

Low-frequency plasma oscillations in the plume of a low temperature magnetic nozzle

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The fluctuations in the light intensity in the plume of a small, partially-magnetized magnetic nozzle are experimentally characterized. A high speed camera with a frame rate of 300 kfps is directed along the axis of symmetry and Fourier analysis is performed to identify the spectral content of the measured oscillations. A low-frequency, 24 kHz and $m=1$ mode oscillation is found to propagate in the azimuthal direction around the center of the plume. In an effort to identify the nature of this mode, a linear dispersion analysis is applied to the measured background plasma properties in the nozzle's plume. This analysis shows that for the measured plasma conditions, an anti-drift like mode can be driven unstable by the strong diamagnetic electron drift in the azimuthal direction. The predicted frequency for this mode is shown to be comparable to the observed frequency of oscillation. The onset of this mode and its potential influence on the nozzle's near-field plasma properties are discussed.

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Nomenclature

η_{AN} = Anomalous collision frequency	E = Electric Field
v_{de} = Relative electron-ion drift	B = Magnetic Field
m_e = Electron mass	r_L = Larmor radius
τ_{eff} = Effective Momentum transfer time	ν_{en} = Electron-neutral collision frequency
$\mu_{\perp}$ = Cross-field mobility	ν_{in} = Ion-neutral collision frequency
Ω_e = Electron cyclotron frequency	ω_{ce} = Electron cyclotron angular frequency
ϕ = Plasma potential	T_i = Ion temperature
I = Pixel intensity	T_e = Electron temperature
$\tilde{I}_0$ = Complex magnitude of pixel intensity	k_{θ} = Azimuthal wavenumber
n = Number of frames	k_z = Axial wavenumber
i = Frame index	v_D = Diamagnetic drift velocity
m = Azimuthal mode	$v_E = E \times B$ drift velocity
ω = Angular frequency	ω^* = Diamagnetic drift frequency
θ = Azimuthal position	c_S = Ion sound speed
$\mathcal{F}$ = Fourier Transform	c_e = Electron thermal speed
$\Delta\phi$ = Phase Offset	m_i = Ion mass
r = Radial position	m_e = Electron mass
N = Number of azimuthal bins	ω_R = Real angular frequency
j = Bin index	ω_I = Imaginary angular frequency
MDX = Magnetic Detachment Experiment	f = Observable frequency
ADI = Anti Drift Instability	γ = Growth rate

I. Introduction

Magnetic nozzles consist of an axially diverging magnetic field used to accelerate a heated plasma.^{1–10} They are used as a component of a variety of thrusters, notably in conjunction with helicon or electron cyclotron resonance sources. One fundamental and unsolved question regarding these devices is that of detachment. The field lines that comprise a magnetic nozzle return to the thruster, allowing magnetized particles to reverse trajectories. If electrons remain attached to a field line, they will follow it back to the spacecraft and incite charging issues. In stronger magnetic fields, ions can also remain attached, returning to the craft and limiting thruster performance. Of particular interest is the case of magnetized electrons and unmagnetized ions, as commonly found in small magnetic nozzles.^{10,11} In such devices, electrons may remain attached, diverging outward and creating electric fields that redirect the ions into also following field lines.

The question of whether or not ions remain attached in these partially-magnetized devices has been largely resolved both experimentally and numerically. Indeed, it has been shown that ion trajectories ultimately deviate from the confining magnetic field.¹ It is still unclear, however, if and how the lighter electrons detach from the nozzle. There have been many proposed mechanisms for this process. These propositions include classical resistive detachment,¹² the effects of finite electron inertia,^{13,14} electron de-magnetization,¹ and stretching of magnetic field lines by induced currents in the plasma.² While all of these drivers have been explored in different measure, one relatively new proposed mechanism for these partially-ionized sources is the onset of plasma instabilities.^{5,8,15} The physical reasoning behind this concept is that instabilities can lead to anomalous cross-field transport across the confining magnetic field, thereby initiating an effective electron detachment.

To date, there have been several studies devoted to examining the role of plasma transients in magnetic nozzle-like plasmas. A prominent example is the work done by Olsen⁵ on the fully-magnetized plasma produced by the VASIMR VX-200 nozzle. As the plasma expands through the nozzle in this device, Olsen

showed that the ions ultimately detach due to finite Larmor radius effects. The detachment of the electrons, on the other hand, is in part dictated by the onset of a lower hybrid drift instability. While this work was insightful in showing the potential role oscillations may have to play in detachment for magnetic nozzles, the VASIMR plasma was not representative of most smaller nozzle sources. The ions were fully magnetized and hot, with temperatures on the order of 30 eV.⁵ These differences suggest that the same instabilities and detachment mechanism Olsen observed may not be present in smaller, partially-magnetized sources. With this in mind, a more representative experimental study is the work performed by Light et al., who conducted an experiment to detect instabilities in a lower temperature, partially magnetized plasma.⁸ They found instabilities inside the plasma source exhibiting characteristics of the resistive drift and Kelvin-Helmholtz instabilities oscillating at frequencies of tens of kHz. They performed this experiment on a variety of propellants and ultimately identified these modes as contributors to anomalous cross field transport to the source wall. This work thus provided strong evidence that we should anticipate the onset of transients in partially-magnetized, low-temperature plasmas. However, we note that this previous work focused on examining instabilities in the plasma source itself as opposed to in the downstream region where detachment must occur. There is thus an open question as to whether or not transients exist in the plume of partially magnetized nozzles. Given the potential role these oscillations may have in dictating detachment in these types of plasmas, the need is apparent for experimental and theoretical studies of the transients in these devices. The goal of this investigation is to provide an initial survey of these modes and to attempt to identify any measured dispersion.

