Accurately Determining the Plasma Potential Using Emissive Probes

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Emissive probes are commonly used in electric propulsion devices to measure the plasma potential, which determines particle flows and confinement. The floating potential method is most commonly used, but is subjected to errors from a variety of effects including space-charge, geometry, and electron temperature. These effects and the most common emissive probe methods are reviewed. A discussion on how best to account for errors in plasma potential measurements cautions and guides the reader so the most accurate measurements can be made.

Nomenclature

e = electron charge
m_e = electron mass
m_i = ion mass
n_e = particle density
n_i = ion density
n_s = density at the sheath edge
T_e = plasma electron temperature
T_{em} = emitted electron temperature
u_B = Bohm velocity
v_e = electron thermal speed
V_w = potential difference between the sheath edge and a surface or wall
Γ = particle flux
Φ = normalized sheath potential
Θ = plasma electron temperature to emitted electron temperature ratio

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I. Introduction

In electric propulsion (EP) devices, electric fields accelerate ions to generate thrust. This makes measurements of the plasma potential crucial for understanding the performance and operation of Hall thrusters and other EP devices. The emissive probe is a proven diagnostic for measuring the plasma potential in a wide variety of plasmas, including the acceleration region of Hall thrusters (see Ref. 1 and references therein). The most common emissive probe technique, the floating point technique,[2] is commonly used to measure the plasma potential, but many phenomena that are often ignored affect the measurement. This conference paper addresses how space-charge effects, geometric effects, and kinetic effects can cause errors in emissive probe measurements and what can be done to correct for them.

Emissive probes are Langmuir probes which are heated until they emit electrons thermionically. Figure 1 depicts a generic emissive probe consisting of a "hairpin loop" of wire. Current run through the probe via the electrical leads heats the filament. When the probe bias potential is below the plasma potential, the emitted electrons are accelerated through the sheath and into the plasma. When the probe is biased below the plasma potential, the sheath traps the emitted electrons at the probe and does not let them reach the bulk plasma. This division is useful for determining the plasma potential.

II. Emissive Probe Theory

A. Space-Charge Effects

Electrically isolated surfaces, such as dielectrics or floating metal surfaces, requires a zero net current through the sheath. In this situation, assuming a Boltzmann distribution with temperature T_e for electrons, the ion Bohm flux, $\Gamma_i = n_s u_B$, equals the electron flux $\gamma_e = \frac{1}{4} n_s v_e \exp(-eV/T_e)$, where n_s is the plasma density at the sheath edge, v_e is the electron thermal velocity, and V is the potential with respect to the sheath edge. For a cold, planar, collecting surface that is electrically floating the potential at the surface is

$$V_w = \frac{T_e}{e} \ln\left(\sqrt{2\pi \frac{m_e}{m_i}}\right) \tag{1}$$

where V_w is the potential difference between the sheath edge and the emitting surface or wall and m_e and m_i are the electron and ion masses, respectively. This value is $\sim 3 - 5T_e$, depending on the ion species. Since electrons are much more mobile than ions ($m_e \ll m_i$) this boundary is always negative compared to the plasma potential.

Emitted electrons reduce the space-charge surrounding the planar surface, causing the sheath potential to shrink. If the emitted electron current is large enough (on the order of the collected electron current) the electric field becomes zero at the surface and the sheath potential becomes

$$V_w \approx \frac{T_e}{e}. \tag{2}$$

That condition is called space-charge limited (SCL) emission. Any further increase in electron emission is prevented from reaching the plasma by the formation of a virtual cathode, a non-monotonic potential structure which reflects the excess electrons back to the probe (see Fig. 2). The potential difference between the virtual cathode minimum and the emitting surface is typically small.[3]

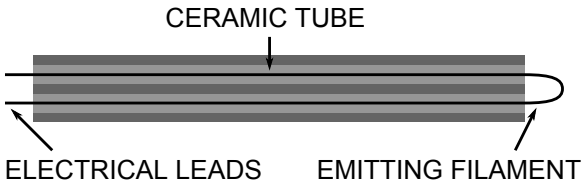

Figure 1. A schematic diagram of a generic emissive probe.

