

Measurement of the Dielectric Wall Erosion in Helicon Plasma Thrusters: an Application to the VASIMR[®] VX-CR Experiment

IEPC-2013-188

*Presented at the 33rd International Electric Propulsion Conference,
The George Washington University, Washington, D.C., USA
October 6–10, 2013*

Juan Del Valle,¹ Jose A. Castro,² Norberto Arce,³ Erick Chinchilla,⁴ Esteban Echeverría,⁵ Daniel Lezama,⁶
Carlos Martínez,⁷ Jorge Oguilve,⁸ Allan Rivera,⁹ Max Rodríguez,¹⁰ Juan Valverde,¹¹ and Rónald Chang Díaz¹²
Ad Astra Rocket Company, Liberia, Guanacaste, Costa Rica

and

Christopher S. Olsen,¹³ Andrew V. Ilin,¹⁴ Matthew Giambusso,¹⁵
Mark D. Carter,¹⁶ Jared P. Squire,¹⁷ and Franklin R. Chang Díaz¹⁸
Ad Astra Rocket Company, Webster, Texas, USA

Abstract: An experimental proposal to characterize erosion phenomena in the cylindrical dielectric tubes of helicon plasma thrusters is presented and discussed. A simplified theoretical framework is described first, identifying simple approximations which can support the design of experiments. Erosion will be quantified by calculating of the volumetric change in the cylinder's inner surface after operation of the plasma source. The required surface maps will be produced using a coordinate measuring machine (CMM) with probes able to reach the complete length of these elements. Knowledge of the distribution and magnitude of plasma-surface interaction phenomena can help in understanding their underlying physical mechanisms, and possible mitigation factors to increase the operational lifetime of the thrusters.

¹Research Engineer, juan@adastrarocket.com

²Chief Scientist, jose.castronieto@adastrarocket.com

³Research Technician, norberto.arce@adastrarocket.com

⁴Research Engineer, erick.chinchilla@adastrarocket.com

⁵Research Engineer, esteban.echeverria@adastrarocket.com

⁶Research Technician, daniel.lezama@plasmathruster.com

⁷Research Technician, carlos.martinez@adastrarocket.com

⁸Director of Operations, jorge@adastrarocket.com

⁹Research Engineer, allan.rivera@adastrarocket.com

¹⁰Research Engineer, max.rodriguez@adastrarocket.com

¹¹Software Developer, jjose@adastrarocket.com

¹²Executive Director, ronald@adastrarocket.com

¹³Senior Research Scientist, chris.olsen@adastrarocket.com

¹⁴Computational Research Lead, andrew.ilin@adastrarocket.com

¹⁵Research Scientist, matthew.giambusso@adastrarocket.com

¹⁶Director of Technology, mark.carter@adastrarocket.com

¹⁷Director of Research, jared.squire@adastrarocket.com

¹⁸Chief Executive Officer, info@adastrarocket.com

Nomenclature

B_0	= intensity of the axial magnetic field inside the helicon source dielectric tube [T]
c_s	= Bohm sonic velocity of the ions [m/s]
CMM	= coordinate measurement machine
e	= elementary charge [C]
EP	= electric propulsion
ICH	= ion cyclotron heating
ISS	= International Space Station
k	= wave number of the induced helicon wave [m ⁻¹]
k_B	= Boltzmann constant [$k_B = 1.3806488(13) \times 10^{-23}$ J/K]
L	= nominal length of the helicon source dielectric tube [m]
L_{ant}	= length of helicon antenna [m]
$\dot{M}$	= estimated average erosion rate [kg/s]
ΔM	= mass difference within the tube's inner surface after exposure to the plasma [kg]
m	= mode number of the induced helicon wave
m_a, m_i, m_e, m_t	= atomic mass, ion mass, electron mass, target surface atomic mass [amu]
$\dot{m}_g$	= propellant mass flow rate [mg/s]
n_e	= electron density [m ⁻³]
P_{RF}	= helicon RF power [W]
R	= nominal radius of the helicon source dielectric tube [m]
RF	= radio frequency
r_i	= calculated radial distance for points measured in the inner surface [m]
R_{ant}	= radius of helicon antenna [m]
s_i	= calculated difference in the radial distance after exposure to the plasma [m]
ω	= frequency induced by the radiofrequency generator on the helicon antenna [Hz]
Δt	= accumulated length of time of operation of the helicon plasma source [s]
T_e	= electron temperature of the plasma [K]
u	= magnitude of particle velocity [m/s]
ΔV	= tube volumetric difference after exposure to the plasma [m ³]
VASIMR [®]	= VArable Specific Impulse Magnetoplasma Rocket
VF-200 TM	= VASIMR [®] flight prototype, 200 kWe power level
VX-CR TM	= VASIMR [®] experiment, Costa Rica
Z_t	= atomic number of impacting ions
$\bar{Z}_t$	= weighted atomic number for the simplified sputtering model
γ_{sput}	= sputtering rate [kg/s]
ε_{thr}	= surface threshold energy for sputtering [V]
ε_t	= electric potential at the surface [V]
ε_i	= energy of the impacting ions [V]
μ_0	= permeability of free space [$\mu_0 = 4\pi \times 10^{-7}$ H/m]
ω	= helicon source RF frequency [Hz]
ρ_{tube}	= density of dielectric tube material [kg/m ³]
θ_i	= calculated azimuthal angle for points measured in the inner surface