With this objective in mind, this paper is organized in the following way: Section II outlines the details of the experiment that we performed to measure the plasma properties, Section III discusses the interpretation of the measurements, Section IV details the theoretical explanation of the instabilities we observe, and Section V discusses the broader implications of the observation of this instability in the context of detachment from field lines in magnetic nozzles.

II. Experimental Setup

First, we will discuss the experimental procedure we implemented during this test. The Magnetic Detachment Experiment (MDX) is a small, radio-frequency plasma source at the Plasmadynamics and Electric Propulsion Laboratory at the University of Michigan that we used for this experiment, and it is described in more detail by Collard et al.¹¹ MDX is comprised of a quartz liner with a coil antenna within a 149-turn

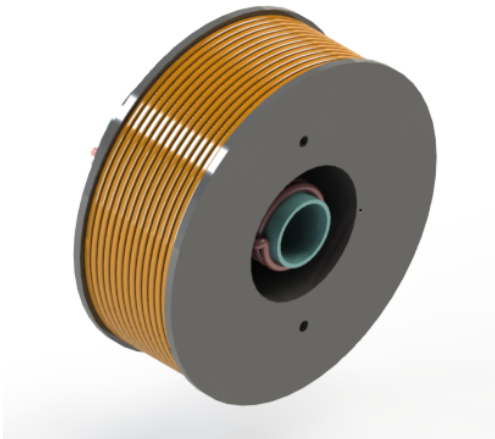


Figure 1: Graphical rendering of MDX.



Figure 2: High speed camera in front of Junior.

copper wire electromagnet capable of generating a centerline magnetic field of 900 G. We fed propellant through the rear of the device from a pressurized tank and encased the entire apparatus in a copper mesh for Faraday shielding. Figure 1 depicts a 3D rendering of MDX before the Faraday mesh, power lines, and feedthroughs have been attached. We operated MDX in the Junior Test Facility, a 3m long and 1m diameter vacuum chamber capable of reaching a base pressure of 10^{-7} Torr with a nominal pumping speed of 38,000 L/s on xenon. Junior maintains low pressure with a cryopump and a turbopump. In the current experiment,

we ran MDX at flow rate of 3 mg/s on xenon, allowing for an operating pressure of 4×10^{-5} Torr.

First, we mapped the magnetic field using a three-axis Gaussmeter at atmosphere with no plasma. We fastened the Gaussmeter to linear motion stages to perform a full sweep in front of the electromagnet. The resolution of the Gaussmeter is .01 G, and we moved it through a rectangular 40×30 grid with 5mm resolution.

We then found background plasma properties through experiments discussed in more depth by Collard et al.¹¹ We ran MDX in front of a planar double probe and an emissive probe to measure plasma potential and density. To reposition MDX for each test, we fastened it to linear motion stages and held the probes stationary. We first took plasma density measurements with planar double probes. Double probes are necessary for this experiment to negate the background radio frequencies from the antenna. The collection surfaces were 0.25 mm in diameter and were encased by a double-bore alumina tube and positioned 1.3 mm apart measured center to center. We then used an emissive probe to measure plasma potential. The emissive probe had a radius of 1mm and was operated in the high-emission limit.

Finally, we looked at the MDX plume using a high speed camera. We situated a Photron Fastcam SA5, depicted in Figure 2, outside of the chamber directly downstream of the plasma. The Fastcam is capable of recording imagery at 1.5 million frames per second. To capture the entire plume with reasonable resolution, we recorded at 300,000 frames per second, with a resolution of 256×64 pixels. Empty space was cropped out, yielding a 55×55 pixel image of the full plume. We attached a camera lens to the front of the Fastcam to provide a full and clear image of the plume, then took 100 ms of data.