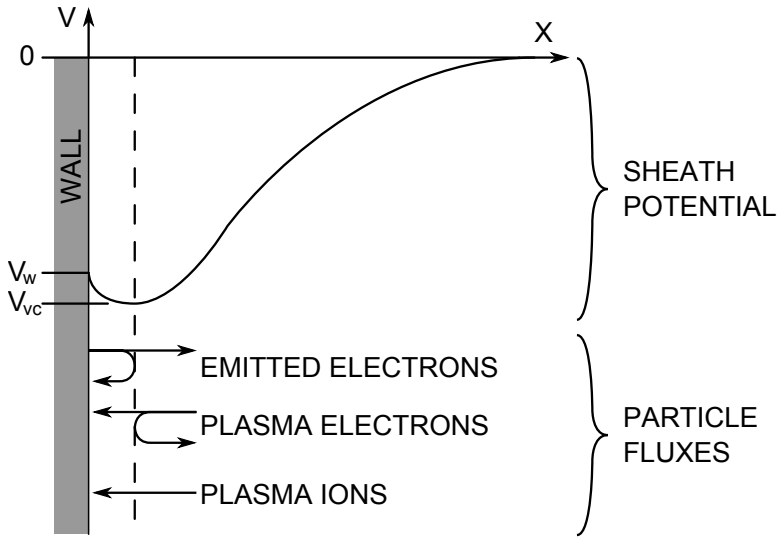

Figure 2. A schematic diagram of the sheath potential (V) of a virtual cathode and the particle fluxes near an electron emitting surface (wall). The dashed line indicates the position of the virtual cathode, X is the distance from the emitting wall, V_w is the potential of the wall, and V_{vc} is the potential of the virtual cathode minimum.

B. Cylindrical Effects

The above theory is for planar emitters. While planar emissive sheaths do exist, typically caused by secondary electron emission from the surface of vacuum vessels, emissive probes are almost always cylindrical (though occasionally spherical[4]). It is, therefore, important to understand how the geometry of the probe affects the sheath that forms around it. Emissive sheath theories in cylindrical geometries have only started to be developed in the last few years. The solution to the cylindrical sheath problem is fundamentally different from the planar sheath problem because the sheath and presheath matching.

Fruchtman et al. solved Poisson's equation in cylindrical coordinates assuming radial motion only, a Maxwellian EEDF, cold ions, and electrons emitted with zero energy.[5] This theory accounts for space-charge effects in cylindrical geometry. For large probe radius (much greater than the Debye length) they were able to find a two-scale solution to the sheath surrounding the probe when the probe was floating and highly emissive. They solved Poisson's equation in the non-neutral sheath and the plasma equation (Poisson's equation with zero electric field) in the quasi-neutral presheath. This solution matched the planar solution given by Hobbs and Wesson. For arbitrary probe radii (a) they solved Poisson's equation numerically, matching the solution far from the probe to the plasma equation (see Figure 7). When the probe radius is small ($\delta = \lambda_D/a$ is large) the sheath potential is smaller, meaning that the probe floats closer to the plasma potential. As ions converge on the cylindrical probe, their density increases because the flux is constant through the sheath. Planar emissive sheaths have a smaller potential drop than collecting sheaths because the emitting surface collects less negative charge. With a cylindrical surface the ion density is greater at the wall which further reduces the sheath potential.

In addition to OML effects on collected charged particles, the OML can alter dynamics of electrons emitted from the probe. Robertson considered the orbital effects of electrons emitted from a wire in a vacuum.[6] Electrons are emitted from the surface with a two dimensional Maxwellian distribution, so there is a velocity component in the azimuthal direction. By considering this aspect of electron emission, Robertson shows that the space-charge build up in the sheath is larger than that predicted by the one dimensional cylindrical model. Increased space-charge increases the curvature of the sheath and results in a larger sheath potential. To understand exactly how orbital motion affects emissive probe, this theory must be extended to emissive sheaths in plasma, but these initial results suggest that orbital motion may offset some of the sheath reduction caused by ion convergence.

