55$ eV from the steady-state solution shown in Figure 3. Both results show that there are approximately 3 modes within the 5-mm domain, which gives a wave number of $3,800 \text{ m}^{-1}$. It is also apparent that in the 1D setup, the ion acoustic wave stays coherent even with the reinjection of ions and electrons. In fact, it was found that the results are insensitive to the injection locations and velocity (temperature). For instance, the electron injection is chosen to be 10 % away from the cathode surface in these simulations, but the results do not change much even at different injection locations.

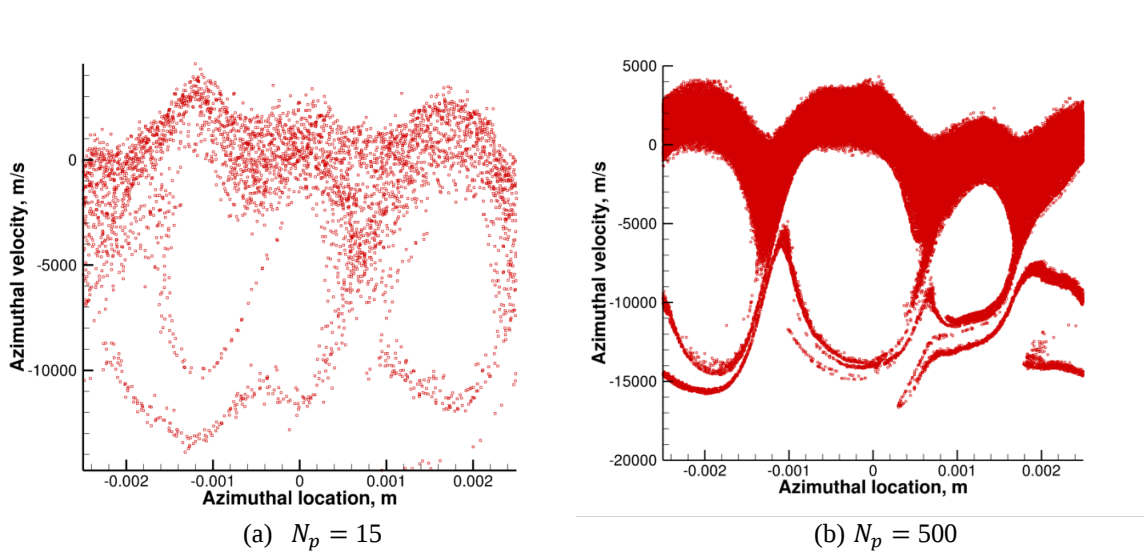


Figure 4. The ion phase space at the 4 μs.

B. Effects of axial boundary condition

Figure 5 shows the electron axial mean velocity with an absorbing and periodic boundary conditions in the axial direction when the ion density is uniform in the azimuthal direction. Here, ion macroparticles are not advanced and ions are assumed to be immobile. Time averaged electron axial mean velocities from Fig. 5 is -16,110 m/s and 2 m/s for axial and periodic boundary conditions, respectively. It is therefore clear from this result that the (artificial) axial boundary condition affects the electron transport in the cross-field direction. Note that the cross-field electron mobility can be given as

$$\mu_{\perp} = -\frac{\langle u_{ez} \rangle}{E_z} \quad (2)$$

where $\langle u_{ez} \rangle$ is the temporally and spatially averaged electron axial mean velocity and E_z is the prescribed axial electric field. Here, the axial electric field is 20,000 V/m. Hence, the cross-field electron mobilities for the axial and periodic boundary condition cases are 0.8 m²/V-s and 10⁻⁴ m²/V-s, respectively. Lafleur et al. proposed that collisionless fluctuation-based electron mobility is proportional to $\langle n_e E_y \rangle$, where $\langle Q \rangle = \frac{1}{\tau} \int_0^{\tau} \frac{1}{L_y} \int_0^{L_y} Q dy dt$ for an arbitrary quantity Q , τ is an arbitrary time for sampling, and L_y is the azimuthal length. For a one-dimensional system with periodic boundary condition in the azimuthal direction, using the Poisson's equation $\frac{dE_y}{dy} = \frac{e}{\epsilon_0} (n_i - n_e)$,

$$\langle n_e E_y \rangle = \langle n_i E_y \rangle - \left\langle \frac{\epsilon_0}{e} \frac{d}{dy} \left(\frac{1}{2} E_y^2 \right) \right\rangle. \quad (3)$$