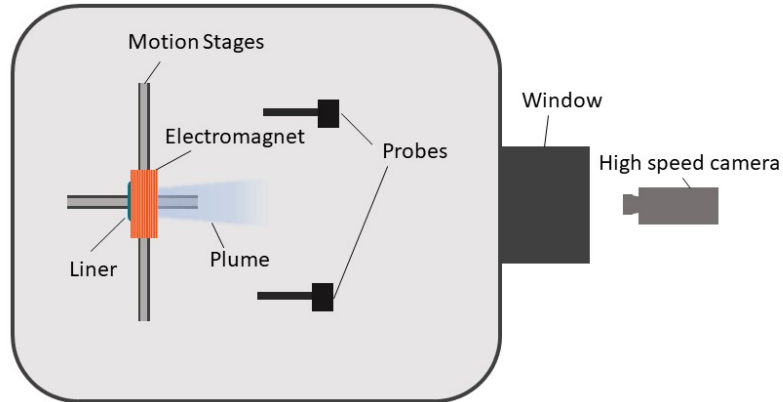


Figure 3: Experimental setup, not to scale. MDX is situated on two linear motion stages with the Fastcam outside of the chamber and electrostatic probes to the side.

III. Results

We now present the results of our measurements on MDX. We first present measurements of the background magnetic field, plasma potential, and density as discussed by Collard et al.¹¹ The magnetic field, potential, and density maps allow us to determine the presence of electron drifts that may provide an energy source for oscillations. We then discuss results from the high speed imagery to determine the character of the observed waves.

Background Plasma Parameters

Figure 4 shows the results from the Gaussmeter measurements. The field profile shows that a magnetic nozzle exists and provides a means of evaluating electron drift velocities when coupled with plasma parameters. The peak field occurs 2.5 mm into the channel and falls to half its peak value 1.2 mm downstream of the exit plane. Both profiles are shown in Figure 4, originally depicted by Collard et al.¹¹ and reproduced here. The exit plane is located at 0 mm, and the peak of the magnetic field on axis is within the liner.

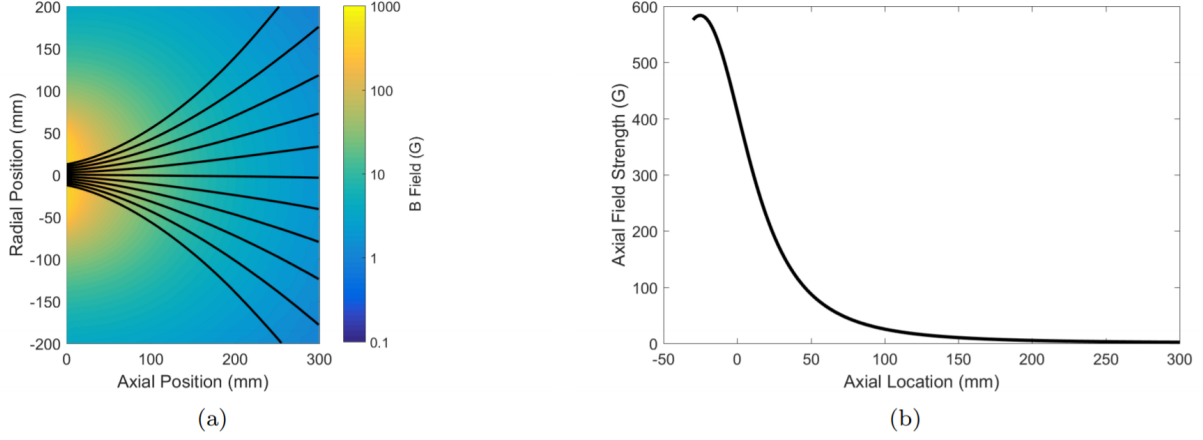


Figure 4: Magnetic field profile of the 30 A condition in MDX. a) 2D map of field strength on a logarithmic scale with field lines in black. b) Magnitude of magnetic field strength on axis with positive distances representing distance downstream of the exit plane. Figure reproduced from Collard et al.¹¹

Next, we find the potential profile to determine $E \times B$ drift velocity. Figure 5 shows the radial potential profile throughout the plume in 8mm axial increments. The gradient of the potential can be found to determine the magnitude of the $E \times B$ drift velocity, which, if significant, may be an energy source of the wave we observe. The potential profile predictably spreads out downstream as the plume expands to cover more area. A well also forms off axis as the plasma progresses downstream.

To find the diamagnetic drift velocity, we need plasma density and electron temperature. Figure 6 depicts the density profile in the plume as measured from the double probe. We also infer the electron temperature from the double probe trace, and we find it to be nearly constant throughout the plume at 6 eV. The gradients evident in Figures 5 and 6 force electron drifts that can drive instabilities. We present the analysis of the drifts caused by these gradients in section IV.

