

Development of a Hall Thruster Fully Kinetic Simulation Model Using Artificial Electron Mass

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A fully kinetic simulation model using artificial electron mass was developed for Hall thrusters. The numerical speeding up technique of artificial mass ratio was employed as the only-one unphysical manipulation. Unlike the previous study, the mass of electrons but not the heavy particles was changed and a new original mass ratio model was used. The simulation of a laboratory model Hall thruster with different mass factor cases were conducted, and the result of plasma property distributions and electron energy distributions (EEDF) were investigated and discussed. The results suggests that the simulation results were consistent with different mass ratio cases; the non-Maxwellian nature of EEDF was captured; and the mass ratio model can be further improved in accuracy and self-consistency by the consideration of energy relaxation of electrons.

Nomenclature

Roman:

B	=	Magnitude of magnetic flux density
B_e	=	Magnitude of magnetic flux density applied for electrons in the mass ratio model
C	=	Constant
E	=	Magnitude of electric field
f	=	Mass acceleration factor
f_B	=	Magnetic flux density correction factor for the mass ratio model
h	=	Channel width
k	=	Boltzmann constant
L	=	Characteristic length of acceleration zone
m	=	Mass
m_e	=	Electron mass
m_i	=	Ion mass
N	=	Number of macro simulation particle
r_L	=	Larmor radius
T_e	=	Electron temperature

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t_E	=	Characteristic time of electron $\mathbf{E} \times \mathbf{B}$ drift
U_d	=	Discharge voltage
v	=	Speed
v_{th}	=	Electron thermal speed
v_E	=	Characteristic speed of electron $\mathbf{E} \times \mathbf{B}$ drift
Greek:		
α	=	Ratio of Bohm diffusion collision frequency against scattering one
$\mathcal{E}$	=	Electron kinetic energy
ν^*	=	Non-dimensional characteristic relaxation frequency
ν_{Bohm}	=	Bohm diffusion collision frequency
$\nu_{Scatter}$	=	Scattering diffusion collision frequency
ν_{tot}	=	Total diffusion collision frequency
σ_{en}	=	Total cross-section for electron-neutral collisions
Ω_e	=	Electron Hall parameter

I. Introduction

NUMERICAL modeling of Hall thrusters is critically important for optimization of a thruster and understanding the physics. Namely, the measurement of plasma properties inside the discharge channel is uneasy either by probe or by spectroscopy because of the high energy density, steep spatial gradient, and measurement interference. Particularly, numerical simulation is effective for Hall thruster lifetime study due to the difficulty of near-wall measurement and immense cost of long time endurance test.

A number of numerical works have been performed and different models have been developed for Hall thruster modeling. So far, the most successful and most used numerical model is Hybrid Particle-In-Cell (PIC) model. Especially the quasi-one-dimensional hybrid PIC model called HPHall¹ was employed by different institutes because of its accuracy and moderate computational cost. Recently, even a fully-fluid model was developed and applied to simulation of long-life Hall thruster². However, it is necessary for these fluid-based models to assume quasi-neutrality and Maxwellian electron energy distribution function (EEDF). These assumptions make it impossible to conduct self-consistent wall sheath modeling, despite of their significant impact on the discharge and wall erosion.

On the contrary, different kinetic approaches had also been developed to overcome the drawbacks of fluid-based models. Fully Kinetic PIC Direct Simulation Monte Carlo (DSMC) model is one of the most widely used kinetic approach, and is the target of this study. In fully kinetic PIC DSMC models, all species of the particles including the electron is tracked and simulated directly, which makes the simulation extremely computationally expensive. Consequently, the computational cost is the major drawback of full-PIC models which prevent them from being widely used despite their effectiveness. Specifically, it is notable that there was no successful Hall thruster lifetime simulation performed so far. Therefore, the purpose of this study is to develop a r - z 2D3V PIC-DSMC model which has the capability of fully kinetic numerical modeling of Hall thruster lifetime.

Different acceleration techniques were developed in previous studies to reduce the full-PIC model's computational cost. The artificial mass ratio and permittivity manipulation are the most used speeding-up technique³. On the other hand, Taccogna⁴ suggested a simulation acceleration model by scaling down the size of the thruster according to a self-similarity law. Each technique reduces the computational cost at the expense of neglecting certain physics, and it is well known that implementing multiple techniques simultaneously can cause severe unrecoverable change of physics. Because it is considered to be impracticable to accomplish the full-PIC lifetime simulation within an acceptable computational cost without implementing any acceleration technique, we adopted the artificial mass ratio as the only-one unphysical manipulation in this study.

There are two ways of implementing artificial mass ratio: one is to reduce the mass of heavy particles to accelerate the convergence of simulation³; and the other is to magnify the electron mass to reduce the plasma frequency and afford greater numerical time-step⁵. The former one is widely used, though it was not adopted in our model because the wall-hitting ion loss flux is theoretically unpreserved. Therefore, the artificial electron mass model was adopted in this study using a new physics recovering model.

Simulation was conducted for a laboratory model magnetic layer type Hall thruster with different mass ratio cases, and the plasma property distributions and EEDF were investigated and discussed in this paper. The detailed explanation of the model, boundary treatments, and simulation results of thrust performances can be found in Ref. 6.