It is clear from Eq. (3) that the cross-field electron mobility must be zero for $n_i = \text{constant}$, because $\left\langle \frac{dQ}{dy} \right\rangle = \frac{1}{\tau} \int_0^{\tau} \frac{1}{L_y} \int_0^{L_y} \frac{dQ}{dy} dy dt = \frac{1}{\tau} \int_0^{\tau} \frac{1}{L_y} [Q]_0^{L_y} dt = 0$ due to $Q(y=0) = Q(y=L_y)$ from periodic boundary condition in the azimuthal direction. Thus, $\langle n_i E_y \rangle = n_i \langle E_y \rangle = 0$. A non-zero electron mobility obtained using a non-periodic absorbing boundary condition in the axial direction is a consequence of artificially moving the gyrocenter before and after being absorbed by and injected from the boundaries. In other words, this is equivalent to introducing artificial collisionality.

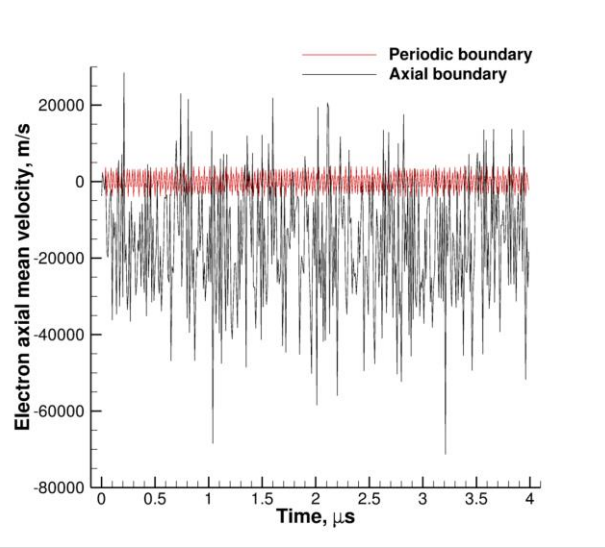


Figure 5. Cross-field electron transport using 1D azimuthal simulations with periodic and non-periodic (absorbing/injecting) boundary conditions in the axial direction.

The electron phase space (axial location vs. axial velocity) is shown in Figure. 6. As the electric field in the axial direction is not solved for, the ion and electron density variation in the axial direction does not affect the solution for the present azimuthal simulations. From the periodic boundary condition case, it can be seen that the electron velocity distribution function (VDF) follows a Maxwellian distribution that is identical to the initial condition ($T_e = 1$ eV). However, when an absorbing/injecting boundary conditions are used, the electron VDFs are broadened and the

electron injection at $z = 9$ mm can be seen. The electrons are absorbed in the upstream ($z = 0$ mm), so there are no electrons that are emitted from that plane, i.e., there are no electrons with positive axial velocity at $z = 0$ mm. Note that the ions are immobile throughout the simulation, hence one can expect $\langle n_i E_y \rangle = 0$. The simulation results suggest that the axial boundary condition proposed here adds artificial collision. For a large ExB drift, the ion acoustic wave generated due to the ExB drift is much more dominant than the artificial axial boundary condition. However, it is important to note that the axial boundary condition may artificially induce electron current for smaller ExB drift.

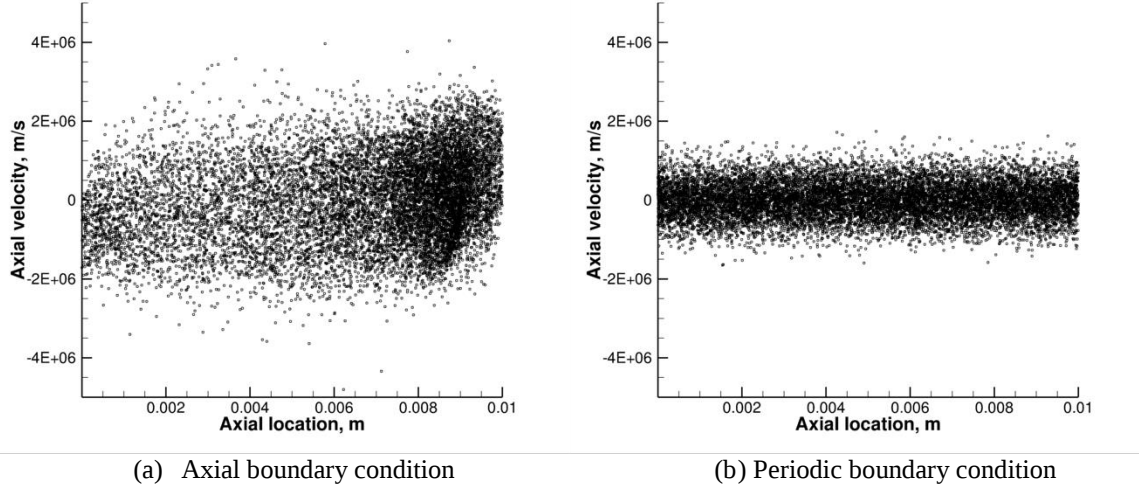


Figure 6. Electron phase space (axial location vs. axial velocity) at final time step. Dots shown represent individual macroparticles in the simulation.