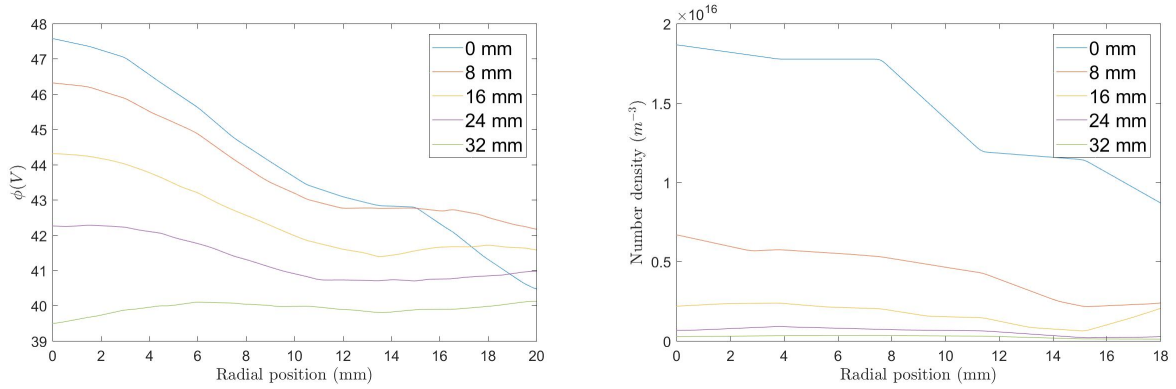


Figure 5: Potential profile at 8mm axial increments. Figure 6: Density profile at 8mm axial increments.

High Speed Imagery

Next, we look at the imagery we took from the Fastcam shown in Figure 7. Assuming that changes in pixel intensity can be equated to fractional changes in density, $\tilde{I}/\bar{I} = \tilde{n}/n_0$, we can use high speed imagery to detect oscillations in the MDX plume.^{16,17} Figure 7 depicts a color image from a standard camera (left) and a grayscale image from the Photron high speed camera (right). We took 0.1 s of data at 300,000 frames per second. To remove noise, we separated this dataset into thirty sets of 1,000 images each and averaged our results over all of the sets. Because of the low light emission outside of the liner radius, we were unable to take reliable data above the noise floor in that region. All imagery analysis takes into account only the radii below the liner radius.

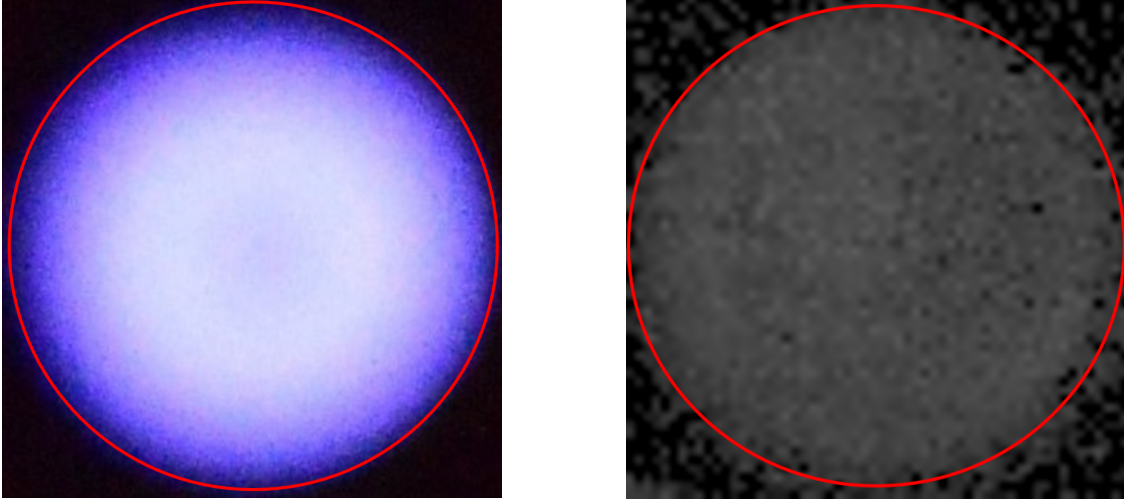


Figure 7: Left: Axial view of the MDX plume with liner location in red. Right: Example high speed camera image with liner location in red.

To detect the presence of instabilities from the high speed imagery, we apply spatial and temporal Fourier analysis, taking our methodology from McDonald et al.^{16,17} Since our primary interest was wave motion, we first averaged out each pixel, dividing each pixel intensity by $\sum_{i=1}^n \frac{I(r,\theta,i)}{n}$, where $I(r,\theta,i)$ is a pixel's intensity at frame i and $n = 1000$ is the total number of frames in each set. To determine if a frequency component exists, we can apply a Fourier Transform to each pixel's intensity over time and examine the power spectral density (PSD) of each pixel:

$$PSD(m, \omega) = \left| \sum_{j=1}^N \mathcal{F} \langle I(r, \theta, t) \rangle \right|^2 \quad (1)$$

We represent the normalized PSD of an arbitrary pixel in the upper-left quadrant in Figure 8.