I. Introduction

HELICON plasma sources^{1,2} are increasingly being researched by the electric propulsion (EP) community due to their production of high-density plasmas using moderate magnetic fields. In contrast with other EP devices, they do not have electrodes or grids in contact with the plasma stream but instead use a helicon

antenna surrounding a dielectric ceramic tube. The absence of plasma-facing metal components is perceived as a factor that increases the lifetime of the thruster in comparison with other competing technologies. However, little research has been performed on the topic to support this hypothesis.

The VASIMR[®] thruster³ is a high-power (> 100 kWe) electric propulsion engine being developed by Ad Astra Rocket Company. Its main advantages are its high power density, constant power throttling while adjusting its specific impulse,⁴ radial confinement of the plasma using electromagnetic fields, and linear acceleration of the ion plume using a magnetic nozzle.⁵ The main components of the thruster are depicted in Fig. 1a. There are three interlinked electromagnetic cells. A helicon plasma source ionizes neutral propellant creating a cold plasma, which is then energized using ion cyclotron heating, and ejected from the thruster through a magnetic nozzle. A better understanding of the lifetime-limiting factors in each one of these stages, including the helicon plasma source, will help to define the life expectancy of the whole design.

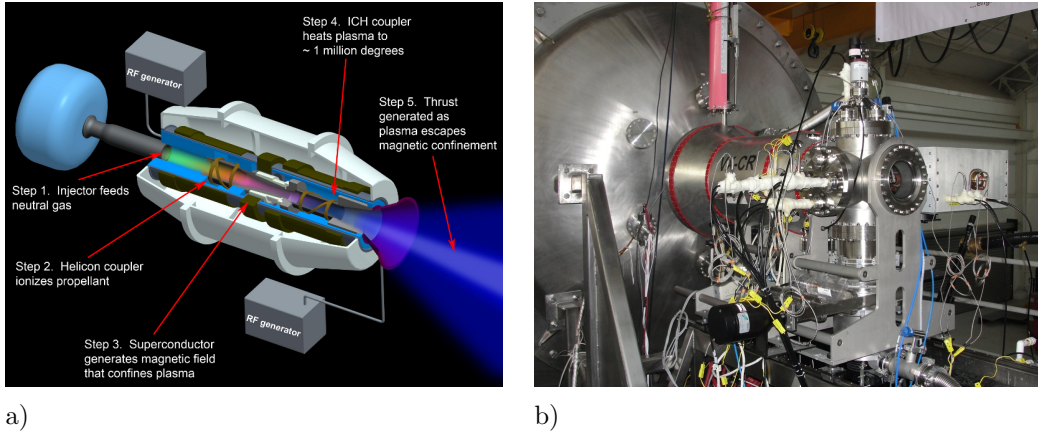


Figure 1. a) The VASIMR[®] thruster. b) The VX-CR experiment at Ad Astra's Costa Rica facility. Source: Ad Astra Rocket Company.

This paper presents a review of the current state of the research on plasma-surface interaction phenomena in helicon sources, focusing on the undesired erosion and deposition effects on the inner surface of the cylindrical dielectric tube. A simplified first-principles model will be presented, describing the main parameters influencing the diffusion and interaction of ions with the confining surfaces. Based on this analysis, an experimental research plan for the VASIMR[®] VX-CR device will be proposed and discussed. A description of the expected results and their relevance will be presented.