These results are also consistent with Fig. 5 because the Joule heating increases as the axial electron mean velocity increases, while the axial electric field is kept constant. The axial boundary conditions introduce an asymmetric profile in the particle dynamics since the electrons are absorbed at the anode surface, while being injected from the cathode side. This asymmetry causes the electron axial current to be non-zero near the anode and therefore throughout the system.

IV.2D results

Two types of 2D simulations with different initial conditions were performed. One is with a uniform plasma density and the other is with a cosine plasma density profile. The former is to mimic the plasma inside the discharge channel while the focus of the latter is to assume a plume type ion distribution, in which the ion beam is located in the channel centerline and the plasma density decreases in the radial direction experiencing ambipolar diffusion. Here, we report the results of the cosine plasma density case.

A. Time dependent results

Figure 7 shows the ion density profile at different times for the cosine plasma density case. The plasma is shielded from the channel walls, because of the large plasma density gradient. For the first few hundreds of nanoseconds, the ExB drift instability (EDI) occurs in the middle of the channel where the plasma density is largest. The plasma wave establishes in the center because the radial electric field and hence the influence of the radial transport are not effective. Thus, the center part can be considered to quasi-1D in the azimuthal direction. At $2 \mu\text{s}$, the plasma wave shows three modes, similar to the 1D case, as shown in Fig. 4. The ion acoustic wave generated due to the EDI is similar to the 1D case for the present case ($E_z = 20000$ V/m, $B_r = 0.02$ T).

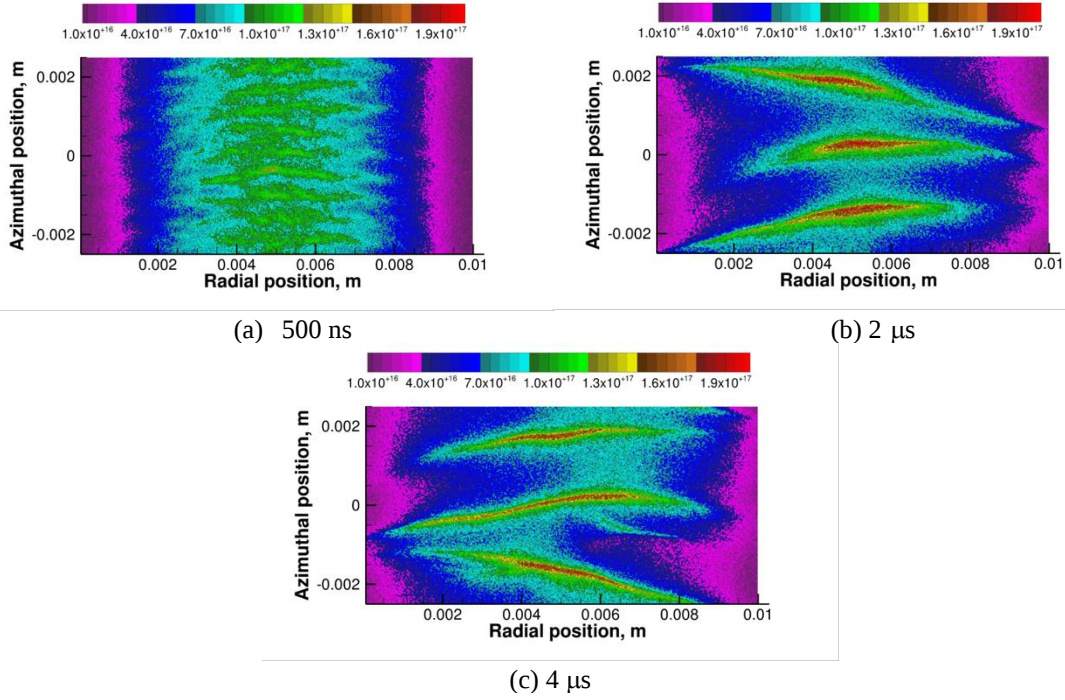


Figure 7. Temporal evolution of ion density [unit: m^{-3}] using a 2D PIC simulation with cosine ion density as initial condition.

