

Kinetic Modeling of Plasma Plumes using Multi-GPU Forest of Octree Approach

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Three-dimensional particle-in-cell simulations are performed to study the characteristics of xenon plasma plumes and their interaction with the neutralizer electrons. To study the effect of electron source location on the plume dynamics and electron kinetics, two configurations are modeled, one with co-located electron and ion sources and the second with a shifted electron source. In the first case, the plume achieved a quasi-neutral state by the electrostatic trapping of electrons within the confined xenon ion plume. The electron temperatures were observed to be anisotropic and the plume characteristics were found to be radially symmetric about the plume axis. For the shifted electron source, however, no such symmetry was observed. As the xenon ion plume evolved, the electrons with high drift velocities experienced a bouncing effect and resulted in counter-streaming electron populations. A two-stream instability was observed for this case and was supported by observations of holes in the electron phase-space as well as bi-modal electron velocity distributions.

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Nomenclature

ρ	= Charge density distribution, C/m ³
e	= Elementary charge, C
n	= Number density, m ⁻³
ϕ	= Electric potential, V
ϵ_o	= Permittivity of free space
$\vec{E}$	= Electric field, V/m
m	= Mass of the charged species, kg
T	= Temperature, eV
λ_{do}	= Initial Debye length, m
ω_{pe}	= Electron plasma frequency, rad/s
R	= Radius of the source, m
Δt	= Timestep, s
u	= x-component of velocity, m/s
v	= y-component of velocity, m/s
w	= z-component of velocity, m/s
Subscripts	
e	= electron
i	= ion
o	= initial time
x	= x-component
y	= y-component
z	= z-component

I. Introduction

Electric propulsion (EP) devices generate thrust by accelerating ionized propellant using an electric or magnetic field, generating high exhaust velocities.¹ The exhaust plasma plume consists of high-speed ions and unionized neutrals, which undergo charge-exchange collisions to generate slow CEX ions.¹ These CEX ions are influenced by the electric field induced between the plume and the spacecraft surface, resulting in their backflow. Consequently, the backflow CEX ions impinge on solar panels and spacecraft surfaces, altering their surface properties as well as causing surface erosion and eventual failure. To prevent charge build-up within the plume, an external hollow cathode is mounted on the spacecraft to emit a beam of electrons for neutralizing the plume. The inclusion of these electrons in the modeling of plasma plumes is essential as it modifies the induced electric field and in turn affects plasma-spacecraft interactions. Furthermore, the location of the hollow cathode also plays an important role on the plume dynamics.² The focus of this work, therefore, is to study the effect of electron source location on the plume dynamics as well as the electron kinetics in a new computational framework.

The plume plasma density is small,¹ on the order of 10¹⁶/m³ and therefore, the continuum assumption becomes invalid and a kinetic approach is required to model the plasma and collision dynamics. Particle-In-Cell (PIC)³ is a well established kinetic approach used to model the

movement of charged species and their interactions with a self-consistent electric field. In this method, the charged and neutral species are modeled as computational macroparticles, wherein each macroparticle represents a large number of real ions, electrons, or neutrals. The computational domain is first discretized into grid cells and the charge density distribution is determined by mapping particles to the cells. The electric field is obtained by solving the Poisson's equation, and the magnetic field, if present, is evaluated by solving Maxwell's equations. Finally, the force exerted by the fields move the charged particles to new positions, generating a new charge distribution every timestep. The PIC approach has been widely used for modeling of ion and hall thruster plumes.⁴⁻⁹ Traditionally, in the PIC simulations of plasma plumes, the ions have been modeled as particles, and electrons are treated as a fluid,^{4,9,10} where the electron temperature is assumed to be isothermal. But, this assumption is inaccurate as per the anisotropic electron temperature observed from the fully kinetic simulations performed by Hu et al.¹¹ Modeling electrons as particles requires a small timestep, thereby increasing computational time. Therefore, efficient parallelization strategies and numerical approaches are required to optimize the simulation run-time of such kinetic approaches.

Previously, PIC tools have used a uniform Cartesian grid¹² to discretize the domain and compute the self-consistent electric and magnetic field. For problems with multiple length-scales, such as, the expanding plasma plume as well as plasma-surface interactions, the Debye length is smaller near the thruster exit or in the sheath region compared to the far-field. The use of a uniform grid with cell size less than the smallest Debye length will unnecessarily increase the number of cells in the domain and the computational cost. Therefore, an adaptive mesh refinement (AMR) or octree approach is employed¹³⁻¹⁶ to refine cells in the regions with small Debye lengths and coarsen in the regions where the Debye length is large. Solving Poisson's equation accurately on such AMR/octree grids is crucial to computing electric field in PIC, but, it requires strategies, different from the uniform grid approach, to accurately resolve the gradients across cells with different sizes. A number of Poisson solver libraries, such as, PETSC,¹⁷ Dendro,¹⁸ Deal.II¹⁹ are available for use on octree structures. A finite volume approach has previously been used to solve Poisson's equation on an octree structure.^{13,20,21} These approaches have used multiple distributed-memory CPU systems for parallelization by employing the MPI paradigm. The ability of GPUs to accelerate the performance of particle-based methods is well known. GPUs have also been used for solving Poisson's equation for PIC methods,²²⁻²⁵ but they have not been exploited for PIC modeling of plasma plumes with kinetic approach for electrons. In this work, we have extended the in-house MPI-CUDA solver, initially developed for flow through porous media, called Cuda-based Hybrid Approach for Octree Simulations (CHAOS), to perform 3D PIC simulations on an octree structure. This will allow us to understand electron kinetic behavior and plasma plume dynamics over many length scales.

Typically, the emission velocities of the ion plume from ion thrusters and the electrons from the hollow cathode are such that $(v_{te} \gg v_{ibeam} \gg v_{ti})$,²⁶ where v_{te} , v_{ibeam} , and v_{ti} , are the electron thermal, ion beam, and ion thermal velocities, respectively. Wang et al.,²⁶ and Hu et al.,^{11,27} have modeled such mesothermal plumes using 2D PIC simulations with a reduced ion-to-electron mass ratio of 1836, and co-located ion and electron sources. But, EP thrusters use xenon as propellant and the electron source is shifted from the thruster exit. Usui et al.²⁸ have modeled a mesothermal plume with a shifted electron source and observed an electron bouncing motion as well as non-Maxwellian electron velocity distribution. However, as mentioned earlier, the ions in these simulations were modeled with a reduced mass equal to that of a proton. For modeling of Hall

thruster plumes, there are still remaining challenges with respect to modeling electron transport across the magnetic-field. Recent work by Lafleur et al.²⁹ suggests that the classic fluid-electron formulation needs to be modified to take into account the collisionless wave-particle contribution.