II. Research background

A. Helicon Plasma Sources

Figure 2 shows a simplified diagram of a typical helicon plasma source. Propellant gas is injected at a given mass flow rate $\dot{m}_g$ into a cylindrical dielectric tube of length L and radius R . A helicon antenna of length L_{ant} and radius R_{ant} surrounds this tube. Several types of helicon antennas are available.⁶ The antenna is driven at a frequency ω by an external radio frequency (RF) generator. A magnetic field of intensity B_0 and coaxial with the dielectric tube is created, typically using solenoid magnet coils or permanent magnets.

The purpose of the magnetic field is to magnetize the plasma and confine it towards the center of the dielectric tube. However, particles will always come into contact with the tube's inner wall or the end backplate where the propellant gas is injected. If these particles have sufficient energy, they may interact with the surface they collide with. One of the possible effects is the sputtering of atoms off the surface; a high rate of material removal could compromise the performance of the source and/or shorten its operational lifetime.

B. Previous research

Plasma-surface interaction studies, although common for most EP technologies, are scarce in helicon plasma literature. Previous work has focused mostly on the use of these plasma sources for materials processing,⁷ or

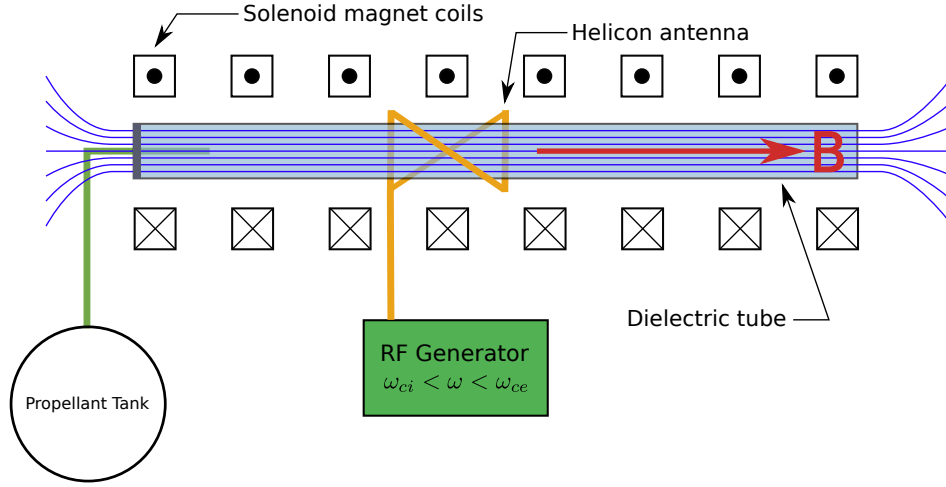


Figure 2. Simplified diagram of a helicon plasma source.

the execution of power balance experiments in these devices^{8,9}. The work by Berisford et al.^{10,11} included an initial exploration of the erosion of the dielectric wall confining the plasma in helicon sources. The experimental results from this study suggest that capacitive coupling in the regions underneath the helicon antenna straps is the only external energy source capable of accelerating the ions to levels where sputtering of wall material can occur.

C. The VASIMR[®] VX-CR Experiment

The VX-CR device¹² is the main experimental plasma source of Ad Astra Rocket Company's Costa Rican research facility. It is a modular helicon source designed as a testbed of thermal management and lifetime issues for the first stage of the VASIMR[®] thruster. VX-CR's maximum RF power level is 13 kWe, lower than Ad Astra's VX-200 experiment (which can operate with 40 kWe on its helicon first stage). However, this value is similar to the 15 kWe power level of the helicon stages of each of the two VF-200 engines included in Ad Astra's proposed Aurora experimental ISS platform.¹³ VX-CR's magnetic field intensity is also lower than other VASIMR[®] prototypes, in order to provide worst-case confinement conditions to the helicon source.

The VX-CR is equipped with a prototype thermal management system that allows the source to operate at thermal steady state for extended periods of time. Table 1 shows some typical operating parameters of this experiment. More recent VX-CR results are presented in a separate paper.¹⁴

Table 1. Typical VX-CR operating parameters for thermal steady-state operation.