II. Numerical Methods

A. Mass ratio model

The mass ratio of the heavy particles and the electrons were artificially changed to speed up the simulation as:

$$\left(\frac{m_i}{m_e}\right)_{real} = f^2 \left(\frac{m_i}{m_e}\right)_{artificial} \quad (1)$$

Because the plasma of Hall thrusters is magnetically confined only for electrons but not for ions, there are two different ways of implementation. One is to reduce the mass of heavy particles by the factor of $1/f^2$ to accelerate the convergence of simulation, and the other is to magnify the electron mass by the factor of f^2 to reduce the plasma frequency and afford larger time-step. Both methods need physics recovering model, and have their own merits and demerits. Note that the intermediate model (e.g. $1/f$ times heavy particle mass and f times electron mass) was yet to be investigated.

Because we would like to preserve the electron-driven nature of Hall thruster plasma, our choice is the later one to magnify the electron mass. The physics recovering model⁶ can be written as:

$$B_e \rightarrow \dot{B}_e = f_B B_e \quad (2)$$

$$\sigma_{en} \rightarrow \sigma'_{en} = f \sigma_{en} \quad (3)$$

$$\nu_{scatter} \rightarrow \nu'_{scatter} = (f_B/f)^2 \nu_{scatter} \quad (4)$$

Where, B_e is the magnetic flux density only applied for electrons, σ_{en} is the total cross-section for electron-neutral collisions, $\nu_{scatter}$ is the electron-neutral scattering collision frequency, and the dashed variables denotes the ones scaled by the mass ratio model. Different from original model⁵, the factor f_B was introduced for the preservation of electron mobility, and its value can be decided to satisfy the fundamental requirements of Hall thrusters in the scaled system:

$$\dot{\Omega}_e = \frac{e \dot{B}_e}{\dot{m}_e \nu'_{scatter}} = \frac{1}{f_B} \Omega_e \gg 1 \quad (5)$$

$$\dot{r}_L = \frac{\dot{m}_e \dot{v}_\perp}{e \dot{B}} = \frac{f}{f_B} r_L \ll L \quad (6)$$

Where, Ω_e is the electron Hall parameter r_L is the electron Larmor radius, and L is the characteristic length of the acceleration zone. The basic idea of this model is to preserve the physics of electron transport across the magnetic field in the scaled system at the expense of unpreserved electron Hall parameter and Larmor radius with limited change. Note that the model goes back to the conventional one by employing $f_B/f = 1$, which can result in the violation of Eq. 5 at the near-anode region of magnetic layer type Hall thrusters.

B. Collisions

Inter-particle collisions were modeled by Direct Simulation Monte Carlo (DSMC). Four kinds of major particles; ions, doubly charged ions, electrons, and neutrals were modeled, and corresponding Xe – e⁺ ionization collisions, Xe⁺ – e⁺ ionization collisions, Xe – e⁺ excitation, and Xe – e⁺ elastic scattering were taken into account. The collision process were implemented as follows. First, the collision cross-section was magnified as Eq. 3 (this mean that the collision cross-section is calculated by the real mass of electrons but not the scaled mass). Second, if the collision event is elastic, then its frequency is reduced as Eq. 4 by a random number. Finally, if the event is inelastic, then the energy of the corresponding particle was changed as usual, and it is decided whether the direction of velocity should by changed or not again by a random number according to Eq. 4.

The concept of Bohm diffusion was used in this simulation. According to the previous study, the virtual collision frequency ν_{Bohm} accounts for the Bohm diffusion can be written as

$$\nu_{Bohm} = \frac{1}{16} \frac{eB}{m_e} = \frac{\Omega_e}{16} \nu_{scatter} \quad (7)$$

Where, $1/16$ is the assumed Bohm diffusion coefficient. In the system scaled with the mass ratio model, the Bohm diffusion frequency becomes:

$$\nu'_{Bohm} = \frac{\dot{\Omega}_e}{16} \nu'_{scatter} = \frac{f_B}{f^2} \nu_{Bohm} \quad (8)$$

C. Hall thruster

A laboratory model magnetic layer type Hall thruster with outer channel diameter of 62 mm was modeled in this study. Figure 1 illustrates the geometry of the thruster. The black stream traces represents the topology of applied magnetic field and the black area denotes the Boron Nitride channel wall. Three points where the EEDF was sampled are also displayed by red squares. The specific position of the positions of the points are respectively, $r = 0.0275$ m, $z = 0.016$ m for point 1, $r = 0.024$ m, $z = 0.021$ m for point 2, and $r = 0.0275$ m, $z = 0.026$ m for point 3. The detail of the thruster can be found in Ref 6 and 7.