In this work, we perform three-dimensional PIC simulations of a mesothermal xenon plasma plume with a kinetic approach for electrons, using a linearized octree structure for evaluating the electrostatic Poisson's equation. The PIC approach and parallelization strategies implemented in CHAOS are briefly discussed in Sec. II. To study the effect of electron source location on the plume dynamics, we have performed two test cases. In the first case, the electron and ion sources are co-located and in the second case, the electron source is shifted from the ion source. The problem set-up for the two cases are given in Sec. III. The plume characteristics obtained from the two cases are compared in Sec. IVA. In Sec. IVB, the electron kinetics for the co-located case is discussed, where we will see that the electron velocity distribution is anisotropic, even for a xenon ion plume. Finally, in Sec. IVC, the electron kinetics for the plasma plume simulation with a shifted electron source location is presented, that show the existence of a two-stream instability.

II. Particle-In-Cell Method using Multiple GPUs

The main steps in the PIC method include, computation of the charge density distribution, ρ , the self-consistent electric potential, ϕ , the electric field, $\vec{E}$, and finally, the forces that accelerate particles to new positions. The set of equations to model these physical processes are given below.

$$\rho = e(Zn_i - n_e), \quad (1a)$$

$$\nabla^2 \phi = -\frac{\rho}{\epsilon_o} \quad (1b)$$

$$\vec{E} = -\nabla \phi \quad (1c)$$

$$m \frac{\partial \vec{v}}{\partial t} = q \vec{E} \quad (1d)$$

The charge density is computed using Eq.(1a), where, e is the elementary charge, Z is the charge state (for example, Xe^+ has a single charge and therefore $Z=1$), n_i and n_e are the respective number densities of the ions and electrons. The electric potential, ϕ , is calculated by solving Poisson's equation, shown in Eq.(1b), where, ϵ_o is the permittivity of free space. The induced electric field, $\vec{E}$, is evaluated using Eq. 1c, and particles are moved to new positions using Eq. 1d. To solve the above equations, the computational domain is spatially discretized into a forest of octrees (FOT).³⁰

An octree is a three-dimensional hierarchical structure constructed to group nearest neighboring particles into leaf nodes.³¹ For large-scale simulations, the computational effort to construct the octrees is distributed among multiple processors, by first dividing the domain into roots and assigning one or more roots to each processor. The processors, in turn, perform recursive subdivision on their assigned root until a refinement criteria is satisfied, to form a forest of trees (FOT). The final leaf nodes are analogous to the cells in a uniform grid. For the PIC simulations, the plasma Debye length serves as the refinement criteria, such that, the final leaf node size is less than the local Debye length.

Additionally, to accurately solve the Poisson's equation, a 2:1 balance constraint^{32,33} is imposed on the leaf nodes of the FOT. This spatial constraint ensures that the size of any leaf node is at

maximum only one level coarser than any of its neighbors. In CHAOS, we impose the 2:1 criteria by comparing the leaf level with the face neighbor leaf level.³⁰ If the leaf node is two levels coarser than its finest face neighbor, it undergoes refinement until the 2:1 balance is satisfied. Face neighbors are the neighboring leaf nodes that share a common interface and in a balanced octree³⁴ every leaf node has a maximum of six faces. The FOT is linearized using a Morton Z-curve, meaning that, only the final leaf nodes of the FOT are stored. A unique Morton Id is assigned to each leaf node to identify its location in the vectorized data structure without storing the entire tree.³⁴ All the leaf nodes in the forest are equally distributed among the processors for load balanced domain decomposition,³⁴ after which, the leaf nodes with face neighbors in a different processor are flagged as *Z*-boundary leaf nodes and their respective face neighbors are flagged as ghost face neighbors.

Poisson's equation in Eq. 1b is discretized on the 2:1 FOT using the finite volume approach and transformed into a set of linearized equation of the form $Ax = b$, the details of which are given in Ref. 30. The charge distribution, ρ , is computed by mapping the ions and electrons to the leaf nodes using fast bit-wise computations, efficiently parallelized using GPUs.³⁴ A preconditioned conjugate gradient (PCG) approach^{35,36} is implemented to iteratively solve the linearized set of equations, using multiple GPUs. In CHAOS, the diagonal elements of the matrix, $Diag(A) = M^{-1}$, are used as a preconditioner in the PCG algorithm.³⁶ To save memory, only the non-zero elements of the matrix are stored on the GPU in a compressed sparse row matrix format. Each GPU stores the part of the matrix that corresponds to the leaf nodes in its sub-domain. Therefore, to perform sparse matrix vector operations required by the PCG algorithm, hybrid MPI-CUDA communications are invoked to transfer data between the *Z*-boundary leaf nodes and their respective ghost face neighbors.³⁰ The iterative PCG procedure is performed until the residuals are less than 1.0×10^{-5} , to obtain a converged solution for the self-consistent electric potential. The electric field is computed by evaluating Eq. 1c on the leaf nodes, using the central difference scheme.³⁷ Particle acceleration is determined based on the electrostatic force computed for the leaf node it is mapped to, and finally particles are moved to generate a new charge distribution. The details of the parallelization strategies implemented in CHAOS for the PIC module are described in our recent work.³⁰ The optimizations performed have resulted in near-ideal scaling efficiency with increase in the number of GPUs.³⁰ Even though electrons are modeled as particles in this work, the above 2:1 strategy and Poisson solver implementation can also be used for a fluid-electron model, where the mass, momentum and energy conservation equations are reduced to a Poisson-like equation.¹⁰

III. Problem Set-Up

To study the effects of hollow cathode location on the xenon plume and the electron behavior, we perform two test cases. In case 1, the ion and electron source are co-located as shown in Fig. 1(a), while in case 2, the electron source is shifted above the thruster exit, in the *y*-direction, as shown in Fig. 1(b). The initial simulation parameters for cases 1 and 2 are given in Table 1. For both the cases, the electrons are initialized with a temperature of $T_{eo} = 2$ eV and an initial number density, n_{eo} , of $1.0 \times 10^{13}/\text{m}^3$. This results in an initial Debye length, λ_{do} , of 3.32×10^{-3} m and electron plasma frequency, $\omega_{peo} = 1.78 \times 10^8$ rad/s. The radii, R , of the ion and electron sources in both the configurations, shown in Fig. 1, is equal to $18.5\lambda_{do}$, i.e., 0.0625 m, representative of real thruster devices,¹ and the size of the three-dimensional domain is chosen to be equal to $500\lambda_{do}$ to avoid boundary-effects.²⁶ For both the cases, the ion source center is at (0.8,0.8,0.0) m, and only for case 2, the electron source, with its center at (0.8,0.925,0.0) is shifted from the ion source center