Parameter	Symbol	Value
Argon flow rate (mg/s)	$\dot{m}_g$	6
Helicon RF power (W)	P_{RF}	2000
Helicon RF frequency (MHz)	ω	13.56
Magnetic field intensity within the dielectric tube (T)	B_0	0.1
Typical peak electron temperature (eV)	T_e	3.0
Dielectric tube inner diameter (mm)	$2R$	90
Dielectric tube length (mm)	L	226
Length of helicon antenna (mm)	L_{ant}	70
Diameter of helicon antenna (mm)	$2R_{ant}$	100

III. Simplified Model of Helicon Plasma Sources

This section presents a simplified first-principles model of the performance of a helicon plasma source, unifying previously-published results into a single description of the formation of the plasma, the transport of different species, and the interaction with its surrounding surfaces.

A. Plasma ionization

The ionization of plasma into the helicon mode has been studied extensively in the past, and the exact energy damping mechanism is still under debate. The derivation of the dispersion relation for helicon plasmas has been described by Chen^{15,16}; and the simplified relationship found in equation 1 can help to describe the behavior of helicon modes $m = 0$ and $m = 1$.

$$\frac{\omega}{k} \approx \left(\frac{3.83}{R} \right) \left(\frac{B_0}{n_e e \mu_0} \right) \quad (1)$$

This expression, when combined with the parameters expressed in Table 1, suggests that the maximum attainable electron density in the VX-CR should be close to $n_e \approx 6.32 \times 10^{24} \text{ m}^{-3}$.

B. Ion radial diffusion

The second component of the model comprehends the 2D equations that describe the diffusion of a cylindrical, magnetized plasma. These derivations are strongly based on the work by Ahedo et al.^{17,18,19}, who built upon the previous work published by Sternberg et al.²⁰, and Fruchtman et al.²¹.

Some observations and results regarding this 2D model are listed below,

1. The model assumes a constant electron temperature T_e in the whole plasma column.
2. The ion radial and axial velocities present a singularity at the value $u_r = u_z = c_s = \sqrt{k_B T_e / m_i}$, which corresponds to the Bohm sonic velocity present at the entrance to the Debye sheath at the vicinity of the dielectric tube inner wall and the transition to the detailed plasma-surface interaction model.
3. The model also imposes as a boundary condition that the ions must reach the sonic Bohm velocity at the rear wall and at the downstream end of the dielectric tube as they flow out of the system. This restriction produces a peak in the plasma density profile in the regions close to the tube's center and the rear lateral wall, where the ionization rate has a peak value. The ion velocity increases towards the limit value c_s as the geometric boundaries of the system (upstream injection plate, tube's inner surface, tube's downstream end) are approached.

One practical implication of this 2D model is that ions within the source will not exceed the sonic velocity boundary value, which using the values of Table 1 would be equivalent to $c_s = 2.68 \text{ km/s}$ in the VX-CR experiment. This 2D model can be used to approximate the ion flux towards the inner wall of the dielectric tube, and then use this value within the sputtering equations of the third component of the model to predict the expected erosion rate.

C. Plasma-wall interaction

The 2D diffusion model from the previous subsection defines the ion sound speed c_s as the condition for the ion velocity at the confining surfaces within the helicon source, as well as the exit velocity of the ions through the downstream end of the dielectric tube. This limit corresponds to the Bohm sheath criterion and marks the transition from the bulk plasma described by the 2D model to the non-neutral Debye sheath present near the boundary surfaces. The third component of the simplified model is a 1D set of relationships describing the impact energies of the accelerated ions to the wall, and the conditions required to sputter material off the inner surfaces of the helicon source.

A complete description of the plasma sheath and the Bohm criterion can be found in the literature²². The description adapted for this model is derived from the textbook by Lieberman and Lichtenberg²³ and the approach used by Berisford.¹⁰ The sputtering yield γ_{sput} , the number of surface atoms released per

impacting ion, is a function of the energy of the incoming ion, the angle of impact with respect to the surface plane, and the surface threshold energy ε_{thr} . For simplicity, the ions impacting the surface will be modeled as having their velocity vector normal to the surface since the velocity component normal to the wall is expected to be dominant after traversing the Debye sheath.