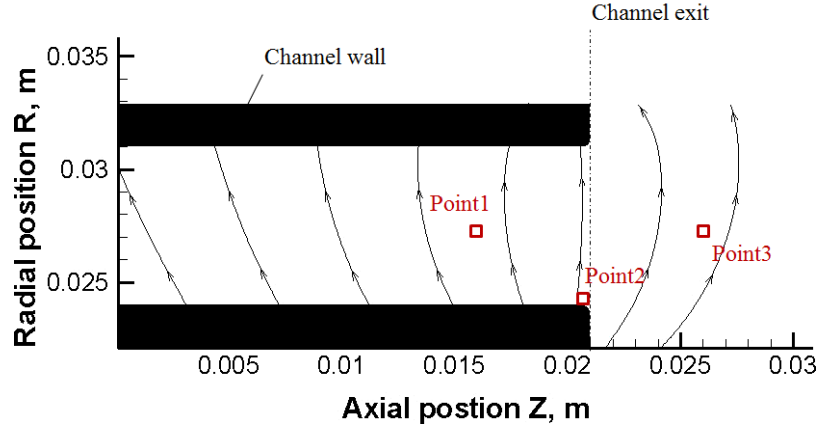


Figure 1. Geometry of the simulated laboratory model magnetic layer type Hall thruster. The black stream traces represents the magnetic topology and the black area denotes the ceramic channel wall. Three points where the EEDF was sampled are also displayed by red squares.

III. Results and Discussion

A. Plasma Property Distribution

The simulation result of plasma property distributions for different mass ratio cases are presented by Fig. 2 and 3. Figure 2 illustrates the plasma potential and electron temperature distribution. The potential drops from the imposed discharged voltage of 300 V to the assumed cathode voltage of 0V. Note that because the cathode itself was not included into the computational area so that the potential drop for the electron beam extraction at the cathode exit was not taken into account in this simulation. Although the equipotential lines show the same direction of inclination as the magnetic field lines, they do not follow ones strictly especially near the channel wall due to the electron pressure and the wall effect⁸. Another feature can be found is that the potential did not dropped to cathode voltage immediately after the channel exit, but had 5 to 10 mm gradual dropping zone outside the channel. This diffusive result was caused by the Bohm diffusion assumption which makes the decrease of electron mobility much slower than that of the classic one. The shape of the distribution of electron temperature generally follows the potential distribution. Temperature inside the discharge chamber was relatively low and ranged from 5 to 15 eV, which is considered to be reasonable considering the energy consumptions by ionization, excitation, and wall loss. On the other hand, electron temperature outside the channel was as high as around 40 eV in maximum, which is considered to be caused by the low energy consumption rate and the electrostatic acceleration by the electric field leaked out from the channel.

Figure 3 shows the electron number density and the electron Hall parameter distribution. Because of the macroscopic quasi-neutrality, the electron number density also approximately represents the ion number density. The number density distribution suggests that the plasma is densest near the channel exit center and decreases toward the anode, the plume boundary, and the wall. The presented electron Hall parameter is the corrected value by the mass ratio model, so the real value can be derived by multiplying the factor f_B . This artificial Hall parameter was presented to confirm that the fundamental requirement of $\Omega_e^2 \gg 1$ was satisfied in the most computational area.

The discrepancies between the different mass ratio cases were limited. The most prominent difference can be found at the near-anode region that the high potential area was broader and more extended toward the downstream side for the cases with smaller mass acceleration factor f . Consequently, the electron temperature for the smaller f cases were also lower at that region because of the less potential drop there. Obviously, this was caused by the insufficient electron mobility preservation for the large f cases that the $\Omega_e^2 \gg 1$ criterion was not fully satisfied at the vicinity of anode as shown by Fig. 3 b). Nevertheless, the magnitude of the potential loss there itself was much less than the discharge voltage. In addition, the discrepancy of the distribution was converging to zero because the case $f=20$ and $f=10$ had almost the same contour shape. Furthermore, the observed discrepancies were limited to the tangential direction against the magnetic field lines that there were no significant difference found for the wall sheath structure.