The difference between the 1D azimuthal and 2D radial-azimuthal cases is the multidimensional effects. For instance, as shown in Fig. 7(b), the ion density front is tilted in the r - θ plane. It is likely that this is caused because the coupling between the bulk plasma and the dielectric walls. The corresponding potential structures are shown in Fig. 8. During the EDI regime (0.5 μ s), a small wave number plasma wave is generated on top of the large wave number waves that correspond to the EDI. The maximum potential difference is about 200 V, which is due to the increase in electron temperature. Note that a reference potential is set to zero at the northeast corner of the simulation domain.

The overall trend follows the feature of multidimensional plasma waves, in which plasma wave bows out. The growth rate of the ion acoustic instability is largest in the center because the plasma density is largest. It can be seen from Fig. 7(a) to Fig. 7(b) that there are regions where the plasma density extends locally to the walls, which can cause charge accumulation locally. Some dissipation of the plasma wave can also be observed at later time in Fig. 7(c) at $t=4\mu\text{s}$, but the ion acoustic wave continues to propagate coherently for a few μs similarly to the 1D case.

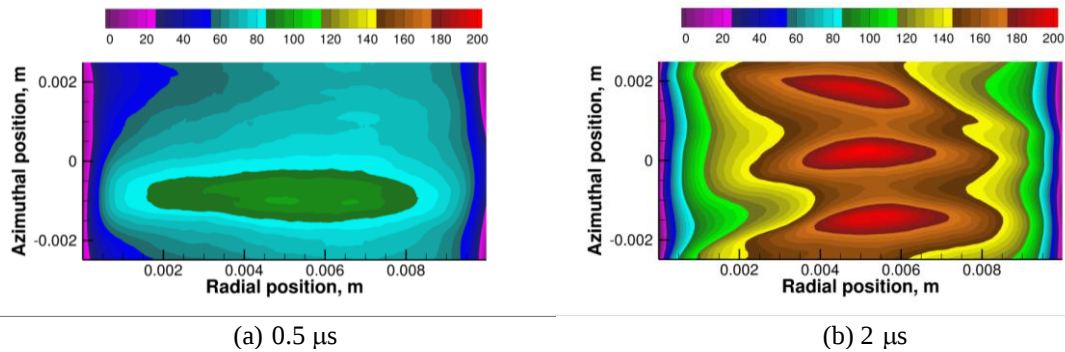


Figure 8. Snapshots of potential [unit: V] using a 2D PIC simulation.

Figure 9 shows the axial electron flux at 0.5 μs and 2 μs that correspond to Figs. 7 and 8. At 2 μs , electron current is conducted where the plasma density is largest. This is consistent with previous studies that showed this effect with segmented anodes²⁰. Near-wall conductivity is not present in these simulation results because the plasma density near the wall is an order of magnitude smaller than that in the center of the channel.

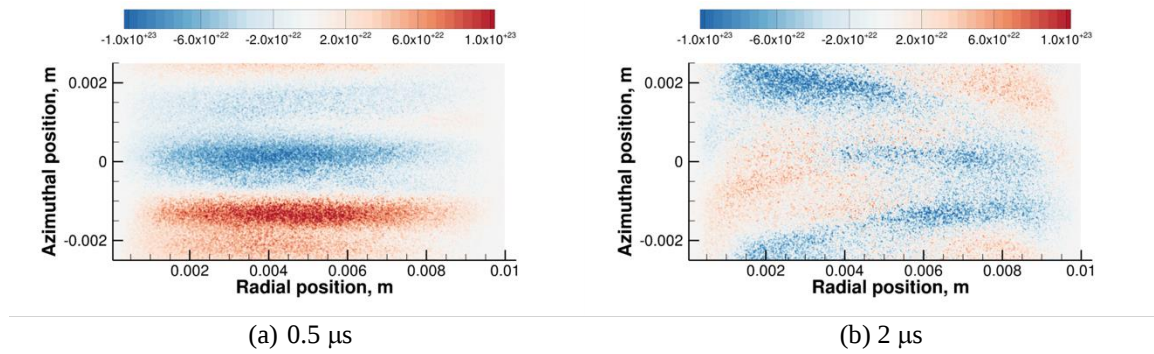


Figure 9. Snapshots of axial electron flux [unit: m^2s^{-1}] using a 2D PIC simulation. White color is zero values.

Figure 10 shows the azimuthal ion mean velocities at 2 μs and 4 μs that correspond to Figs. 7 and 8. The plasma wave with a larger wave number can be observed during the EDI stage. It can be clearly seen that the trapped ion region due to the plasma wave is curved accordingly to the plasma density profile.