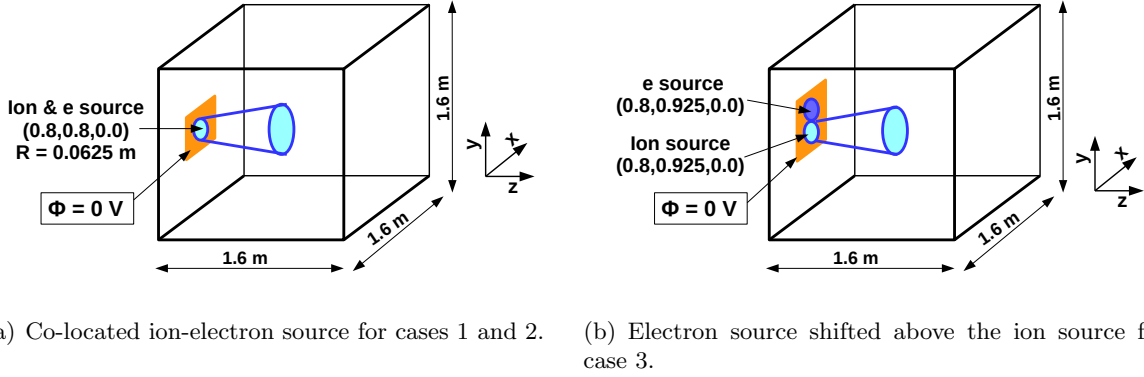


Figure 1: Simulation set-up for the mesothermal plasma plume cases with domain size of $(1.6 \times 1.6 \times 1.6)$ m and location of ion and electron sources. A Dirichlet boundary condition of $\phi = 0$ V is implemented in the highlighted orange region at the inlet plane, surrounding the radial sources. At all other boundaries and within the circular source region, a homogeneous Neumann boundary condition is applied.

by one diameter length in the y-direction, as shown in Fig. 1(b).

The ratio of initial ion temperature, T_{io} , to initial electron temperature, T_{eo} , used in the simulations is 0.01 and the beam velocity, v_{ibeam} is taken to be $0.05v_{teo}$, where v_{teo} is the thermal velocity of the electrons for an initial temperature of T_{eo} , similar to the mesothermal set-up performed by Wang et al.²⁶ The initial ion number density for case 1 is such that the ion and electron current density at the co-located source is equal. For case 2, on the other hand, the initial ion and electron number densities are equal such that the initial charge at the respective sources are equal. The simulations are performed for a duration of $t\omega_{peo} = 3500$, in order to resolve the electron time scales,²⁶ and a uniform timestep resolution of $\Delta t = 2.8 \times 10^{-10}$ s is used for both, ions and electrons, such that, $\Delta t \times \omega_{peo} = 0.05$.²⁶ At every timestep, ions are emitted with a Maxwellian distribution corresponding to temperature T_{io} , and a bulk velocity, v_{ibeam} , along the z-direction. The electrons, however, are introduced every timestep with velocities sampled from a stationary Maxwellian distribution with temperature T_{eo} , and no bulk component. To model the thruster surface surrounding the ion and electron source, a Dirichlet boundary condition with $\phi = 0$ V is applied in the region surrounding the sources, as shown by the highlighted orange region in Fig. 1. On all the other boundaries, a homogeneous Neumann boundary condition, $d\phi/dn = 0$, is implemented to model the zero electric field for plume expansion into the vacuum of space. The electric potential, electric field, and the resulting acceleration of the charged particles are computed on the E-FOT using the methodology discussed in Sec II.

IV. Results and Discussion

The unsteady evolution of the plume dynamics for cases 1 and 2, obtained from the CHAOS PIC simulations are compared in this section. The effect of electron source location on their velocity distributions are also analyzed.

Table 1: Parameters for Mesothermal Plasma Plume Simulations

Cases	2 - Xenon Plasma Co-located e⁻ Source	3 - Xenon Plasma Shifted e⁻ Source
n_{i0} (/m ³)	4.0×10^{13}	1.0×10^{13}
m_i/m_e	239,669.5	239,669.5
v_{beam} (m/s)	30,000	30,000
$n_{eo}=1.0 \times 10^{13}/\text{m}^3$, $\omega_{peo}=1.78 \times 10^8\text{rad/s}$, $T_{eo}=2$ eV, $v_{teo}=592,892$		

A. Effect of Electron Source Location on the Plume Dynamics

The instantaneous macroparameters of the unsteady plume are extracted along the y-z plane passing through the center of the domain. Figure 2 shows the transient evolution of the ion and electron charge density distribution at plasma times $t\omega_{peo}=1000$, 1500, and 3000 for the co-located electron-ion source plume. The leading edge of the ion beam, called beam-front, is located at $z=0.18$ m at $t\omega_{peo}=1000$, as shown by the dotted line in Fig. 2(a). Initially, the electrons with a higher velocity than the ions, travel beyond the beam-front, leaving behind a positively charged plume. This positively charged plume electrostatically traps the negatively charged electrons introduced in the subsequent timesteps and these trapped electrons now move along with the ion beam. The electron charge density distribution at the corresponding plasma time, $t\omega_{peo}=1000$, shows that the electrons are electrostatically confined by the ion beam and only a few energetic electrons escape beyond the ion beam-front. As time progresses, the ion beam-front advances to $z=0.28$ and 0.6 m at times $t\omega_{peo}=1500$ and 3000 , as shown by the dotted line in Figs. 2(b) and 2(c), respectively. From the corresponding electron charge density distribution shown in Figs. 2(e) and 2(f) we observe that the electrons move along with the ion beam. At plasma time, $t\omega_{peo}=3000$, the total charge density distribution within the plume is found to be quasi-neutral.

The electric potential induced by the distribution of the ion and electron charge densities, at plasma times $t\omega_{peo}=1000$, 1500 and 3000 for case 1 is shown in Fig. 3. At plasma time $t\omega_{peo}=1000$, the potential within the plume near the thruster exit, $z < 0.04$ m is nearly 5 V, which is lower than the potential at the plume-edge, 20 V, as shown in the zoomed-in view in Fig. 3(a). This drop in potential is caused by the electrons that are electrostatically trapped in this region by the xenon ion beam. As time increases to $t\omega_{peo}=1500$, the maximum potential of the plume increases to 25 V and the electrons that were trapped move along with the xenon ion plume. At plasma time $t\omega_{peo}=3000$, the electric potential is maximum, 80 V, near the thruster exit and decreases as we move downstream towards the beam-front, as shown in Fig. 3(c). At all plasma times, the charge distributions and electric potential are symmetric about the plume center, $z=0.8$ m, as observed in Figs. 2 and 3, respectively.