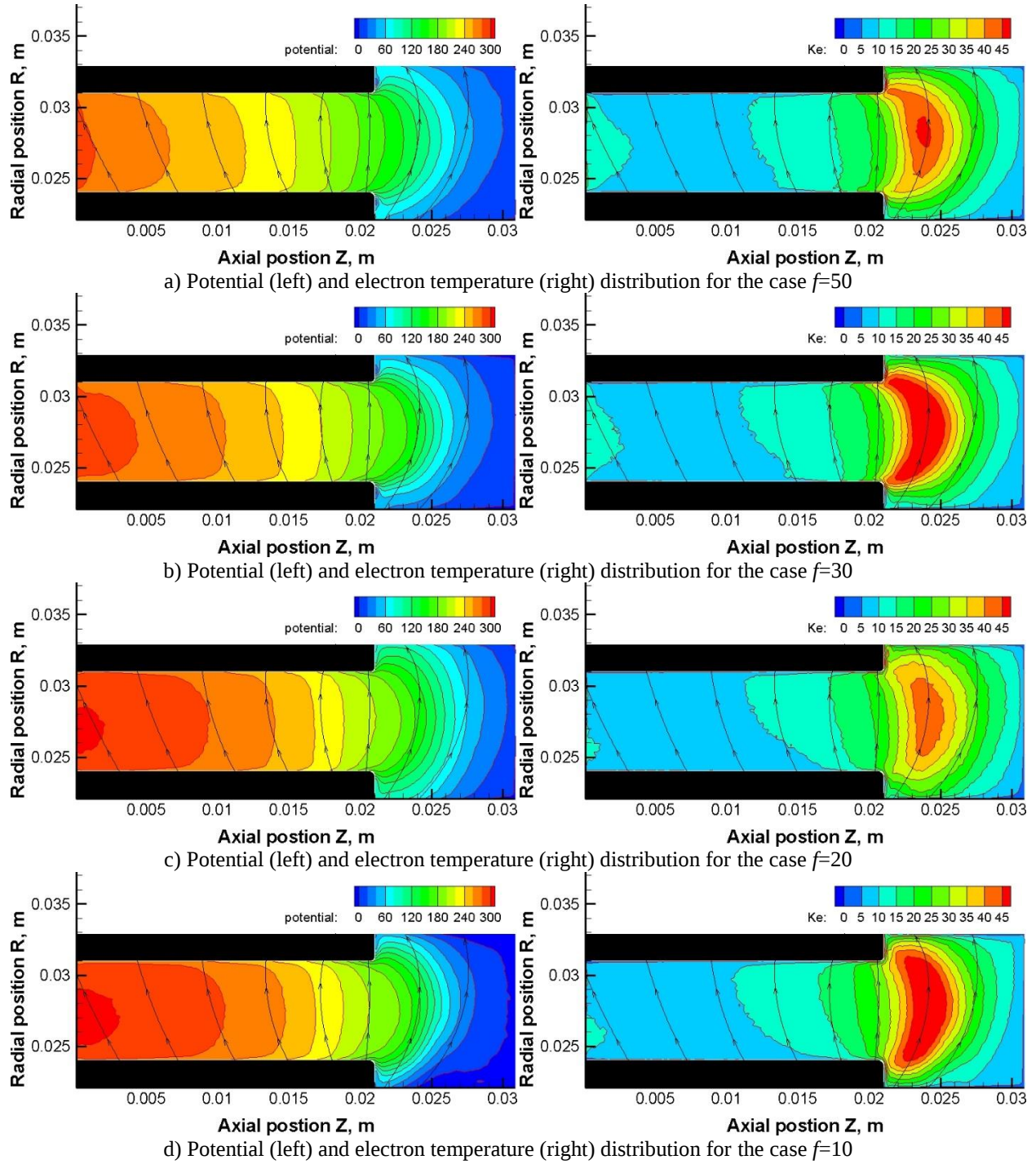


Figure 2. Simulation result of potential and electron temperature distribution. Black lines represents the applied magnetic field and the factor f is the artificial mass acceleration factor that the mass of electrons were numerically multiplied by the factor of f^2 .