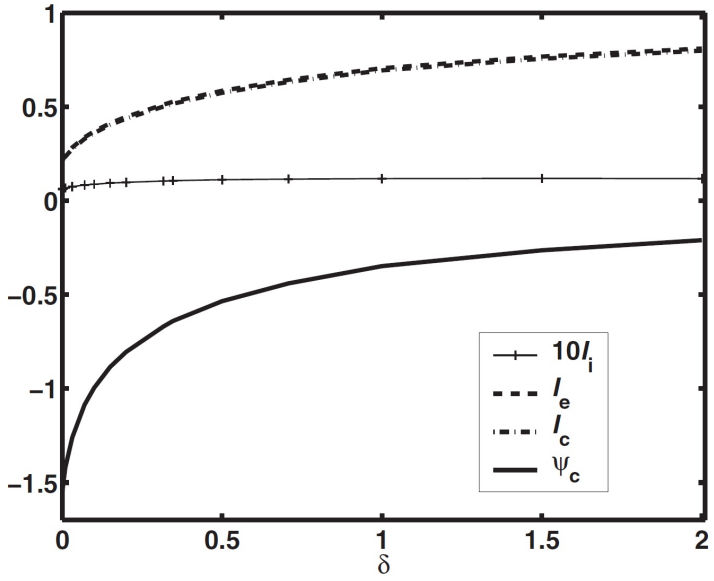

Figure 3. Normalized collected ion flux (I_i), normalized collected electron flux (I_e), normalized emitted electron flux (I_c), and the sheath potential normalized to electron temperature (ψ_c) versus inverse probe size ($\delta = \lambda_d/a$).

C. Kinetic Effects

In the fluid theory of emissive sheaths the emitted electrons were assumed to appear at the surface with zero energy. In reality, they have some velocity distribution which can have a significant effect on the floating potential of an emissive probe. Figure 4 shows the emissive sheath potential normalized to the plasma electron temperature versus the plasma electron temperature to emitted electron temperature ratio, the results of the kinetic theory.[7] In the figure, $\Phi = -e\phi/T_e$ and $\Theta = T_e/T_{em}$ where T_{em} is the temperature of the emitted electrons. When the plasma electron temperature is much larger than the emitted electron temperature the kinetic effects are small and the sheath potential is close to that predicted by fluid theory. However, when the plasma electron temperature is within an order of magnitude of the emitted electron temperature the sheath potential is significantly reduced. Since most emissive probes emit electrons with a temperature of ~ 0.2 eV, in a plasma with $T_e = 1$ eV, the sheath potential is half of that predicted by fluid theory, so the kinetic effects can be quite significant.

These results were corroborated with PIC codes.[7] Experiments were performed in the afterglow of an RF plasma to measure the emissive sheath potential as a function of electron temperature ratio. During the afterglow the plasma cooled from a few eV to less than 0.1 eV, making it a good setup for investigating these kinetic effects. It was found that the sheath potential did drop off significantly, but only when the plasma electron temperature was less than 4 times that of the emitted electrons. For ratios around 5, the sheath potential was unexpectedly large. This effect may be caused by a two-stream instability between the emitted and collected electrons and warrants further study.

III. Methods

A. Floating Point with Large Emission

The first major step in the application of emissive probes was developed by Kemp and Sellen in 1966.[2] They made use of the fact that as emission increases, the floating potential of an emissive probe approaches the plasma potential.[8] The floating potential of a probe is the intersection of the I-V curve with the load line, the current-voltage curve of the electronics used to measure the current. As the emission of the probe is increased from no emission, the floating potential of the probe rises rapidly at first, but plateaus at the plasma potential.[2] After this point, increasing the emission only slightly changes the floating potential due to space-charge effects.[3] Kemp and Sellen claim that a good measure of the plasma potential is the linear