The sputtering yield can be approximated through the following expression²³,

$$\gamma_{sput} \approx \frac{0.06}{\varepsilon_t} \sqrt{\bar{Z}_t} (\sqrt{\varepsilon_i} - \sqrt{\varepsilon_{thr}}) \quad (2)$$

where ε_t is the electric potential at the surface, ε_i is the energy of the impacting ions and $\bar{Z}_t$ is a weighted atomic number calculated from²³

$$\bar{Z}_t = \frac{2Z_t}{\left(\frac{Z_i}{Z_t}\right)^{2/3} + \left(\frac{Z_t}{Z_i}\right)^{2/3}} \quad (3)$$

Z_i is the atomic number of the impacting ion species, and Z_t is the atomic number of the surface atoms under analysis. These expressions are valid for ion and surface atoms with large atomic numbers ($Z_t, Z_i \gg 1$) and similar within the range $0.2 < Z_t/Z_i < 5$. When the ion and surface atom masses fall within the range $m_i/m_t \geq 0.3$, the sputtering threshold energy ε_{thr} can be approximated using²³

$$\varepsilon_{thr} \approx 8\varepsilon_t (M_i/M_t)^{2/5} \quad (4)$$

IV. Experimental Proposal for the Study of Inner Wall Erosion

Based on the estimations provided by the mathematical framework of the previous section, an experiment plan will be proposed in order to quantify the erosion rate occurring inside the dielectric tube after exposing it to the VX-CR plasma discharge. Similar to previous erosion rate research performed in other EP devices²⁴, the geometry of the inner surface of the ceramic tube will be mapped before and after operation of the helicon plasma source. These measurements will be matched and compared, in order to estimate the volumetric change occurred after the test. This value will be then transformed into an equivalent loss of mass, which can be used to estimate an average rate of erosion using the length of time the source operated.

The cylindrical configuration of the ceramic tube can interfere with the accuracy of optical measurement systems, particularly for elements with a large length-to-radius (L/R) ratio. Therefore, instruments having a small enough measuring element installed on a thin, long support are better suited for mapping the entirety of the tube's inner surface. An industrial coordinate measurement machine (CMM) was chosen to fit these requirements. This instrument is owned by Costa Rica's Instituto Nacional de Aprendizaje and is shown in Fig. 3a. Its main technical specifications are listed on Table 2.

In order to compare data obtained before and after a particular experiment, the measurement coordinates of the CMM instrument must be correlated between both sets. An aluminum rectangular fixture will be used to hold the dielectric cylinder in a vertical position. The holder has a pin that aligns with a circular groove carved in the outer surface of the ceramic piece. The CMM instrument can then align its coordinate axis with the flat surfaces in the fixture, therefore different measurements will use the same coordinate system. This part is shown in Fig. 3b.

When performing a set of measurements for a particular cylinder, the first step is the alignment of the CMM's horizontal coordinates with the predetermined lateral surfaces of the fixture piece; the Z axis will always be the vertical one. Then the origin of the coordinate system is translated to the point on the axis of the cylinder and located at the same vertical height as the uppermost edge of the tube. This can be achieved using preprogrammed routines in the CMM control software which find the center of the cylinder's circumferential cross section, and then translate the zero point on the Z axis to match a horizontal surface. From this new frame of reference, a preprogrammed routine is loaded into the CMM instructing it to move the measuring probe radially outwards to a predetermined set of points on the inner surface. When the probe detects contact with the material, it stops the movement, records the current coordinates and then proceeds to the next measuring point. Once all points have been recorded at the current height, the probe