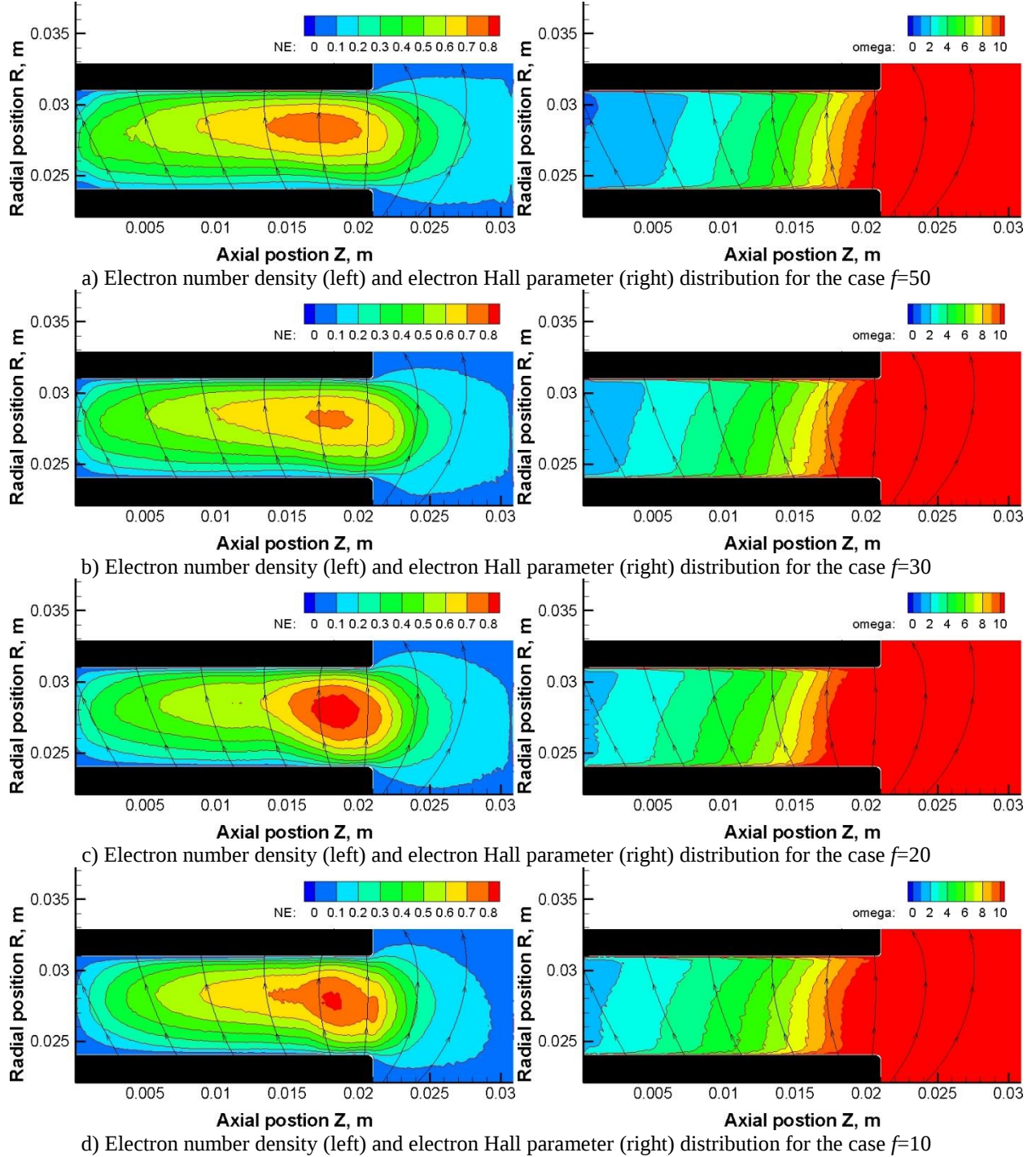


Figure 3. Simulation result of potential and electron temperature distribution. Black lines represents the applied magnetic field and the factor f is the artificial mass acceleration factor that the mass of electrons were numerically multiplied by the factor of f^2 . The shown electron Hall parameter is the corrected value by the mass ratio model.

B. Electron Energy Distribution Function

Maxwellian distribution of electron speed can be written as:

$$f(v) = 4\pi \left(\frac{m}{2\pi kT_e} \right)^{3/2} v^2 \exp \left(-\frac{mv^2}{2kT_e} \right) \quad (9)$$

This can be converted to energy distribution function by

$$\varepsilon = \frac{mv^2}{2k} \text{ and } d\varepsilon = \frac{mv}{k} dv \quad (10)$$

Resulting

$$f(\varepsilon) = \frac{2}{T_e} \sqrt{\frac{\varepsilon}{\pi T_e}} \exp\left(-\frac{\varepsilon}{T_e}\right) \quad (11)$$

In particle simulation, ε is the kinetic energy of individual super particle, electron temperature T_e was calculated in each cell as

$$T_e = \frac{2}{3k} \frac{N}{N-1} \left(\overline{v^2} - \bar{v}^2 \right) \quad (12)$$

Where, N is the number of macro particles and the over-line denotes the average manipulation. The distribution function can be derived by counting up the number of super particles within each energy range.

The EEDF were respectively sampled from a single cell. Because of the limited number of super particles within one cell (~ 200), the distribution function was sampled over 1,000 time step (0.5 μ s), which is short enough compared with the breathing mode oscillation. The positions of the sampling already displayed in Fig. 1 are respectively, channel center in radial position and 5 mm upstream from the channel exit in axial position for point 1; adjacent to inner channel wall in radiation position and channel exit in axial position for point 2; and channel center in radial position and 5 mm downstream from the channel exit in axial position for point 3. The result of the EEDF is shown by Fig. 4. The distribution function was normalized by the local temperature T_e , and compared with the Maxwellian distribution using T_e . The sensitivity limit of the sampled EEDF was 2.E-6 in normalized value. Distribution function lower than the limit could not be captured because of the limited number of sampling particles in the PIC simulation.

The simulated EEDF at the point inside channel center shown by Fig. 4. a) suggests that electrons were sufficiently thermalized by high-density neutrals, and can be approximated by Maxwellian distribution there. However, the log-scale view of the graph reveals that the high energy tail of the sampled distribution function has much higher value than the Maxwellian distribution. The relative difference was greater than an order of magnitude at the energy level $>10 T_e$. It is to be noted that the absolute value of the sampled distribution was greater than the sensitivity limit of the sampling by an order of magnitude at that energy level, which suggests that the observed difference was non-trivial and was not numerically generated by so called statistic noise of PIC simulation. Qualitatively, the existence of the long high energy tail of EEDF within the discharge chamber agrees with Morozov's theoretical analysis and probe measurement⁸, who suggested the existence of the "escaping electrons".

It is well-known that the EEDF inside a Debye layer of Hall thrusters is non-Maxwellian. Indeed, the simulated EEDF at the point inside wall sheath shown by Fig. 4. b) was significantly different both from that probed at the bulk plasma region and the Maxwellian distribution. The low energy electrons with their energy lower than the thermal temperature occupies the majority of the distribution, whereas the high energy tail of the distribution has no less value than that of the Maxwellian distribution. It is considered that this distribution was formed due to the deceleration by wall sheath and low energy secondary electron emission from the ceramics.

The simulated EEDF at the point outside the discharge chamber presented by Fig. 4. c) was non-Maxwellian, and was also different from the other two points described above. The sampled EEDF had beam-like shape and shown almost no existence of electrons with energy higher than $4 T_e$ to the accuracy of sensitivity limit of the sampling. The reasons of this beam-like shape was considered to be as follows. First of all, the relaxation of energy outside the discharge chamber is significantly insufficient because of the low collision frequency and the short resident time of electrons without locking by dielectric wall. Secondary, the electrons are equally accelerated by electric static field leaked out from the discharged chamber gaining much higher kinetic energy (30-40 eV) than their initial injection thermal energy (2 eV in Maxwellian distribution, assumed). Consequently, the electrons are equally energized but not sufficiently thermalized before either entering the chamber or passing through the computational area experiencing only a few collisions and energy losing events.