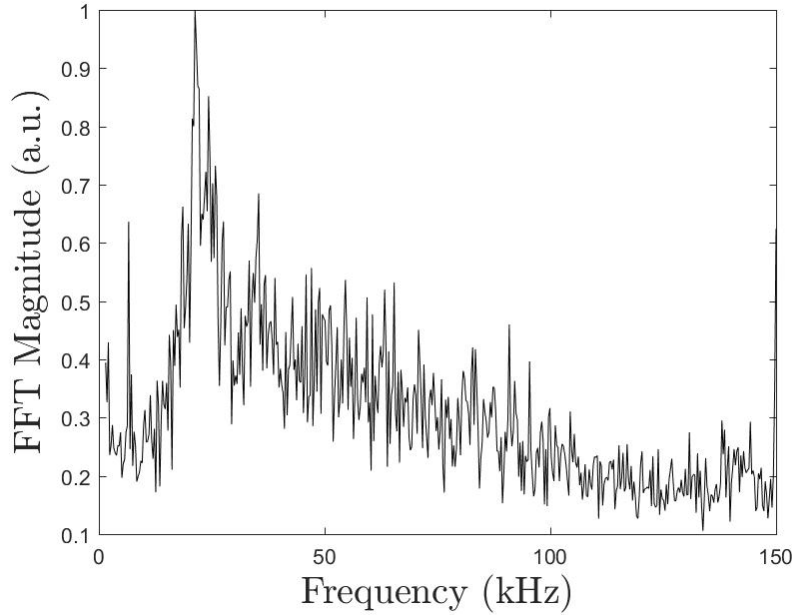


Figure 8: Normalized PSD of arbitrary pixel in plume.

As evidenced in Figure 8, we observe a peak between 20 and 30 kHz. While we cannot replicate the PSD of each pixel here, those elsewhere in the plume show a similar pattern to that depicted in Figure 8. The existence of this oscillation may imply that an instability is present in the plume. However, this analysis does not account for any spatial character the wave may have. To perform a spatial analysis, we assume that each pixel's intensity can be represented as a sum of frequencies in the azimuthal direction:

$$I(r, \theta, i) = \sum_{\omega} \tilde{I}_0(\omega) e^{i(m\theta - \omega t)} \quad (2)$$

where m is the mode. The cylindrical symmetry of the plasma forces m to be an integer. Thus, for a given ω , $m\theta$ provides the phase offset for the frequency in question. We can then qualitatively locate an $m = 1$ mode by sweeping frequencies and finding where a clear circular pattern in the phase offsets can be seen.¹⁸ To map this value, it is necessary to decompose pixel intensity into its frequency components:

$$I(r, \theta, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathcal{F}(I(r, \theta, t)) e^{i(\phi(\omega) - \omega t)} \quad (3)$$

where $\mathcal{F}$ represents the Fourier transform with respect to time. We can then find the phase difference between two pixels for a certain frequency:

$$\Delta\phi(\omega) = \phi(\omega) - \phi_0(\omega) \quad (4)$$

where $\phi = \tan^{-1} \left(\frac{\text{Im}\mathcal{F}(I)}{\text{Re}\mathcal{F}(I)} \right)$. We can visualize the rotating mode graphically. Figure 9 contains the representation of the phase offset map of our data at 24 ± 2 kHz. A pixel in the upper left quadrant was chosen arbitrarily as a reference where $\Delta\phi = 0$. We can see from the image that the phase offset has a rotational quality with one full period around the plume, implying the existence of an $m = 1$ mode at this frequency.

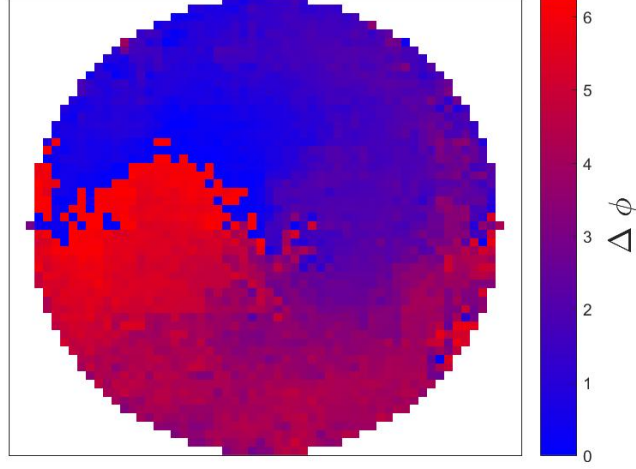


Figure 9: Relative phase offset at 24 kHz after averaging the FFT over ± 2 kHz and applying a 5x5 mean filter on the 55x55 pixel image. The $m=1$ mode is discernible by the rotational nature of the phase offset.

To further quantify the existence of this mode, we now apply the technique first proposed by McDonald et al.^{16, 17, 19, 20} We define a set of N azimuthal bins, bounded by $\theta_j = 2\pi \frac{j}{N}$, where $1 < j < N$. We can then average each bin,

$$\langle I \rangle_j = \frac{N}{\pi r_{max}^2} \int_0^{r_{max}} \int_{\theta_j}^{\theta_{j+1}} I(r, \theta, t) r dr d\theta. \quad (5)$$

We now have a set of data that we can input into a two-dimensional Fourier transform to determine the spatial and temporal PSD of our data. The results of applying Equations 1 and 5 are plotted in Figure 10. While choosing N azimuthal bins allows us to find the first $N/2$ modes, there was no significant signature above $m = 1$, so only the first two are shown.