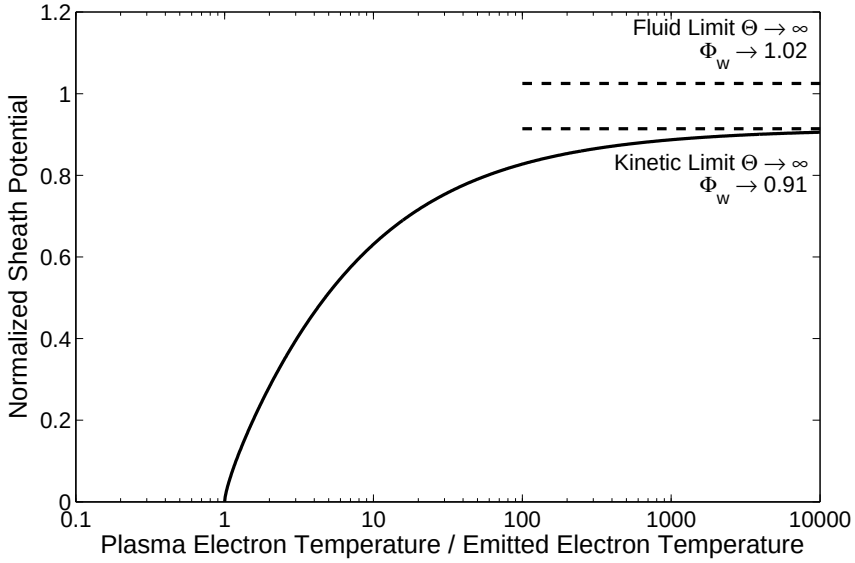

Figure 4. The emissive sheath potential normalized to the plasma electron temperature (Φ) as a function of plasma electron temperature to emitted electron temperature ratio (Θ).

extrapolation and intersection of the lines in the two regimes. Generally when using this technique, rather than reading multiple floating potentials at different emissions, the emission is increased until the floating potential saturates. The technique is known as the floating potential in the limit of large emission and many assume the value of the floating potential at saturation to be the plasma potential.[2,9–11]

The largest source of error in this technique comes from the fact that the floating potential of a highly emitting probe is, in fact, significantly less than the plasma potential. The size of this difference may be between Te/e and 2Te/e and depends on the details of the EEDF. Hobbs and Wesson considered a floating emitting surface in a plasma and solved Poisson's equation with Bohm's criterion modified for an emitting surface and determined that the potential of the floating surface was approximately Te below the plasma potential in the limit of large emission.[12] Numerical simulations show that a floating probe in the limit of large emission (emission current greater than collection current) will float $1.5T_e$ below the plasma potential.[13] Additionally, in plasmas with large fractions of energetic electrons, such as beam plasmas or double plasma devices, an emissive probe in the limit of large emission can float near the energetic electron energy and will not provide an accurate measure of the plasma potential.[14,15] If the EEDF is expected to be constant for all measurements, then the differences between the plasma potential measurements (which are often of greatest interest) would be unaffected although the absolute measurements would be affected. To determine if the floating potent measurement is seriously affected by this offset, the floating potential technique can be compared to the inflection point technique (see following section).

B. Inflection Point in the Limit of Zero Emission

The inflection point method was developed by Smith et al. in an attempt to reduce the space-charge effects associated with the floating point method.[16] To use this technique multiple I-V traces must be taken, each with a different value of the temperature limited emission current. The inflection point of each I-V trace is determined as a function of the temperature limited emission current (the maximum emission current for a given filament temperature). By extrapolating the inflection point to zero emission current the plasma potential can be determined without the errors introduced by space charge effects.

The uncertainty of this technique is more complex than the others. Theory predicts that it is accurate to within $T_e/10e$, which is typically less than the uncertainty from noise, so the method can be assumed to be accurate.[17] To accurately determine the inflection point, the I-V trace data must have a high enough signal to noise ratio for differentiation to yield meaningful data. A second source of uncertainty is extrapolating the emission current versus inflection point data to zero emission. Here is it important to take data over a

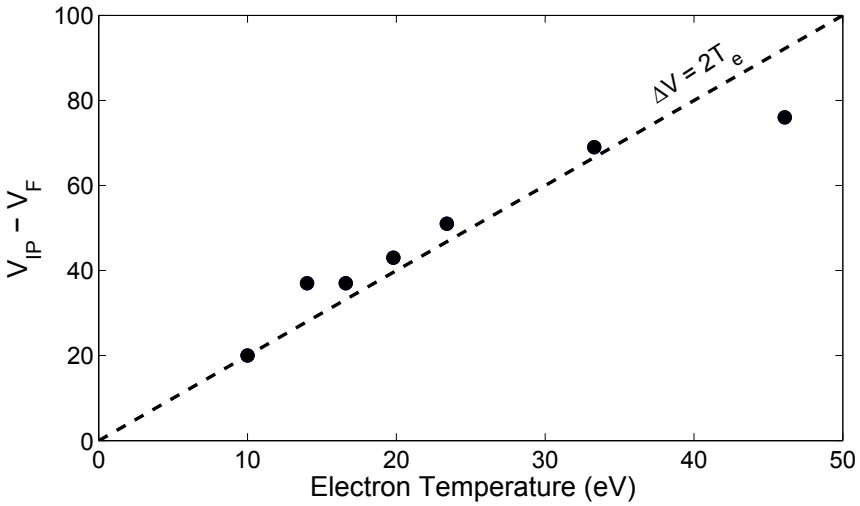

Figure 5. The circles show the difference between the inflection point in the limit of zero emission technique (V_{IP}) and the floating point technique (V_F) and the dashed line is a reference line indicating where a difference of $2T_e/e$ would fall.

wide range of emission currents.