The transient electron and ion charge density distribution for case 2 at plasma times $t\omega_{peo}=1000$, 1500, and 3000 are shown in Fig. 4. The electron source location for case 2 is shown by the grey line on the inlet plane, and the ion source location is highlighted by the black line, and is shown in all the subsequent figures for case 2. The beam-front is located at $z=0.2$ m at plasma time $t\omega_{peo}=1000$, as shown by the dotted line in Fig. 4(a). As soon as the electrons are emitted from their source location, they are electrostatically attracted towards the ion beam. This is shown by an increase in the negative charge density close to the ion source compared to the electron

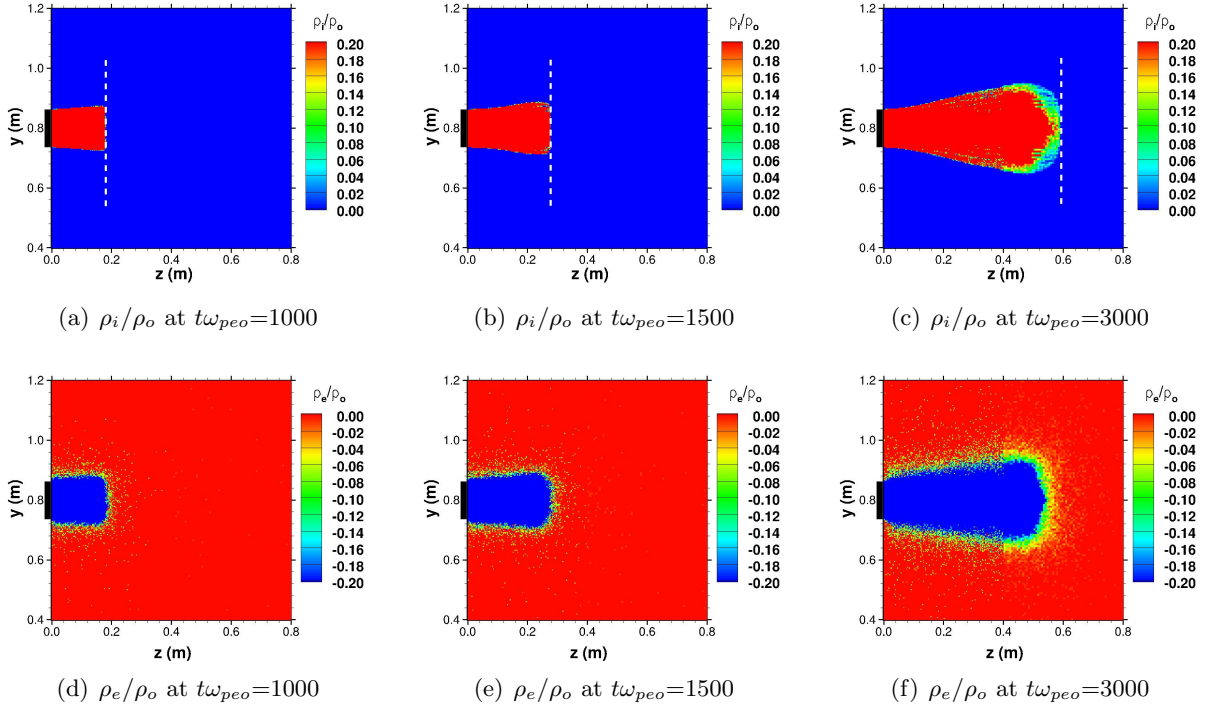


Figure 2: Transient ion and electron charge density distribution for case 1, along the y-z plane extracted at the center of the domain, normalized by $\rho_o = en_{eo}$.

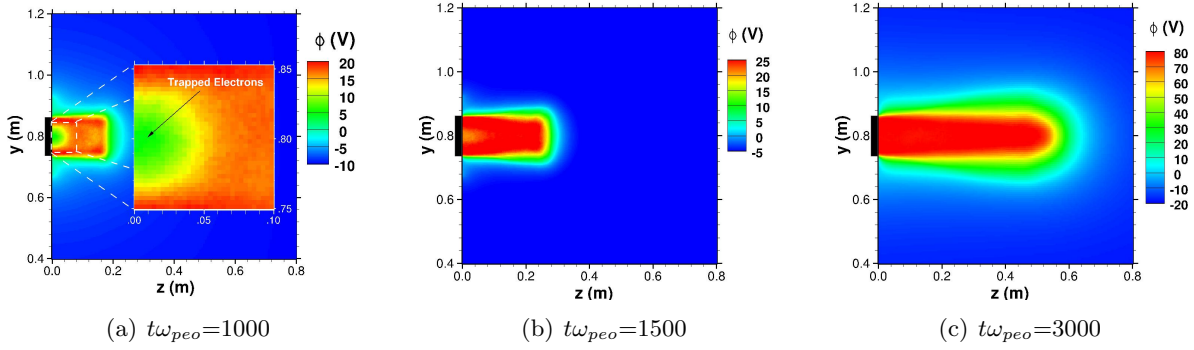


Figure 3: Spatial distribution of electric potential obtained for case 1 along the Y-Z center-plane.

source, in Fig. 4(d). Due to the initial electrostatic force in the $-y$ direction exerted by the induced electric field, the electrons gain a high velocity and overshoot the radial edge of the xenon beam, resulting in a positively charged plume center. This positively charged plume center then attracts the electrons from the radial edge, reversing the electron direction along the positive y -direction. Due to their high y velocity component and low mass, the electrons constantly overshoot the plume center, and the corresponding changes in the electrostatic forces continuously attract the electrons towards the center. This bouncing motion causes the electron concentration to increase at the edge