Finally, comparing the different mass ratio acceleration factor f simulation cases with each other, it can be said that the qualitative tendency of the EEDF was the same. In quantitative comparison, the cases of $f=50$, $f=30$, and $f=20$ agreed well with each other, whereas the case $f=10$ shown non-trivial differences with other cases specifically at the points with non-Maxwellian distributions result. Both at the points inside Debye layer and outside the chamber, the $f=10$ result was relatively closer to Maxwellian distribution than the other three cases, which suggests that the energy

relaxation effect was stronger for $f=10$ case than the other three. The reason of this discrepancy is discussed in the next section.

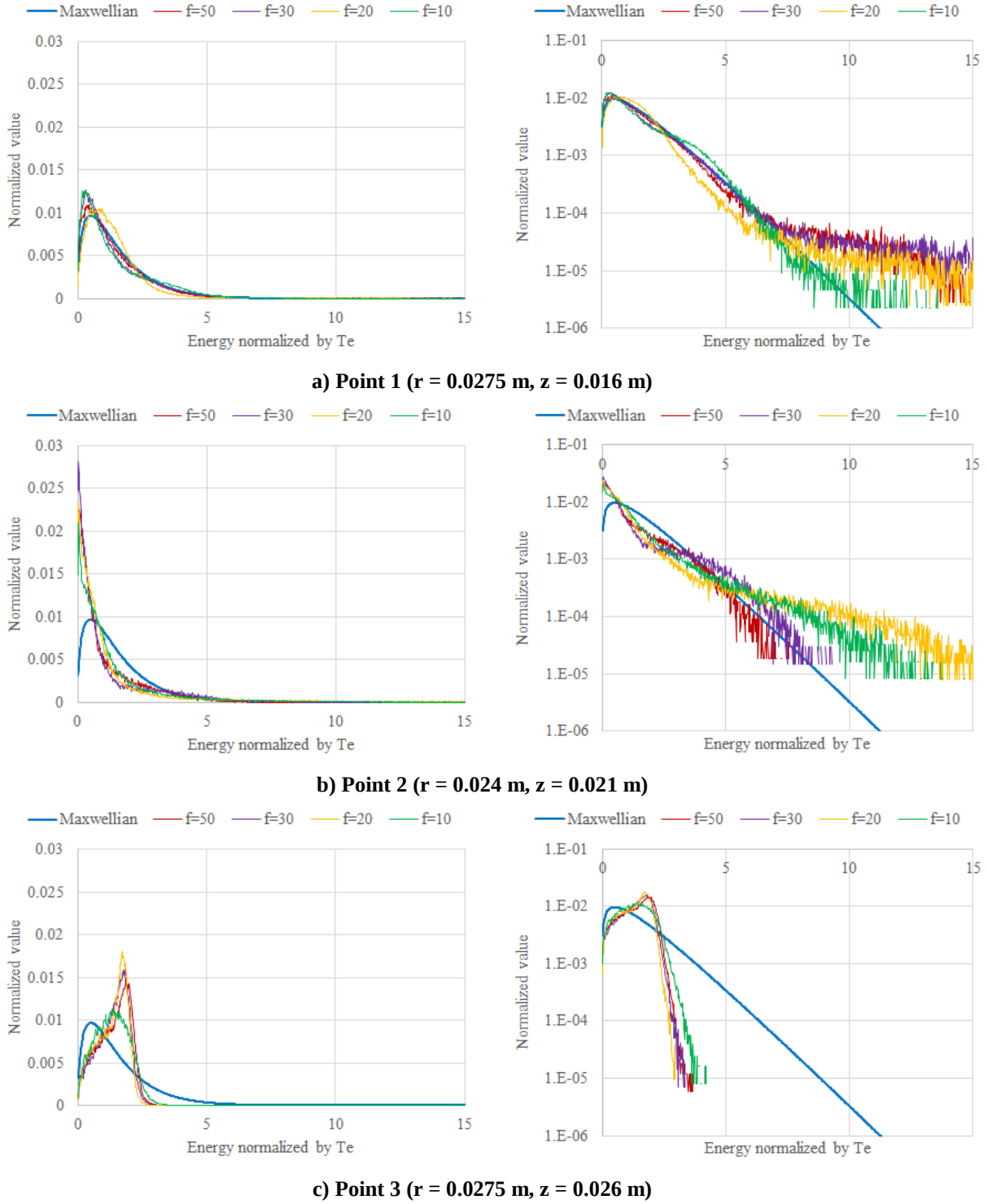


Figure 4. Electron energy distribution function (left) and the same graph in log scale (right). The Maxwellian distribution was constructed by using local electron temperature T_e calculated from particles. Different acceleration factor f represents different mass ratio cases.