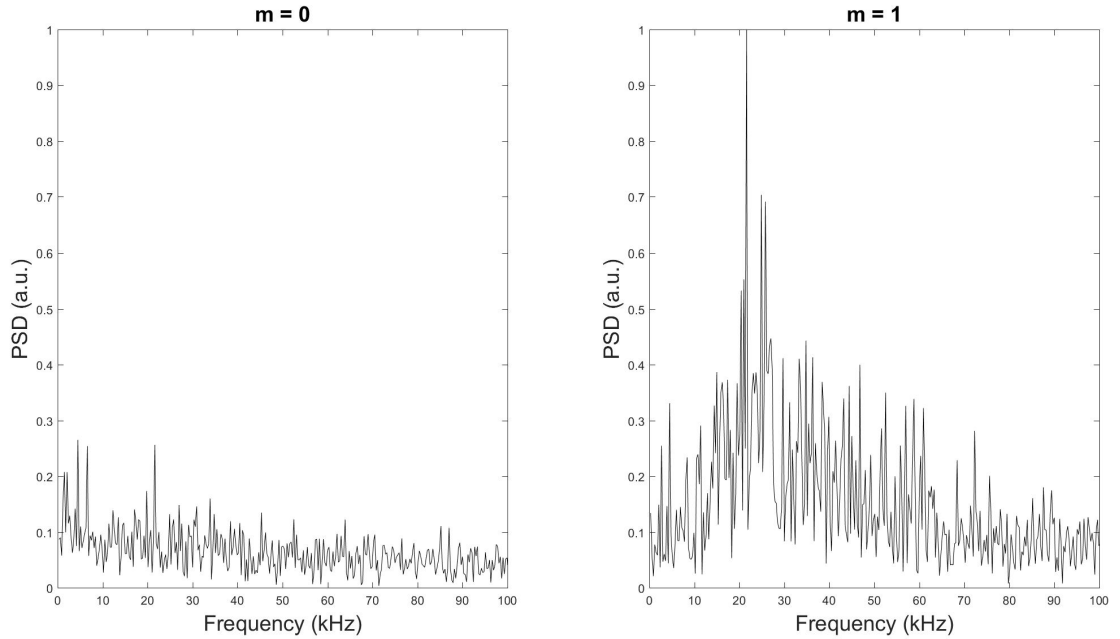


Figure 10: PSD of the $m = 0$ and $m = 1$ azimuthal modes.

The $m = 0$ mode has no clear peaks, implying that no coherent global oscillation exists. However, the $m = 1$ mode has a clear peak between 20 and 30 kHz, implying the existence of such a mode in this range. The intensity of this oscillation can then be plotted as a function of radius by mapping the PSD of each pixel at the frequency in question. This graph is given in Figure 11. The oscillation appears to peak at 5.5 mm from the center of the plume.

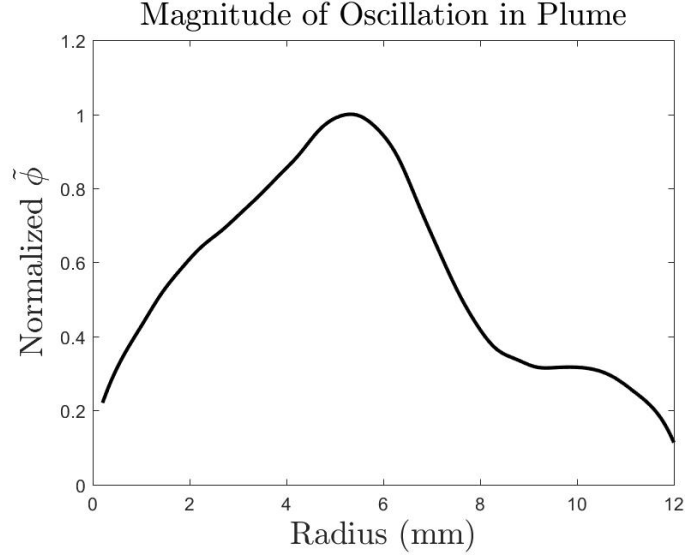


Figure 11: Oscillation magnitude of the 23 kHz wave as a function of distance from the center.

In summary, we have observed an $m = 1$ azimuthal mode oscillating between 20 and 30 kHz through spatial and temporal Fourier analysis. In the next section, we will present a theoretical description of this wave using measured plasma properties.

IV. Theoretical Analysis

In this section, we provide theoretical description for the wave we observe. Waves often pull energy from electron drifts, so we begin by examining the two dominant electron drift modes present in the plume, shown in Figure 12.