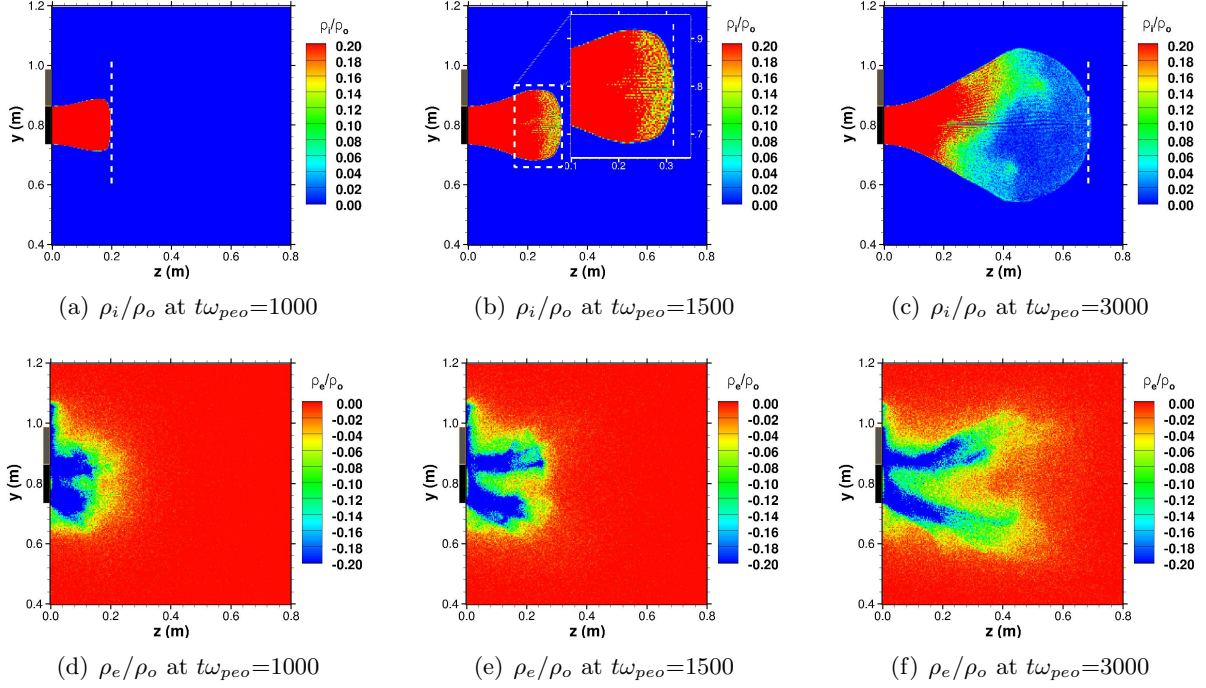


Figure 4: Transient ion and electron charge density distribution for case 2, along the y-z plane extracted at the center of the domain, normalized by en_{eo} .

of the plume as compared to the center, as shown in Fig. 4(d). In contrast, for case 1 at the same plasma time, $t\omega_{peo}=1000$, the electron density is higher at the plume center compared to the plume edge, as shown in Fig. 2(d). Due to the decrease in electron charge density at the plume center, the electrostatic forces, in case 2, attract the electrons towards the plume center and repel the ions away from the plume center, causing the ion beam to split, which can be observed at plasma times $t\omega_{peo}=1500$ and 3000 in Figs. 4(b) and 4(c). On the contrary, the plume structure for case 1 is radially symmetric as shown in Fig. 2(a) to 2(c). The electrons in case 2 continue to overshoot the center increasing the charge density at the plume edge, as shown by bifurcation of the electron charge density distribution in Figs. 4(e) and 4(f).

The electric potential induced by the ion and electron charge density distributions at plasma times $t\omega_{peo}=1000$, 1500 , and 3000 is shown in Fig. 5. The electric potential at plasma time $t\omega_{peo}=1000$ is higher within the plume compared to the plume edge, as shown in Fig. 5(a), unlike the drop in electric potential within the plume observed for case 1 in Fig. 3(a). As time progresses to $t\omega_{peo}=1500$, the maximum potential observed within the plume is 200 V, as shown in Fig. 5(b) and the plume is not axially symmetric about the plume center. At plasma time $t\omega_{peo}=3000$, the potential is observed to decrease beyond $z > 0.2$ m, because the ion charge density splits at the plume center beyond this region, as shown in Fig. 4(c), thereby decreasing the potential.

The spatial variation of the streamwise component of ion velocity, w_{ion} , at plasma time $t\omega_{peo}=3000$, normalized by the initial beam velocity, $v_{ibeam}=30$ km/s, for cases 1 and 2 is shown in Figs. 6(a) and 6(b), respectively. It can be observed that for case 1, the ion velocity is maximum only near the leading edge of the plume, i.e., $z > 0.5$ m, at plasma time $t\omega_{peo}=3000$, and the ion velocity is

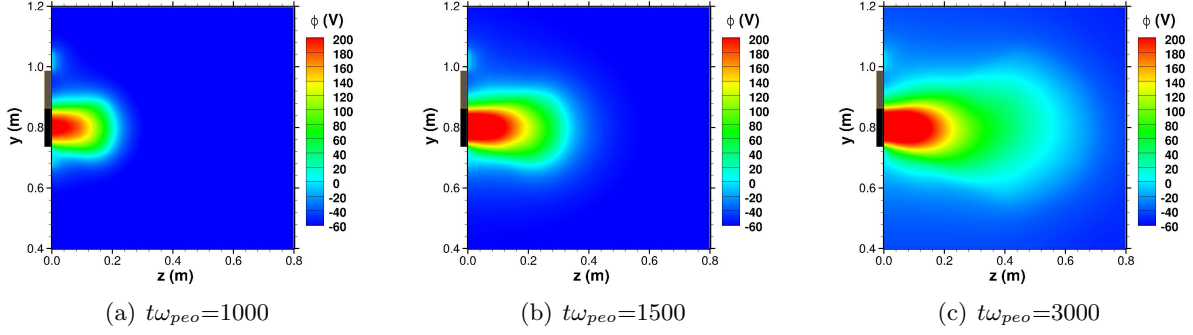


Figure 5: Spatial distribution of electric potential obtained for case 2 along the Y-Z center-plane.

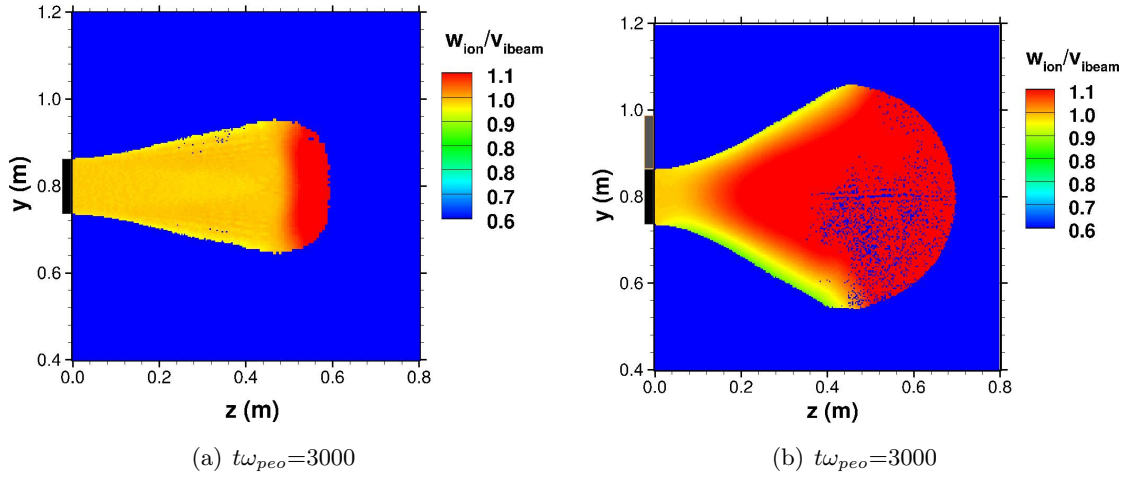
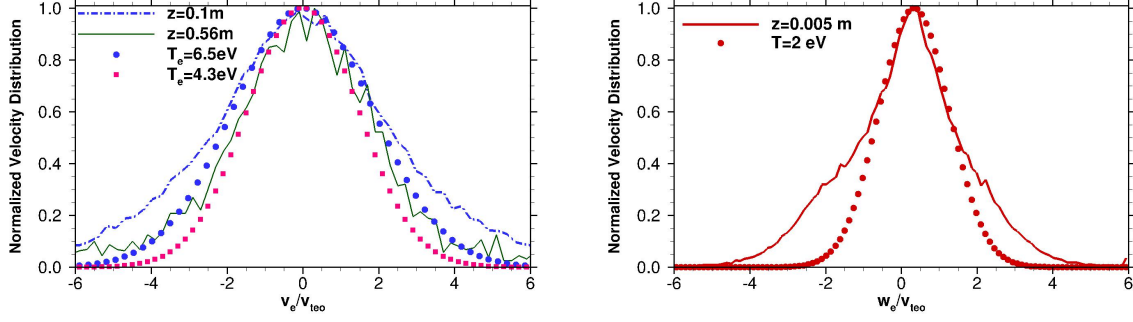