C. Discussions

Obviously, the discrepancy of the degree of energy relaxation between the $f=10$ case and the other three cases was resulted from the artificial mass ratio and its physics recovery model. Therefore, we focus on the impact of mass ratio model on the thermalization process of the electrons. As the simplest model, selecting the guiding center drift speed as the characteristic speed v_E and the channel width h as the characteristic length⁹, the characteristic time t_E of guiding center drift can be written as:

$$v_E = \frac{E}{B} \sim \frac{U_d}{Bh} \quad (13)$$

$$t_E = \frac{h}{v_E} \sim \frac{B}{U_d} \quad (14)$$

Where U_d is the discharge voltage. Note that choosing other length (e.g. Larmor radius) as characteristic length will not cause any change of the conclusion. Thus, the non-dimensional characteristic relaxation frequency ν^* can be expressed as:

$$\nu^* = t_E \cdot \nu_{tot} \quad (15)$$

$$\nu_{tot} = \nu_{scatter} + \nu_{Bohm} \quad (16)$$

Where, ν_{tot} , $\nu_{scatter}$, and ν_{Bohm} are respectively the total, scattering collision and anomalous collision frequency since we assumed the existence of anomalous diffusion. The ratio of $\nu_{Bohm}/\nu_{scatter}$ can be written as:

$$\frac{\nu_{Bohm}}{\nu_{scatter}} = \frac{1}{16} \cdot \frac{eB}{mv_{scatter}} = \frac{\Omega_e}{16} \equiv \alpha \quad (17)$$

Where, $1/16$ is Bohm coefficient and Ω_e is electron Hall parameter. Note that for typical Hall thrusters, the electron Hall parameter ranges from 30 to 300 so that the ratio α roughly ranges from 2 to 20 for Bohm coefficient $1/16$. Thus, finally we get:

$$\nu^* = \frac{B}{U_d} (1 + \alpha) \nu_{scatter} \quad (18)$$

Now we consider the artificial mass ratio and its physics recovering model we used. According to the presented scaling law for the mass ratio model, the corrected characteristic frequency ν^* can be written as:

$$\nu^* = \frac{\hat{B}}{U_d} (1 + \alpha) \nu_{scatter} = \frac{f_B B}{U_d} \left(1 + \frac{\alpha}{f_B}\right) \left(\frac{f_B}{f}\right)^2 \nu_{scatter} = \frac{f_B + \alpha}{1 + \alpha} \left(\frac{f_B}{f}\right)^2 \nu^* \quad (19)$$

Thus, in order to preserve $\hat{\nu}^* = \nu^*$ regardless of the mass acceleration factor f , the magnetic field correction factor f_B must satisfy following equation:

$$\frac{f_B + \alpha}{1 + \alpha} \left(\frac{f_B}{f}\right)^2 = 1 \quad (20)$$

or,

$$f_B^3 + \alpha f_B^2 - (1 + \alpha) f^2 = 0 \quad (21)$$

Assuming there is no Bohm diffusion so that the ratio $\alpha = 0$, this can be solved immediately and results $f_B = f^{2/3}$. If $\alpha > 0$, then the factor f_B preserving the characteristic relaxation frequency can be found either analytically or numerically.

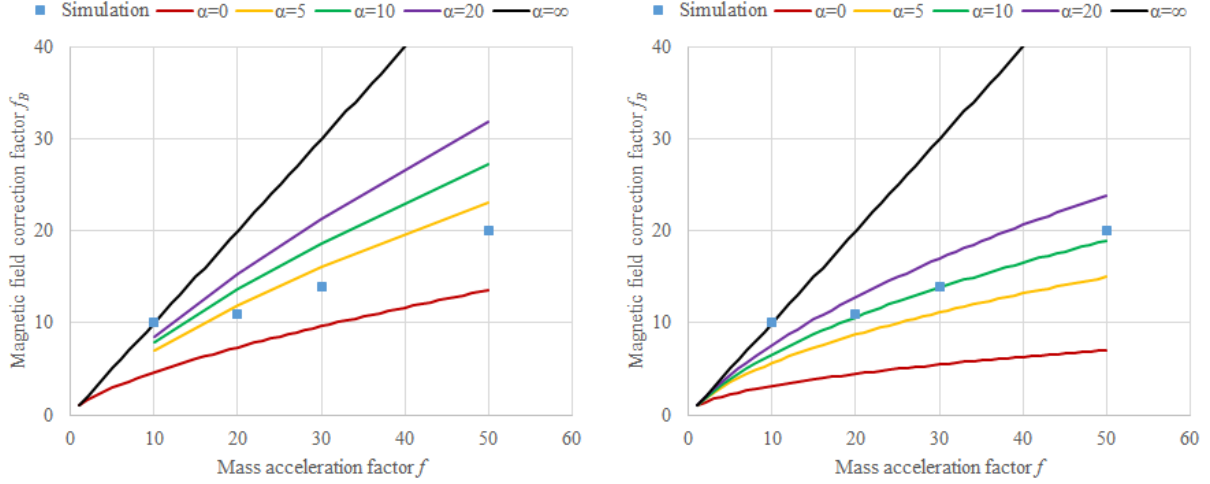
Another approach can be considered by simply selecting the electron thermal speed v_{th} as the characteristic velocity. In this case, since v_{th} scales with $1/f$ by the mass ratio model, Eq. 21 becomes:

$$f_B^2 + \alpha f_B - (1 + \alpha) f = 0 \quad (22)$$

Likewise, if $\alpha = 0$ then we get very simple $f_B = \sqrt{f}$. Or If $\alpha > 0$, we derive:

$$f_B = \frac{-\alpha + \sqrt{\alpha^2 + 4(1 + \alpha)f}}{2} \quad (23)$$

Figure 5 summarizes the optimum f_B in terms of the energy relaxation preservation for different collision frequency ratio α and the choice of the characteristic speed. The graph suggests that first, the existence range of the optimum f_B is $f^{1/2} < f_B < f$ for all possible α and characteristic speed selection. Second, the optimum f_B monotonically increases as the increase of α . because the higher f_B value means the higher collision frequency and



a) Characteristic speed is electron drift speed

b) Characteristic speed is electron thermal speed

Figure 5. Optimum f_B for the preservation of energy relaxation. The factor α is the ratio of scattering collision frequency and Bohm collision frequency. The lines illustrates the optimum value of factor f_B for the preservation of EEDF for different α cases. The dot plots shows the actual value used in the simulation.