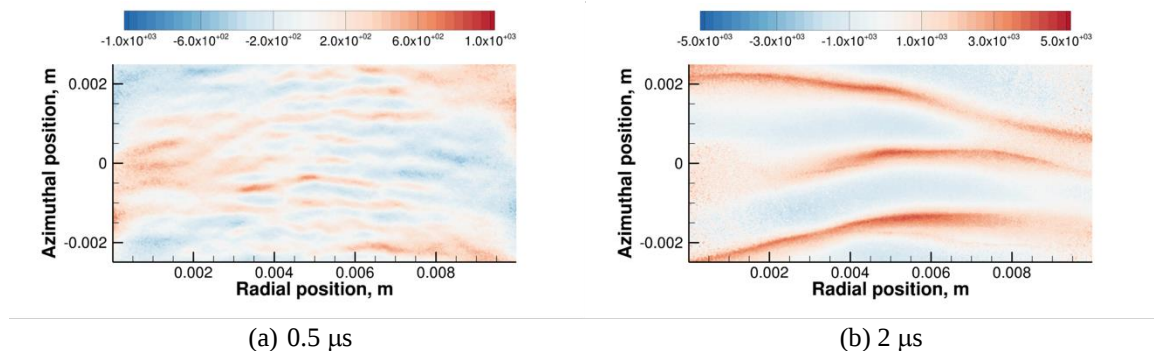


Figure 10. Snapshots of azimuthal ion mean velocity [unit: m/s] using a 2D PIC simulation. Note the scale is different in the two figures.

Figure 11 shows the effects of secondary electron emission (SEE). We have chosen a SEE yield smaller than unity, in which space charge saturation (SCS) does not occur. It is reported that the ion acoustic wave due to the EDI and that induced by SEE can nonlinearly interact and make the system more dynamic²¹. It can be seen from Fig. 11 that the SEE below SCL does not significantly affect the ion acoustic wave, particularly the cross-field electron transport. A slight decrease in the electron temperature is observed in the presence of SEE, which is consistent with standard sheath theories. The sheath potential drop is reduced because of the SEE, allowing more heat flux to the wall.

The effects of plasma wall interaction on the plasma wave generation are small in these simulations mainly because the electron temperature in the radial direction is small (on the order of 1 eV) compared to the electron temperature in the azimuthal direction (on the order of 50 eV) in the current configuration. The sheath potential is smaller than 5 V, assuming the sheath potential is $5.2 T_e$ (for xenon plasma). Therefore, the sheath is a thin layer as can be seen from Fig. 8 (maximum potential is 200 V). For the plasma-material interaction to contribute to the plasma wave damping, the electron temperature in the radial direction needs to be larger. Two mechanisms that increase the radial electron

temperature include collisionless and collisional mechanisms. Collision frequency in the Hall thruster discharge plasma is at most 10 MHz inside the channel. Therefore, for the present simulations, the collision frequency is not large enough to thermalize the electrons.

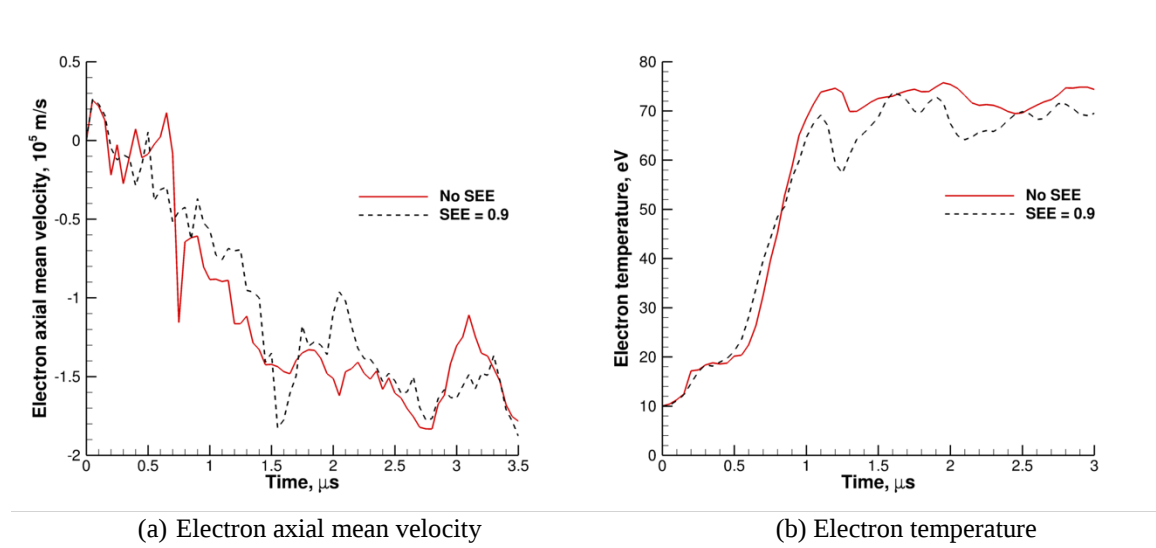


Figure 11. Effects of the secondary electron emission.