Figure 6: Transient streamwise ion velocity variation, w_{ion} , normalized by the beam velocity, $v_{beam}=30,000$ m/s for case 2

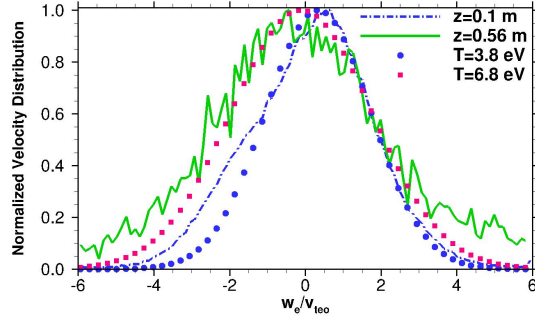
symmetric about the plume axis as well as radially uniform. On the contrary, for case 2, the ion velocity is not symmetric and not radially uniform as shown in Fig. 6(b). The velocity is higher than the initial beam velocity at the plume center because of the repulsion caused by the depression in the electron charge density at the plume center, as shown in Fig. 4(f). Beyond $z > 0.4$ m, the ion charge density at the plume center decreases to zero in some regions, as observed in Fig. 4(c), which results in zero streamwise velocity in these regions, indicated by the blue streaks, (zero ion velocity), at the plume-center in Fig. 6(b)

B. Electron Kinetics - Case 1

The instantaneous electron velocities are sampled at $z = 0.005$, 0.1 , and 0.56 m, within a radius of 0.05 m about the plume axis, for plasma time, $t\omega_{peo}=3000$. In Fig. 7(a), the y-velocity distributions sampled at $z = 0.1$ and 0.56 m are compared with analytical Maxwell-Boltzmann (MB) distributions,



(a) y-velocity component at $z=0.1$, (blue), and 0.6 m, (b) z-velocity component at $z=0.005$, and analytical distribution at $T_e=2$ eV. For the analytical distribution, $T_e=6.5$ and 4.3 eV for $z=0.1$ and 0.56 m, respectively.



(c) z-velocity component at $z=0.1$, (blue), and 0.56 (green) m. For the analytical distribution, $T_e=3.8$ and 6.8 eV for $z=0.1$ and 0.56 m, respectively.

Figure 7: Comparison of case 1 electron velocity distribution sampled from the simulation, (solid lines), at plasma time $t\omega_{peo}=3000$, with the analytical distribution, (dotted symbols), at locations $z=0.005$, 0.1 , and 0.6 m. The temperature used to obtain the analytical distribution is specified in the legend.

such that, the electron temperature used to generate the MB distribution reproduce the sampled distributions. It is observed that T_{ey} at $z=0.1$ is 6.5 eV, which is higher than the initial electron temperature of 2 eV. The electrons are trapped within a confined xenon ion plume, as shown for case 1 in Fig. 2, which causes their thermal spread to increase, thereby increasing the cross-stream component of the temperature, at $z=0.1$ m. Further downstream, the beam expands as observed in Fig. 2(c) and as a result, the T_{ey} decreases to 4.3 eV at the beam-front, i.e., $z=0.56$ m. From Fig. 7(a) it can be seen that the peak of the y-velocity distribution is located at 0 m/s, meaning that the net cross-stream velocity is zero. Additionally, the sampled x and y electron velocity distributions at these locations were found to be equal, suggesting that $T_{ex}=T_{ey}$ for case 1.

The z-velocity distributions, compared with the MB distributions, are ‘near-Maxwellian’ at all the sampled locations as shown in Figs. 7(b) and 7(c). In contrast to the decrease in T_{ey} downstream from the thruster exit, the comparison of the sampled z-velocity distribution with the MB distribution shows that T_{ez} increases from 2 eV at $z = 0.005$ m to 3.8 eV at $z = 0.1$ m and finally to 6.8 eV at $z = 0.56$ m, where the beam-front is located, as shown earlier in Fig. 2(c). Close to the

thruster exit, $z = 0.005$ m, the electron energy from the thermal mode is converted to kinetic energy of the electron in the streamwise direction. As a result, the electrons that were initialized with a zero bulk velocity at the thruster exit, $z = 0.0$ m, now propagate with a bulk velocity of 200 km/s, shown by the velocity corresponding to the peak location in Fig. 7(b). Further downstream, at $z = 0.1$ m, the temperature, T_{ez} , increases due to the electron trapping by the positively charged plume, which reduces the bulk velocity to 140 km/s, shown by the shift in the peak location in Fig. 7(c). At the beam-front, $z = 0.56$ m, the bulk z -velocity of electrons decreases to zero as shown by the MB peak location in Fig. 7(c). Since the bulk z -velocity of the electrons at $z = 0.1$ m was 140 km/s and the beam propagates at only 30 km/s, the electrons that traveled beyond the beam-front are electrostatically attracted back towards the positively charged beam-front, increasing the thermal spread at $z = 0.56$ m and T_{ez} to 6.8 eV as shown in Fig. 7(c). The electron temperature is anisotropic because the sampled velocity distribution resulted in T_{ez} values which are not equal to T_{ex} and T_{ey} .