the higher degree of energy relaxation, This means that the higher the α or the electron Hall parameter, the higher collision frequency is necessary to preserve the energy relaxation process. This requirement is qualitatively *not* in conflict with the requirement of Eq. 5, because the higher the Hall parameter, the higher the f_B value can be afforded. Finally the value of f_B used in the simulation cases nearly coincide with the $\alpha = 2$ line in Fig. 5. a) or $\alpha = 10$ one in Fig. 5. b). with the exception of $f=10$ case. Because higher f_B means higher corrected relaxation frequency ν^* , this result reasonably explains why the EEDF derived in $f=10$ case was always more smother and closer to Maxwellian distribution than the other three simulation cases. Although there is no experimental EEDF result for our thruster to compare with, it still can be determined that the simulation case $f=10$ result was over-relaxed than the real because the conventional $f_B=f$ is only correct for $\alpha = \Omega_e = \infty$ collision-less situation.

Although there is no concrete evidence to conclude whether the electron drift speed of the thermal speed is appropriate for the discussion of energy relaxation, it is rather conservative and simpler to go for the thermal speed. Then, the relationship between the requirement of energy relaxation preservation of Eq. 22 and Hall thruster fundamental requirements of Eq. 5 and 6 can be further discussed as follows. First, for the case $4f \gg \alpha$ or $f \gg \Omega_e/64$, Eq. 23 can be approximated as:

$$f_B \cong \sqrt{f(1 + \alpha)} \quad (24)$$

So that Eq. 5 and 6 becomes:

$$\Omega_e = \frac{1}{f_B} \Omega_e = \frac{16\alpha}{\sqrt{f(1 + \alpha)}} \gg 1 \quad (25)$$

$$r_L = \frac{f}{f_B} r_L = \sqrt{\frac{f}{1 + \alpha}} r_L \ll L \quad (26)$$

For both Hall parameter and Larmor radius, the higher the α means the wider the acceptable range of f . Thus, only the minimum α case for the thruster should be checked if Eq. 5 and 6 are satisfied. If not, smaller acceleration factor f should be used.

Second, for the case $4f = C\alpha$ or $f \sim C\Omega_e/64$ ($C \sim 1$), Eq. 24-26 becomes:

$$f_B \sim \frac{\sqrt{C+1}-1}{2} \alpha \quad (27)$$

$$\Omega_e \sim \frac{32}{\sqrt{C+1}-1} \sim 77.3 \gg 1 \quad (28)$$

$$r_L' \sim \frac{C}{2(\sqrt{C} + 1 - 1)} r_L \sim 1.2r_L \ll L \quad (29)$$

Obviously, these criteria are almost automatically satisfied.

Finally, the case $4f \ll \alpha$ or $f \ll \Omega_e/64$ means the change of mass ratio from the real one is negligibly small, and there is no need to worry about the requirements of Eq. 5 and 6.

In a brief summary, originally, the factor f_B was introduced with an additional degree of freedom to the model for the conservation of electron mobility. The value of f_B was chosen to fulfill the fundamental requirements of Hall thruster electron confinement, namely, the electron Hall parameter and Larmor radius. However, because both of the requirements only restrict the acceptable range of f_B , the choice of its exact value does have some ambiguity. Nevertheless, by the discussion about the preservation of energy relaxation or the EEDF presented above, the optimum value of f_B can be specified without any ambiguity. In this way, the electron mass model can be improved in accuracy of EEDF prediction with improved self-consistency reducing a possible adjustable tuning parameter. The implementation of the strictly fixed f_B strategy can be realized by either calculating electron Hall parameter for each cell and apply local f_B , or simply eliminate Bohm diffusion from the simulation model.

IV. Conclusion

A fully kinetic simulation model using artificial electron mass was developed for Hall thrusters. The numerical speeding up technique of artificial mass ratio was employed as the only-one unphysical manipulation. Thus, one of the most important advantage of the proposed model is that there were no artificial manipulation employed for the permittivity and the Debye length. Unlike the previous study, the mass of electrons but not the heavy particles was changed and an original mass ratio model was used. The basic idea of the new model is to preserve the physics of electron transport across the magnetic field in the scaled system at the expense of unpreserved electron Hall parameter and Larmor radius with limited change.

The simulation of a laboratory model Hall thruster with different mass factor cases were conducted, and the result of plasma property distributions and EEDF were investigated and discussed. The results suggests that first, the simulation results were consistent with different mass ratio cases for both plasma distributions and EEDF with limited discrepancies in-between. Second, the non-Maxwellian nature of EEDF was captured especially at the near-wall and outside-channel region. In addition, even at the inside-channel bulk plasma region, the high energy tail greater than Maxwellian distribution were observed. Finally, it was suggested that the mass ratio model can be further improved in accuracy and self-consistency. The mass ratio model parameter can be specified without any ambiguity by the consideration of energy relaxation of electrons.

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