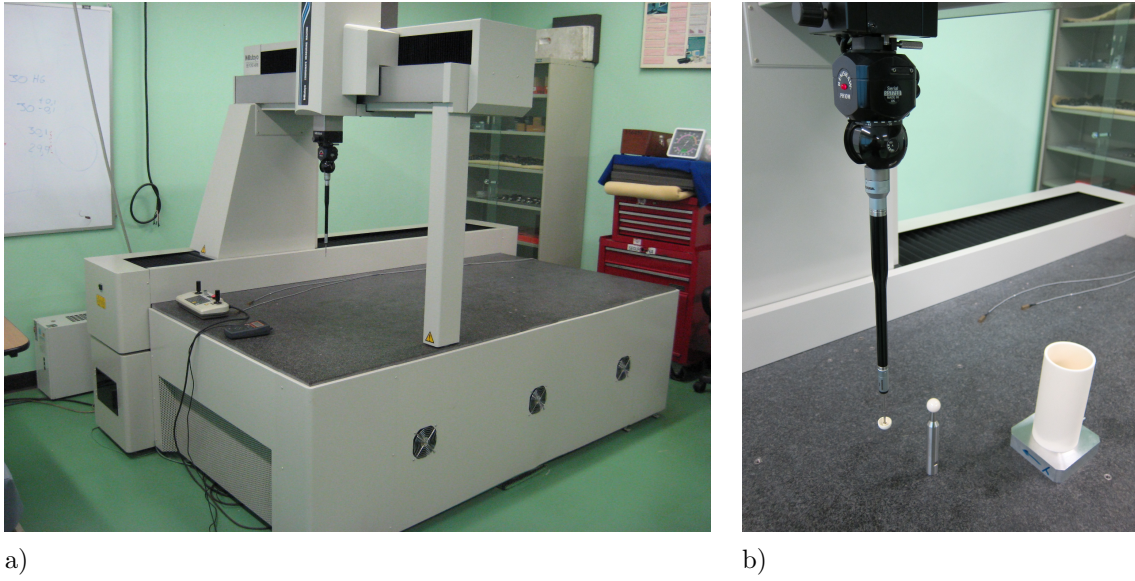


Figure 3. a) Coordinate Measurement Machine (CMM) used to map the inner surface of the dielectric tube. b) Measuring boom of the CMM instrument, with a semi-spherical measuring tip installed. Also shown are a calibration sphere, and a sample ceramic tube inside the aluminum holder piece.

Table 2. Technical specifications of the Coordinate Measurement Machine (CMM).

Brand, model	Mitutoyo, Crysta Apex CRT-A916
Resoution (μm)	0.1
Accuracy (μm , worst-case)	3.4
Measuring range, Z (vertical) axis (mm)	605
Measuring range, X (horizontal) axis (mm)	905
Measuring range, Y (horizontal) axis (mm)	1605

moves downwards to the next vertical value and then begins mapping points in a circumference again. This procedure is repeated until data has been taken at all vertical axial positions within the cylinder.

The map obtained is a discrete representation of the inner surface of the dielectric cylinder. Information on the areas not measured directly must be obtained by an interpolation routine. Therefore, the accuracy of the map depends on the total number of points measured, which at the same time determines the duration of the whole measuring procedure. Tests conducted with a sample VX-CR dielectric tube were able to measure 96 circumferential points at 217 vertical positions (a total of 20832 points) in about 10 hours.

Processing the data will involve transforming the data set from the chosen Cartesian reference system into cylindrical coordinates, calculating a radial distance r_i using the X and Y coordinates of each point and a corresponding azimuthal angle θ computing inverse trigonometric functions on these same values. This procedure is shown in equations 5 and 6.

$$r_i = \sqrt{x_i^2 + y_i^2} \quad (5)$$

$$\theta_i = \arctan\left(\frac{y_i}{x_i}\right) \quad (6)$$

Since the same CMM control routine will be used to measure a tube before and after exposing it to the plasma in the VX-CR, the sequence of measured points should be the same between the two sets of data. If the routines that align the axis and translate the origin of the coordinate systems are performed carefully in both measurements, the center of the coordinate system should be the same point (relative to the metal

fixture) and the difference between the radial distances calculated with equation 7 for each point represent the changes in the surface caused by material sputtered or deposited on it.

$$s_i = r_{i,\text{after}} - r_{i,\text{before}} \quad (7)$$

2D interpolation routines will be applied to the data in order to complete the map throughout the entire inner surface of the tube. Integrating all the values for radial variation s_i over the cylindrical coordinate system formed by θ and z , an estimation of the total amount of volume removed from the surface can be calculated as shown in equation 8.

$$\Delta V = \int_0^L \int_0^{2\pi} s(\theta, z) R d\theta dz \quad (8)$$

Finally, this value can be transformed into a mass difference and then used to compute the estimated erosion rate for the system, as shown in equations 9 and 10. These expressions assume the density of the dielectric tube material is constant and isotropic.

$$\Delta M = \rho_{\text{tube}} \Delta V \quad (9)$$

$$\dot{M} = \frac{\Delta M}{\Delta t_{\text{plasma}}} \quad (10)$$

The accuracy of this differential mapping procedure depends on the resolution of the map (the amount of points measured and their spacing), the uncertainty of each data point, and the precision of the coordinate alignment for distinct sets of measurements. These factors depend on a careful confirmation of the uncertainty of the CMM measurements, and an adequate manufacturing technique for the metal fixture that holds the tube in place during the measurements. The accumulated length of time in which the tube will be in contact with the plasma must be sufficient to produce erosion values with a magnitude greater than the expected uncertainty of the measurements. This value will be calculated using the predicted erosion rate estimations from the simplified model described in the previous section, and after executing calibration tests to verify the uncertainty in the CMM measurements. Previous results suggest that these experiments require at least tens of hours of exposing the tube to the plasma to produce measurable erosion¹².