C. Electron Kinetics and Two-Stream Instability - Case 2

The transient evolution of the electron y -velocity, v_e , normalized by the initial thermal velocity, v_{teo} , at plasma times, $t\omega_{peo} = 1000, 1500$, and 3000 is shown in Figs. 8(a), 8(b) and 8(c), respectively. It can be seen that the electron velocity is higher than the initial thermal velocity, and as the plume advances in the streamwise direction, the electrons bounce in the y -direction with high bulk velocity. This high bulk velocity caused the electron charge density depression at the plume center as shown earlier in Fig. 4(f). This oscillatory motion, known as bouncing effect,²⁸ is also observed for the streamwise velocity component, w_e , shown in Fig. 8(d) to 8(f). The streamwise velocity in Fig. 8(d) is found to be negative at the plume center region close to $z = 0.16$ m, whereas, upstream of this region, the electron velocity is positive, with the same magnitude of $1.5v_{teo}$. This shows that, for case 2, the bulk velocity is larger than the initial electron thermal velocity and that two populations of electrons are traveling in opposite directions with the same intensity, also known as, counter-streaming electrons. As the electrons bounce in both the y and the z directions, the regions with positive and negative streamwise velocity evolve, as shown in Figs. 8(e) and 8(f). The counter-streaming electron populations are more prominent at $t\omega_{peo} = 3000$, shown in Fig. 8(f), where, upstream of $z < 0.18$ m, the electron velocity, w_e , is negative above and positive below $y = 0.86$ m, while downstream, for $z > 0.18$ m, the direction of w_e has flipped about the plume axis. From the analytical stability analysis performed by Stringer,³⁸ it is known that for an electrostatically counter-streaming plasma with $T_e \gg T_i$, the above conditions of v_e and w_e being higher than v_{teo} satisfy the criteria for an electron two-stream instability. Even though, a similar electron trapping was observed in case 1, the net drift velocity of the electrons for the co-located case was much less than the thermal velocity, and therefore did not lead to regions with counter-streaming electron populations, i.e., conditions for the two-stream stability were not achieved.

To further analyze the existence of a two-stream instability, the electron evolution in the phase-space is shown in Fig. 9, where the streamwise drift velocity of electrons, normalized by the initial v_{teo} , is plotted on the y -axis and the position of the electrons is plotted on the x -axis. The electron drift velocity in this work is defined as their bulk velocity, since, the difference between the electron and ion bulk velocities are negligible for the mesothermal xenon ion plume. Figure 9(a) shows that at time $t\omega_{peo} = 1000$, the electrons have a negative velocity lower than $-1.5v_{teo}$ in the region between $z = 0.1$ and 0.2 m. Just upstream of this region, i.e., $z < 0.1$, the net flow of electrons is in the $+z$ -direction, which is also observed from Fig. 9(a). As the electrons bounce in y and z -directions

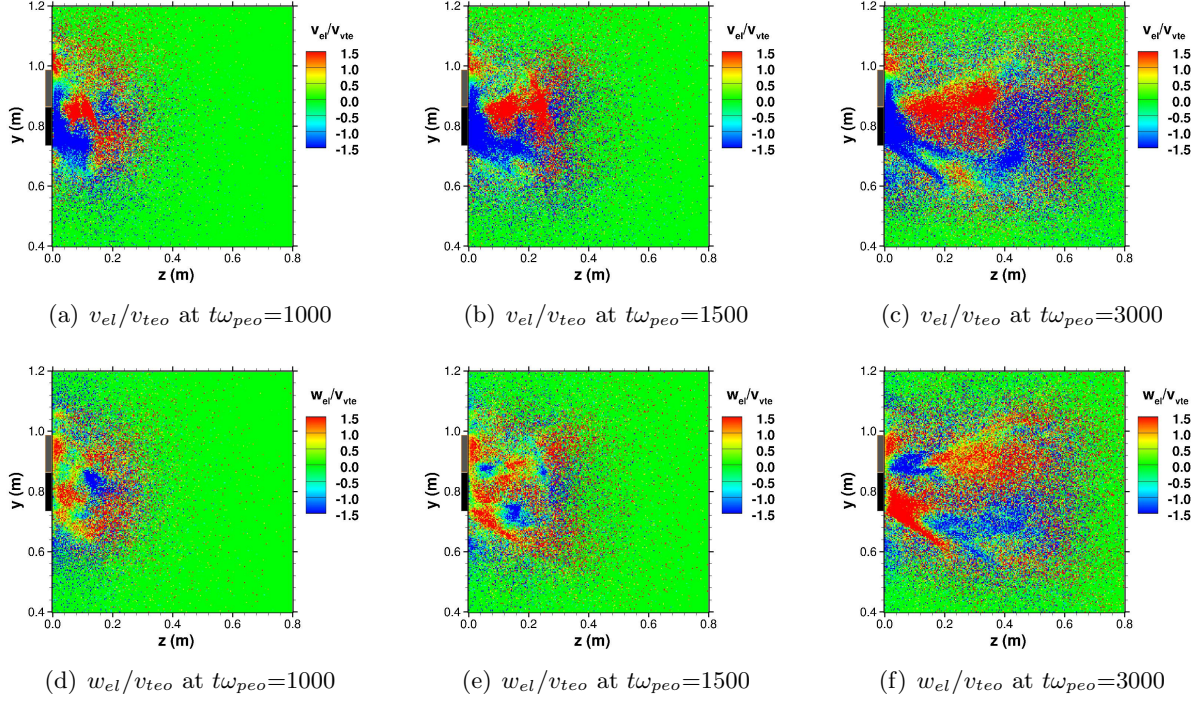


Figure 8: Transient evolution of case 2 electron velocity distribution normalized by electron thermal velocity (v_{te0}), exhibiting oscillations in the y - and z -direction.

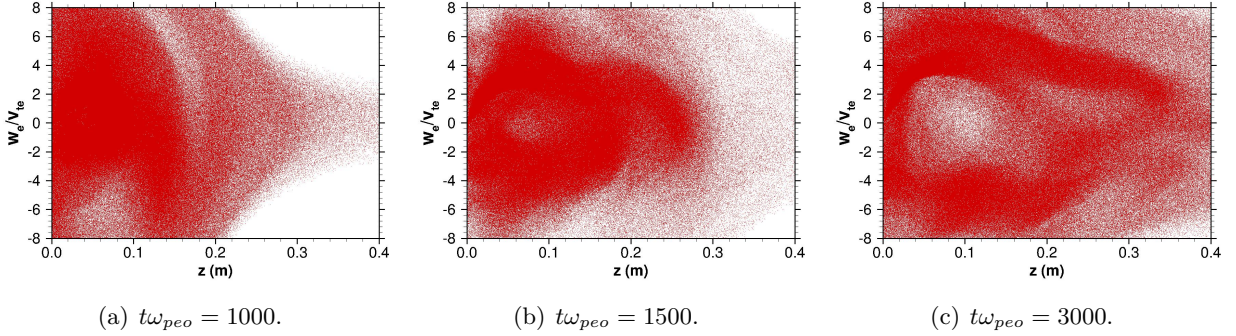
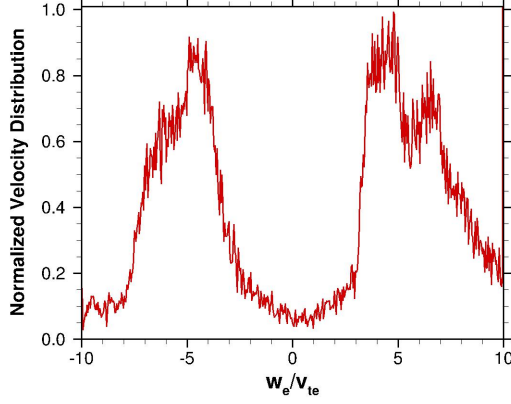