C. Method Comparison

It has been observed that the various emissive probe techniques do not agree with each other.[10,16,17] A dedicated study regarding this question was recently published.[17] The experiments were performed in a 2kW cylindrical Hall thruster plasma with effective electron temperatures (Te,eff) between 10 and 50 eV. Details of the experiment can be found in Ref. 17.

Figure 5 shows the difference between the inflection point method and the floating point method as well as the difference between the separation point method and the floating point method.[17] The trend shows that the inflection point method yields a value $\sim 2T_e/e$ above the floating point method, which is fairly consistent with the $1.5T_e/e$ predicted from fluid theory and PIC simulations.[13] The authors conclude that the inflection point in the limit of zero emission more accurately measures the plasma potential than the floating point method. Additionally, the difference between the separation point method and the floating point method varies wildly compared to the other techniques, suggesting that the separation point technique is an inaccurate method for measuring the plasma potential.

Although the inflection point technique may be the most accurate, it is not the best technique for every experiment (see Sec. 7.2). The inflection point method cannot measure temporal variations easily and is difficult to use in high energy density plasmas. If the electron temperature is small, so will be the difference between the inflection point technique and the floating point technique. Additionally, if the electron temperature is constant, the floating point technique's error is constant, so relative changes in the plasma potential and electric field measurements will still be correct. The authors suggest that the floating point method be compared to the inflection point in the limit of zero emission method before use, though, to ensure the measurements will be accurate.

IV. Discussion

The floating potential technique is the emissive probe method most often chosen for making plasma potential measurements in EP devices such as Hall thrusters. Floating potential measurements can be made more quickly than inflection point measurements, reducing the amount of time the probe spends in the destructive environment of the Hall thruster plasma. Additionally, sweeping the probe bias for the inflection point technique can draw large currents to the probe, melting it in regions safe for the floating point technique. Using a translation stage, the floating potential of a highly emissive probe can be measured

as a function of axial distance from the anode, through the acceleration region, and into the plume. From these data one aims to extract the plasma potential.

In some experiments, the floating potential itself is assumed to be a good approximation of the plasma potential. This is a reasonable assumption in devices with constant (over space) and small electron temperatures ($T_e \ll e\phi$), but such is not the case in Hall thrusters. Considering only planar space-charge effects, the plasma potential is $1.5 - 2T_e$ above the emissive probe floating potential. Due to the temperature gradient in the Hall thruster acceleration region[18] the potential difference between the plasma potential and emissive probe floating potential will vary with position, resulting in an inaccurate measurement of the electric field.

If the electron temperature is known the floating potential measurement can be corrected to approximate the plasma potential. Doing so, however, can result in a non-monotonic potential structure with a potential rise just before the potential drop that accelerates the ions. Conventional wisdom dictates that this potential structure is non-physical. If the true potential structure is indeed monotonic, then the electron temperature scale adjustment over compensates on the anode side of the acceleration region. Debye lengths can easily be on the same length scale as the diameter of emissive probe filaments (10s - 100s of μm) in which case cylindrical effects are important. If the Debye length changes as a function of position in the acceleration region then the electron temperature factor difference between the floating potential and plasma potential changes, making a simple correction factor inaccurate. Kinetic effects are not expected to play a large role because the plasma electron temperature is typically much larger than the emitted electron temperature.

V. Conclusion

Determining the plasma potential is key for understanding ion acceleration in electric propulsion devices. Emissive probes are a powerful tool for making those measurements, though great care must be taken in analyzing the results. A wide variety of factors including space-charge effects and geometry can affect the measurements, especially in plasmas with temperature and density gradients. Accounting for these factors is key for making accurate measurements with emissive probes.

VI. Acknowledgements

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