The proposed experimental plan aims to map and quantify erosion phenomena in the helicon source dielectric tube as a function of the operating parameters of the VX-CR device, mainly the power level in the RF generator. In order to avoid thermal and mechanical problems at the higher values of this parameter, the source will be operated in pulses previously designed to avoid overheating any of its internal components.

V. Conclusions

A procedure to measure the inner wall erosion in the ceramic tubes of helicon thrusters has been presented and discussed. A simplified theoretical framework was introduced to support the design of these experiments and estimate the expected results. The method described can also be used to analyze the distribution of erosion and deposition of material throughout the inner surface of the tube. This effort is part of a larger research project aiming to characterize and understand plasma-surface interaction phenomena in helicon plasma sources.

Acknowledgments

This research was carried out with the collaboration of the Metrology Laboratory at the Francisco J. Orlich campus of Costa Rica's Instituto Nacional de Aprendizaje (INA).

Appendix

The following equations formulate the 2D model for the dynamics of a cylindrical magnetized plasma, as derived by Ahedo et al.¹⁹ A variable separation technique is used to decouple the radial and axial models.

Equations 11 through 14 describe the radial behavior, while equations 15 through 19 represent the axial description.

$$\frac{1}{r} \frac{\partial}{\partial r} (r n_r u_r) = n_r \nu_w \quad (11)$$

$$u_r \frac{\partial u_r}{\partial r} = -c_s^2 \frac{\partial \ln n_r}{\partial r} - \frac{r B_0}{m_i} u_\theta + \frac{m_e}{m_i} \frac{u_\theta^2}{r} - n_n (R_{in} + R_{ion}) u_r \quad (12)$$

$$u_r \frac{\partial u_\theta}{\partial r} = \frac{e B_0}{m_e} u_r - [n_n (R_{en} + R_{ion}) + n_r + R_{ei}] u_\theta - \frac{u_\theta u_r}{r} \quad (13)$$

$$e \frac{\partial \phi_r}{\partial r} = T_e \frac{\partial \ln n_r}{\partial r} + e B_0 u_\theta - m_e \frac{u_\theta^2}{r} \quad (14)$$

$$n_z u_z + n_n u_n = g_0 \quad (15)$$

$$\frac{\partial}{\partial z} (n_z u_z) = n_z (n_n R_{ion} - \nu_w) \quad (16)$$

$$u_z \frac{\partial u_z}{\partial z} = -c_s^2 \frac{\partial \ln n_z}{\partial z} - n_n (R_{in} + R_{ion}) (u_z - u_n) \quad (17)$$

$$u_n \frac{\partial u_n}{\partial z} = -n_z \left[R_{in} (u_n - u_z) + \frac{\nu_w}{n_n} u_n (1 - \alpha_w) \right] \quad (18)$$

$$e \frac{\partial \phi_z}{\partial z} = T_e \frac{\partial \ln n_z}{\partial z} \quad (19)$$