It is to be noted that the current simulations are not realistic in terms of the energy input in the system. In a standard SPT-100 type Hall thruster, the discharge power is 1.35 kW (discharge voltage of 300 V and current of 4.5 A). The electric field is maximum near the location at which the magnetic field is maximum (say, 20000 V/m) and it is observed that the electron axial mean velocity in the acceleration region is < 5000 m/s. If all the plasma parameters are kept constant, e.g., plasma density, this means that power input to the electrons is 10-30 times larger than the realistic power input. Therefore, in reality, because the input power is limited, if the electron current increases via ion acoustic instability, the electric field that can be sustained must decrease, which will in turn weaken the ion acoustic instability. For this, a z- θ simulation is needed, where the current and electric field can be globally determined between a potential drop from the anode and cathode.

B. Static ion density case

Similar to the 1D test case, two static immobile ion density profiles are considered in the 2D PIC simulation. For one simulation, an ion density profile with a sinusoidal distribution in the azimuthal direction is assumed, as shown in Fig. 12(a), and it is kept fixed throughout the simulation. The other is an azimuthally uniform ion density profile,

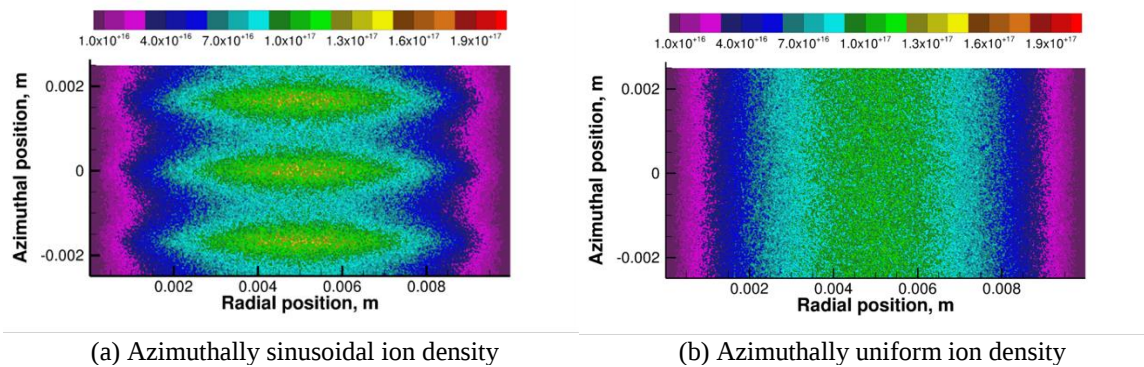


Figure 12. Initial conditions assuming immobile ion density.

shown in Fig. 12(b). Note that in both cases statistical noise is present because the ion density profile is generated by random sampling of the ion macroparticles.

The spatially averaged electron axial mean velocity and the electron temperature are shown in Fig. 13 as a function of time for the two immobile ion density profiles. The temporal averaged electron axial mean velocities are -67700 m/s and -7270 m/s for the azimuthally sinusoidal and uniform ion profile cases, respectively. Thus, the effective cross-field electron mobilities are $3.38 \text{ m}^2/\text{V}\cdot\text{s}$ and $0.36 \text{ m}^2/\text{V}\cdot\text{s}$. Both the 1D and 2D results suggest that the ion density profile is critical in determining the cross-field electron transport. Moreover, in the absence of ion density azimuthal fluctuation, it was observed from the 1D analysis that there must be no electron transport (see Eq. (3)). The oscillation amplitude observed for the uniform case is on the same order as the 1D case, see Fig. 5. Therefore, it can be concluded that the axial boundary condition may affect the electron transport mechanism for this 2D case as well.