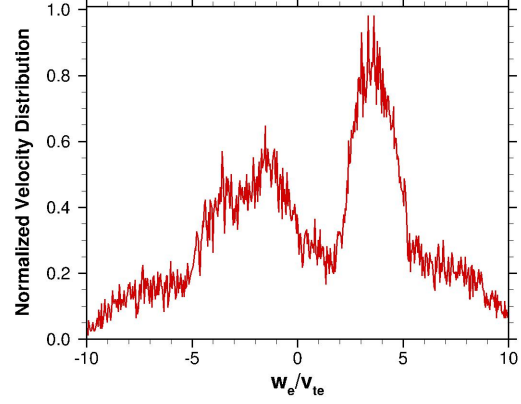
Figure 9: Evolution of electron distribution in phase-space showing the development of holes that indicate a two-stream instability for Case 2.

and circulate with very high velocities, vortices or ‘holes’ in the phase-space are formed, which are typically observed in a two-stream instability.³⁹ This hole can be clearly seen at $z=0.1$ m for plasma time $t\omega_{pe0}=1500$, in Fig. 9(b). At plasma time $t\omega_{pe0}=3000$, we observe two holes, one at $z=0.1$ m and the second at $z=0.26$ m, which strengthens the hypothesis for the existence of a two-stream instability for case 2.

The electron streamwise velocity distribution was sampled at $t\omega_{pe0}=3000$ at the locations of the phase-space hole, $z=0.1$ and 0.26 m, shown in Fig. 9(c). This distribution is bi-modal as shown in

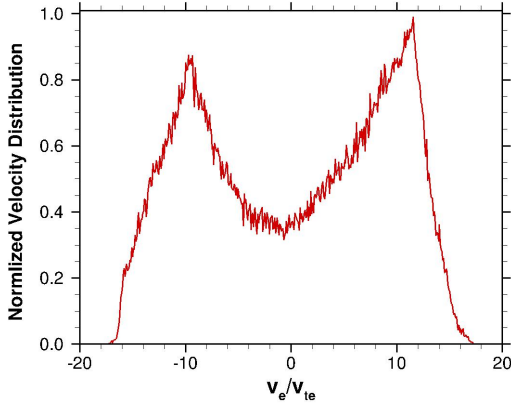


(a) At $t\omega_{peo}=3000$.

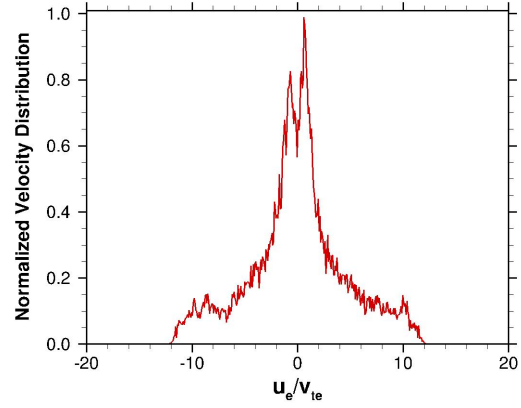


(b) At $t\omega_{peo}=3000$.

Figure 10: Transient evolution of normalized w_e/v_{teo} distribution function at $z=0.1$, where a hole in phase-space is located (Fig. 9).



(a) y-velocity, v_e/v_{teo}



(b) x-velocity, v_e/v_{teo}

Figure 11: Cross-stream velocity-components at $z=0.1$, where a hole in phase-space is located, at plasma time $t\omega_{peo}=3000$.

Figs. 10(a) and 10(b), which is in contrast to the single peak electron velocity distribution obtained for case 1, shown in Fig. 7(c). The peaks of the two modes are located at $w_e/v_{teo} = -5$ and 5 at $z=0.1$ m as shown in Fig. 10(a). This suggests that half the electron population has a bulk velocity of $5v_{teo}$ and the other half is counter-streaming with bulk velocity of $-5v_{teo}$. A similar counter-streaming bi-modal distribution is also observed for the electron streamwise velocity component at $z=0.26$ m, shown in Fig. 10(b). At the second phase-space hole location, i.e., $z=0.26$ m, the peak bulk velocities are lower, i.e., $w_e/v_{teo}=3.5$ and -1.5 as shown in Fig. 10(b), compared to the peak velocities observed for electrons at $z=0.1$ m. This decrease in the peak velocities is also supported by the smaller phase-space hole at $z=0.26$ m compared to that at $z=0.1$ m, as shown in Fig. 9(c).

The electron bouncing effect in the y-direction, observed earlier, results in a bi-modal y-velocity

distribution at $z=0.1$ m, shown in Fig. 11(a) even for plasma time $t\omega_{pe0}=3000$. The x-velocity distribution, however, has a single peak at zero bulk velocity and an enhanced tail, as shown in Fig. 11(b). If the hollow cathode source was shifted in the x-direction, the bouncing effect would have been more pronounced in the x-direction compared to the y-direction. In contrast to the axial symmetry observed for the ‘near-Maxwellian’ velocity distributions for case 1, shifting the electron source location in the y-direction, resulted in significantly different x- and y-velocity distributions for case 2.

V. Conclusion

Mesothermal plasma plume simulations were performed with co-located electron-ion sources and a shifted electron source, to study the effect of electron source location on the plume dynamics and electron kinetic behavior. For the co-located case, the plume achieved quasi-neutrality by electrostatically confining the electrons within the plume. This trapping increased the electron temperature close to the thruster exit compared to the initial temperature of 2 eV. Downstream from the thruster exit, cross-stream electron temperature decreased due to expansion of the plume. However, the streamwise electron temperatures were not equal to the cross-stream components, resulting in anisotropic temperature for the electrons. Similar anisotropy was also observed by Hu et al.²⁷ in their 2D proton plasma simulations, but compared to the proton plume, the xenon ion plume with higher mass is more confined³⁰ and therefore has even higher electron temperatures.

The plume structure for the shifted electron source case showed bifurcation of the ion and electron charge density, where the densities were higher at the plume edge compared to the center. The shift in the electron source location resulted in an initial electrostatic force, that caused counter-streaming electron populations with high drift velocities and a bouncing effect. The electron drift velocities were observed to be higher than the initial thermal velocities, which satisfied the criteria for a two-stream instability in a plasma with $T_e \gg T_i$. Holes in the electron phase-space strengthened the hypothesis of the electron two-stream instability for the second case, which was further supported by non-Maxwellian electron velocity distributions, sampled at the location of the holes. As a next logical step, the PIC module will be coupled with the DSMC module to model both, plasma and collisions dynamics and predict the effect of neutralization on the ion energy distribution in the backflow region.

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