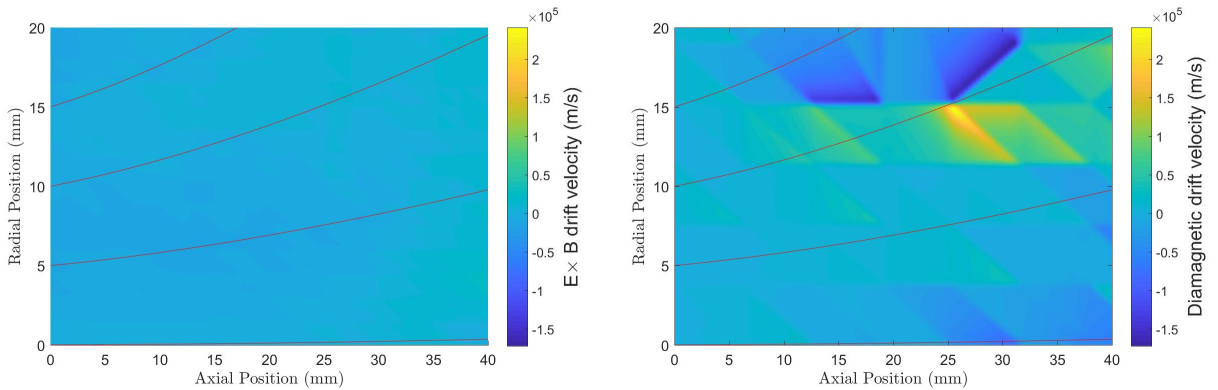


Figure 12: Contour plots of $E \times B$ drift velocity (left) and diamagnetic drift velocity (right). Black lines represent magnetic fields..

The left side of Figure 12 shows that the $E \times B$ drift velocity is on the order of a few km/s, whereas the right side shows that the diamagnetic drift velocity throughout the plume can reach over 100 km/s. It is thus evident that the $E \times B$ drift velocity is at least an order of magnitude less than the diamagnetic drift

velocity, so we will assume that the diamagnetic drift provides the energy for the wave. To find the effect that drives the wave unstable, we look to electron-neutral collisions. While exact neutral pressure profiles throughout the plume have not yet been measured, we can still estimate the character of this instability with theoretically predicted values. With neutral densities between 10^{18} and 10^{21}m^{-3} , electron-neutral collision frequencies will be between 10^7 and 10^{11} Hz, which is significant compared to the observed oscillation frequency. Given the high electron-neutral collision frequency, we will continue with collisional theory.

For our experiment, we can now make the following assumptions:

- Magnetized electrons ($r_{L,e} \ll B/\nabla B$)
- Unmagnetized ions ($r_{L,i} \gg B/\nabla B$)
- Intermediate frequency, $\nu_{in} < \omega < \nu_{en} < \omega_{ce}$
- Magnetic field in the axial direction, $\vec{B} = B_0 \hat{z}$
- Axial invariance of density and magnetic field, $\frac{dB_0}{dz} = 0$
- Cold ions, $T_i \rightarrow 0$
- Dominant diamagnetic drift, $v_D \gg v_E$

The Anti-Drift Instability (ADI) incorporates these assumptions. The ADI is a drift whose frequency is inversely proportional to the drift frequency from which it derives energy. In our case, the ADI is driven unstable by electron-neutral collisions and gains energy from the electron diamagnetic drift. The derivation for the dispersion relation is outside of the scope of this paper, but the reader is referred to Light et al.⁸ and Sakawa et al.²¹ With the aforementioned assumptions, the dispersion relation is

$$\frac{k_\theta c_s^2}{\omega^2} = \frac{\omega^* + ik_z^2 c_e^2 / \nu_{en}}{\omega + ik_z^2 c_e^2 / \nu_{en}}. \quad (6)$$

Here, k_θ is the azimuthal wave number defined as $k_\theta = 1/r_{max}$ with r_{max} being the location of the strongest oscillation, $c_s = \sqrt{\frac{T_e}{m_i}}$ is the ion sound speed, $c_e = \sqrt{\frac{T_e}{m_e}}$ is the electron thermal velocity, and $\omega^* = k_\theta v_D$ is the diamagnetic drift frequency. The solution to Equation 6 will take the form $\omega(k_z, k_\theta) = \omega_R + i\omega_I$, where $f_r = \omega_R/2\pi$ is the frequency we would observe and $\gamma = \omega_I/2\pi$ is the growth rate of the instability. For typical parameters of the MDX plasma, Figure 13 represents the solution as a function of k_z .