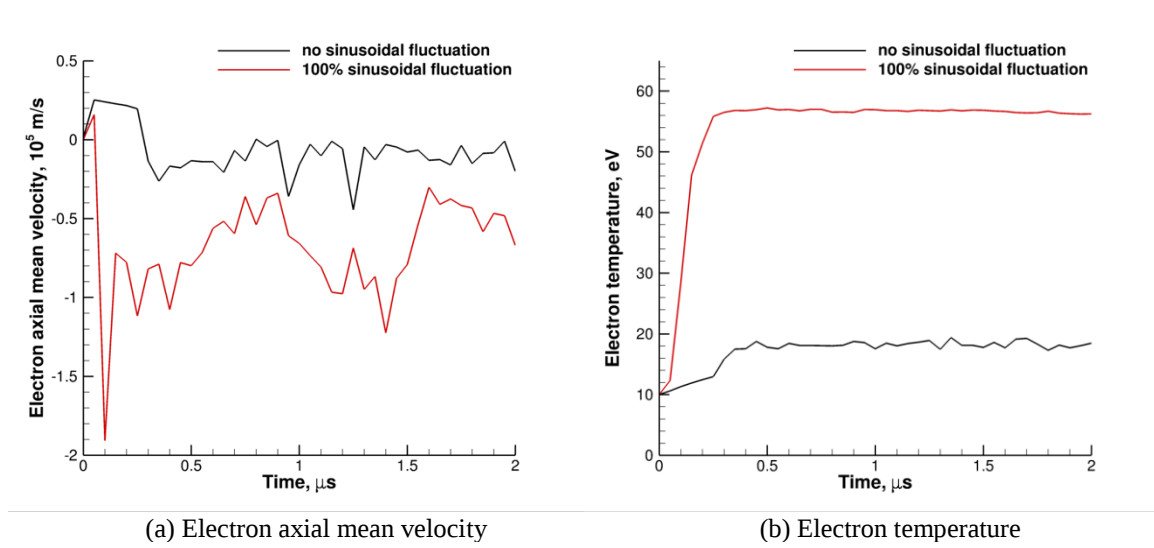


Figure 13. Time evolution of plasma properties assuming immobile ion density study.

Joule heating in each direction (x, y, and z) is obtained by calculating $\langle j_{ej} E_j \rangle$, where subscript $j = x, y$, and z . The results are summarized in Table 2. The sinusoidal case generates a cross-field transport due to the ion acoustic wave and E_z is fixed at 20000 V/m in all the simulations. Hence, the axial Joule heating is dominant in the sinusoidal case, while the Joule heating for the azimuthally uniform case is an order of magnitude smaller. The radial Joule heating also increases about 3 times for the sinusoidal case with respect to the uniform case.

Table 2. Joule heating contributions for immobile ion density cases.

	Sinusoidal	Uniform
Axial (z)	$8.30 \times 10^{25} \text{ V}/(\text{m}^3\text{s})$ [86.2%]	$9.343 \times 10^{24} \text{ V}/(\text{m}^3\text{s})$ [17.7%]
Azimuthal (y)	$1.05 \times 10^{25} \text{ V}/(\text{m}^3\text{s})$ [10.9%]	$4.24 \times 10^{25} \text{ V}/(\text{m}^3\text{s})$ [80.3%]
Radial (x)	$2.85 \times 10^{24} \text{ V}/(\text{m}^3\text{s})$ [2.9%]	$1.05 \times 10^{24} \text{ V}/(\text{m}^3\text{s})$ [2.0%]
Total	$9.63 \times 10^{25} \text{ V}/(\text{m}^3\text{s})$ [100%]	$5.28 \times 10^{25} \text{ V}/(\text{m}^3\text{s})$ [100%]

Figure 14 shows the radial and azimuthal electric fields as well as the axial electron flux for the sinusoidal and uniform ion density profiles. The values of the electron axial flux are large because of the relatively large azimuthal electric field, but these do not contribute to an overall electron transport in the cross-field direction.