References

- ¹Boswell, R. and Chen, F., “Helicons: The Early Years,” *IEEE Transactions on Plasma Science*, Vol. 25, No. 06, 1997, pp. 1229–1244.
- ²Chen, F. and Boswell, R., “Helicons: The Past Decade,” *IEEE Transactions on Plasma Science*, Vol. 25, No. 6, December 1997, pp. 1245–1257.
- ³Chang-Díaz, F., “The VASIMR Rocket,” *Scientific American*, Vol. 283, 2000, pp. 90–97.
- ⁴Longmier, B., Squire, J., Olsen, C., Cassady, L., Ballenger, M., Carter, M., Illin, A., Glover, T., McCaskill, G., Chang-Díaz, F., and Bering, E., “VASIMR VX-200 Improved Throttling Range,” *48th AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit*, 2012.
- ⁵Olsen, C., *Experimental characterization of plasma detachment from magnetic nozzles*, Ph.D. thesis, Rice University, 2013.
- ⁶Light, M. and Chen, F., “Helicon wave excitation with helical antennas,” *Physics of Plasmas*, Vol. 2, 1995, pp. 1084 – 1093.
- ⁷Perry, A., Vender, D., and Boswell, R., “The Application of the Helicon Source to Plasma Processing,” *Journal of Vacuum Science and Technology*, Vol. 9, No. 2, 1991, pp. 310–317.
- ⁸Pucci, J., *An Analysis of Energy Balance in a Helicon Plasma Source for Space Propulsion*, Master’s thesis, Massachusetts Institute of Technology, 2007.
- ⁹Mulcahy, J., Browne, D., Stanton, K., Chang-Díaz, F., Cassady, L., Berisford, D., and Bengston, R., “Heat flux estimation of a plasma rocket helicon source by solution of the inverse heat conduction problem,” *International Journal of Heat and Mass Transfer*, Vol. 59, 2009, pp. 2343–2357.
- ¹⁰Berisford, D., *Thermal Phenomena and Power Balance in a Helicon Plasma*, Ph.D. thesis, University of Texas at Austin, May 2009.
- ¹¹Berisford, D., Bengston, R., and Raja, L., “Power balance and wall erosion measurements in a helicon plasma,” *Physics of Plasmas*, Vol. 17, 2010.
- ¹²Del Valle Gamboa, J., Cortés, S., Fonseca, L., Oguilve-Araya, J., Valverde, J., Martínez, C., Ixcamparij, D., Chang-Díaz, R., Longmier, B., Glover, T., Squire, J., and Chang-Díaz, F., “The VX-CR Experiment: A Thermal and Lifetime Testbed for the VASIMRTM Engine,” *32nd International Electric Propulsion Conference*, No. IEPC-2011-155, 2011.
- ¹³Bering, E., Edeen, G., Antiochos, S., DeVore, R., Thompson, B., Carter, M., Giambusso, M., Olsen, C., Squire, J., Larson, D., McFadden, J., Longmier, B., and Smith, D., “The ISS Plasma Laboratory: A Proposed Electric Propulsion On-Orbit Workbench,” *The 33rd International Electric Propulsion Conference, The George Washington University, USA*, No. IEPC-2013-104, 2013.
- ¹⁴Castro, J., Del Valle, J., Arce, N., Chinchilla, E., Echeverría, E., Lezama, D., Martínez, C., Oguilve-Araya, J., Rivera, A., Rodríguez, M., Valverde, J. and Chang-Díaz, R., Carter, M., Olsen, C., Squire, J., and Chang-Díaz, F., “VASIMR VX-CR Experiment: Status, Diagnostics and Plasma Plume Characterization,” *The 33rd International Electric Propulsion Conference, The George Washington University, USA*, No. IEPC-2013-202, 2013.
- ¹⁵Chen, F., “Plasma Ionization by Helicon Waves,” *Plasma Physics and Controlled Fusion*, Vol. 33, 1991, pp. 339–364.
- ¹⁶Chen, F., “Physics of helicon discharges,” *Physics of Plasmas*, Vol. 3, 1996, pp. 1783–1793.

- ¹⁷Ahedo, E., "Parametric analysis of a magnetized cylindrical plasma," *Physics of Plasmas*, Vol. 16, 2009, pp. 113503.
- ¹⁸Ahedo, E., "Cylindrical model of a helicon-generated plasma," *31st International Electric Propulsion Conference*, 2009.
- ¹⁹Ahedo, E. and Navarro-Cavallé, J., "Helicon thruster plasma modeling: two-dimensional fluid dynamics and propulsive performances," *Physics of Plasmas*, 2013.
- ²⁰Sternberg, N., Godyak, V., and Hoffman, D., "Magnetic field effects on gas discharge plasmas," *Physics of Plasmas*, Vol. 13, 2006, pp. 063511.
- ²¹Fruchtman, A., Makrinich, G., and Ashkenazy, J., "Two-dimensional equilibrium of a low temperature magnetized plasma," *Plasma Sources Science and Technology*, Vol. 14, 2005, pp. 152–167.
- ²²Riemann, K. U., "The Bohm criterion and sheath formation," *Journal of Physics D: Applied Physics*, Vol. 24, 1991, pp. 493–518.
- ²³Lieberman, M. and Lichtenberg, A., *Principles of plasma discharges and materials processing*, Wiley-Interscience, 2005.
- ²⁴Gildea, S., Matlock, T., and Martinez-Sanchez, M. and Hargus, W. J., "Erosion Measurements in a Diverging Cusped-Field Thruster," *32nd International Electric Propulsion Conference*, 2011.