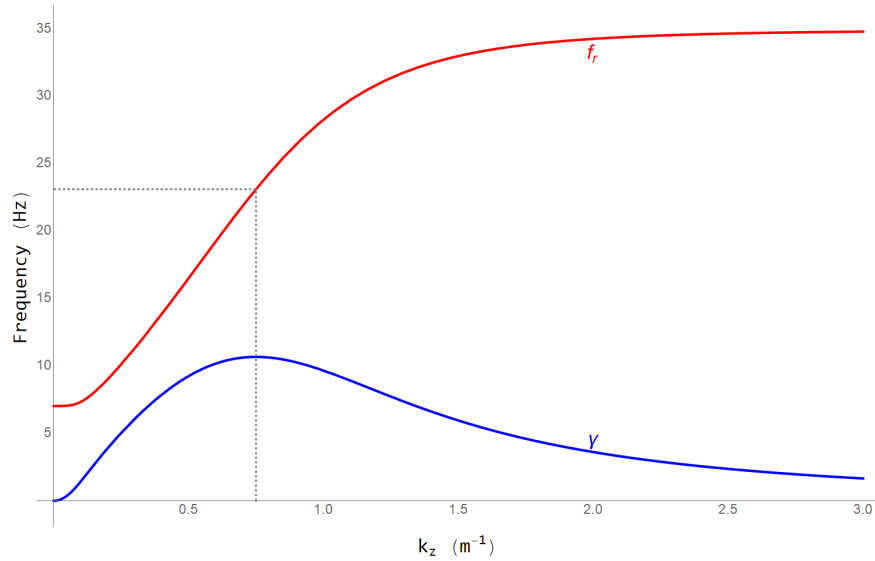


Figure 13: Real part of the frequency f_r (red) and growth rate γ (blue) predicted by the ADI as a function of axial wavenumber. For this model, $T_e = 6$ eV, $n_n = 10^{19}\text{m}^{-3}$, $v_D = 6$ km/s, and $r = 5.5\text{mm}$, corresponding to the peak of the oscillation. Dotted gray lines illustrate the correlation between maximum growth rate and predicted observable frequency.

The observable wave is at the axial wavenumber corresponding to the highest growth rate, i.e. $k_z \approx 0.75\text{m}^{-1}$. Looking at the real component of frequency at this value, this model predicts that we would observe a frequency of $f_r \approx 23\text{ kHz}$. As we have previously discussed, the frequency of the wave we observe experimentally is between 20-30 kHz, so the frequency predicted by our analysis lies within the range we have determined experimentally. Therefore, the collisional ADI we propose here is a strong candidate for the wave observed in MDX.

V. Discussion

The MDX nozzle confines a weakly-ionized plasma that is partially magnetized, highly collisional, and supports gradients in both the plasma potential and background density that can serve as sources of energy to drive instabilities. Our measurements of the background plasma parameters have revealed that the diamagnetic drift in the azimuthal direction is the dominant drift term. Our subsequent linear analysis has shown how this electron drift can provide energy to an anti-drift instability that is driven unstable by electron-neutral collisions. By employing a high-speed camera, we were able to find direct evidence that such a wave—an $m = 1$ mode in this case—propagates in the direction of diamagnetic drift in the nozzle plume. We make this inference with the caveat that, although we saw the azimuthal and radial character of the fluctuation, there were some inherent limitations to the physical insight yielded by our experimental diagnostics. Indeed, using the Fastcam involves a line integration down the axis such that the value of each pixel is not the intensity of a point in the plume, but rather an integration over a line extending from the source to the window. As such, the axial wavenumber remained a free parameter in our analysis, and we do not know at what axial positions the wave existed. It will be necessary in subsequent work to attempt to determine the value of the parallel wavenumber to definitively relate this oscillation to the anti-drift mode.

The physical significance of this mode may have direct implications for the steady-state operation of the nozzle, despite the oscillation being transient in nature. Indeed, this mode grows at the expense of the energy in the gradient-driven azimuthal drift. By conservation of energy, this growth of the instability in turn must act like a drag on the drift motion. Such an azimuthally directed drag in turn could give rise to cross-field electron transport. As cross-field motion is one of the key criterion for electron detachment from magnetic field lines, the ADI mode may in some measure contribute to detachment in the nozzle. In order to explore this possibility, there are a number of additional properties of the oscillation and the background plasma that must be understood. These include the axial character of the oscillation to resolve the uncertainty in the line-integrated visual characterization we have concurrently applied, as well as the application of a more sophisticated description of the oscillation's dispersion. Our work here to date, although yielding accurate predictions of the oscillation properties, has been Cartesian in nature. In reality, the oscillation has a global, cylindrical character. Exploring the implications of a more sophisticated, global model is reserved for a future study.

VI. Conclusion

Detachment is necessary for a magnetic nozzle to generate thrust, and instabilities have been previously shown to yield enhanced cross-field transport that is necessary for detachment.⁸ Determining the criteria for detachment will allow more effective magnetic nozzle-based thrusters to be designed. We have detected the presence of an instability by using a high speed camera to analyze the plasma in MDX. The plasma oscillations we observe exhibit a frequency between 20 and 30 kHz and an $m=1$ azimuthal mode that peaks halfway between the center and the liner radius. We measured plasma properties with electrostatic probes and presented values for $E \times B$ and diamagnetic drifts. We found that the diamagnetic drift is the dominant factor in azimuthal electron motion, and, assuming significant electron-neutral collisions, predicts the presence of an ADI with axial wavenumber of roughly 0.75 m^{-1} . The frequency we observe with the high speed camera matches well with ADI theory under the assumptions appropriate in MDX. We have discussed briefly the implications that this mode may have for promoting cross-field electron transport, and thereby a mechanism for detachment, in these devices. Indeed, the existence of this low-frequency instability in a magnetic nozzle may point to an important role of transients in promoting detachment in these low temperature, partially-magnetized nozzles.

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