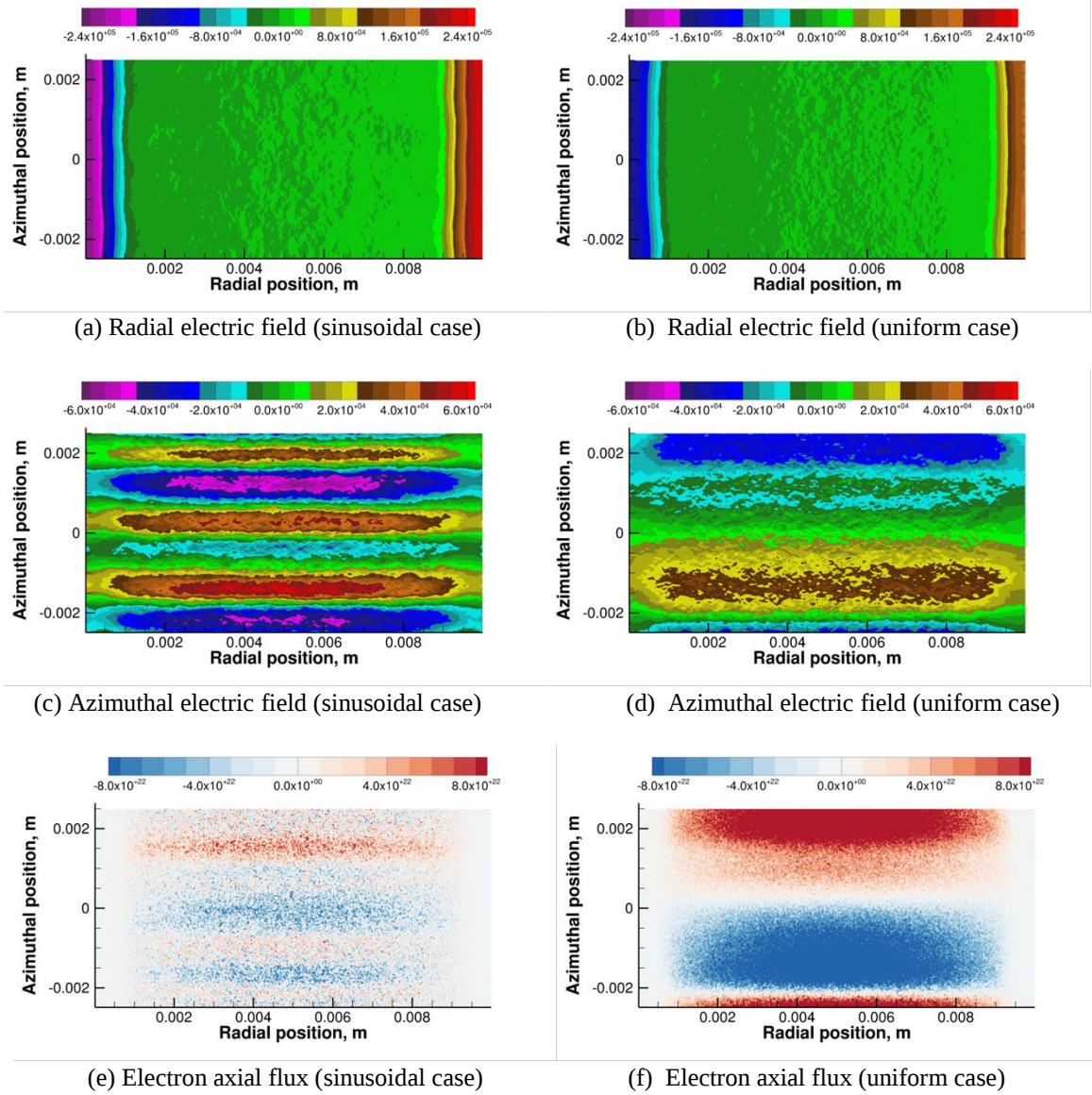


Figure 14. Results at $2\mu\text{s}$ assuming immobile ion density study.

Units for (a-d): V/m, Units for (e-f): $\text{m}^{-2} \text{s}^{-1}$.

V. Conclusions

A 1D azimuthal PIC simulation is first developed to study the ion acoustic wave generated due to the electron ExB drift. The results obtained from the present 1D PIC simulation show good agreement with published data. A numerical convergence study was conducted by varying the number of macroparticles in the domain. In addition, static immobile ion density profile was assumed to investigate the electron transport. The 1D results show that the axial boundary condition numerically affects the electron current in the axial direction, even in the absence of ion dynamics.

A 2D planar PIC simulation that represents a radial-azimuthal configuration in Hall thrusters is developed using MPI and HYPRE software. Verification of the model is conducted and the most efficient numerical method is chosen for the Poisson solve in 2D. The parallelization is built allowing the usage of different number of processors for the

particle module and the electric field subroutine. The multidimensional effects of the plasma wave propagation are observed, namely, the coherent plasma wave with radial dynamics related to the plasma sheaths. The axial boundary conditions act as a source of artificial collision. Hence, the electron current can be numerically induced even in the absence of ion density perturbation. Simulations with immobile ion density profiles show that ion density profile in the azimuthal direction plays a significant role in the cross-field electron transport. This suggests that ion kinetics is equally (or more) important than the electron kinetics.

Finally, the prefixed axial electric field indefinitely provides free energy into the system, which induces anomalous electron transport and hence electron current in the axial direction. As the input power for Hall thruster is limited by the electrical circuit, the electric field must be reduced if the electron current becomes too large. Hence, the current and electric field structure in the axial direction is crucial in understanding the electron transport in the cross-field configuration, which is reserved for future work.

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