

Theoretical Model of a Lanthanum Hexaboride Hollow Cathode

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Abstract: A model to predict the plasma properties inside a thermionic hollow cathode as a function of operational conditions and geometry is presented. The hollow cathode features a lanthanum hexaboride (LaB6) insert, which is capable of emitting current densities as high as 10^5 Am^{-2} at temperatures of about 1900 K, along with a tantalum orifice plate located at the downstream end of the cathode tube. The model self-consistently computes the plasma parameters in both the emitter and orifice regions. A simple semi-empirical relation is suggested to evaluate the plasma penetration depth in the cathode interior, which is of primary importance to establish the plasma conditions. The heat transfer mechanisms and the related temperature gradients along the cathode are evaluated with the aid of a dedicated thermal model, which is coupled to the plasma model and accounts for temperature-dependent material properties. A parametric study of the cathode performance was conducted to assess the dependence of the power consumption and operational lifetime on discharge current and mass flow rate, as well as on the geometry. The results are in good agreement with both theoretical and experimental trends found in the literature as well as with experimental data collected by Alta. Further developments will include a deeper investigation into the cathode erosion phenomena, along with a broader comparison with empirical data.

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Nomenclature

F_{ij}	= view factor from surface i to surface j
I_d	= discharge current
j_{em}	= thermionic current density
j_{er}	= electron thermal current density
j_i	= ion current density
j_{th}	= thermal current density
J	= radiosity
K	= semi-empirical parameter
K_i	= electron-impact ionization rate
k_B	= Boltzmann constant
L_{eff}	= effective emission length
L_o	= orifice length
L_t	= cathode lifetime
$\dot{m}$	= mass flow rate
n	= plasma number density
n_0	= number density of neutrals
p	= plasma pressure
P	= power
q	= electron charge
r_e	= emitter inner radius
r_o	= orifice radius
R	= plasma resistance
R_g	= specific gas constant
T_e	= electron temperature
T_c	= emitter surface temperature
T_o	= orifice surface temperature
V	= voltage
α	= plasma degree of ionization
$\langle \varepsilon_i \rangle$	= average ionization energy loss
ε	= emissivity
μ	= dynamic viscosity
ϕ_{eff}	= effective work function
σ_{SB}	= Stefan-Boltzmann constant

Subscripts

o	= orifice region
e	= emitter region

I. Introduction

THERMIONIC hollow cathodes are employed as sources of electrons in a variety of applications, including electric thrusters and plasma contactors. Both ion thrusters and Hall effect thrusters require the emission of electrons to ionize the propellant and to neutralize the outgoing ion beam. The design of a hollow cathode aims at minimizing the power and propellant consumption while maintaining a lifetime in the order of 10^4 hours. Since the cathode performance depends on the operating conditions and geometry¹, and the small size hinders the plasma diagnostics², simple numerical tools are desirable to guide the cathode design for a given application. At present, several numerical models are available to predict the plasma properties in hollow cathodes, especially for Hall and ion thrusters^{3–5}, and for magnetoplasmadynamic thrusters^{6,7}. However, these models require the input of experimental data or assumptions are made to set the plasma density, the electron temperature or the coupling voltage to the keeper. The model described in this paper is conceived to be self-consistent, thus not relying on experimental data as input. As shown in the following, only one plasma parameter is evaluated by means of a phenomenological method based on the available literature. A reduced-order approach appears to be the most practical compromise between including details and keeping the model flexible and time-saving.

In a single-channel hollow cathode the electrons are directly extracted via thermionic emission from a thin-walled refractory metal tube, at a surface temperature as high as 2500-3000 K. Electrons are emitted from a region of the cathode called active zone, located adjacent to the exit section of the tube⁸. In the so-called orificed hollow cathode, an orifice plate is placed at the downstream end of the tube to increase the internal pressure. The electrons are extracted from an insert made of a low-work function material at a lower temperature with respect to refractory metals, leading to a longer lifetime. The dispenser impregnated cathode features a barium-based emitter which is responsible for the formation of a low-work function (~ 2 eV) superficial layer⁹, providing current densities as high as 10^5 Am^{-2} at a surface temperature of about 1300 K. Since chemical reactions are involved in the establishment of the emitting layer, this type of cathode is highly susceptible to oxygen contamination. This problem is mitigated by the adoption of a rare-earth emitter, such as lanthanum hexaboride (LaB_6), where the emission comes from the bulk material. A major drawback of LaB_6 is the relatively high work function¹⁰ (~ 2.7 eV), resulting in an operating temperature of about 1900 K for a current density of 10^5 Am^{-2} . Given the reactivity of LaB_6 with many refractory metals at high temperatures, care must be taken in the design of the cathode body. Nevertheless, the evaporation rate¹¹ is lower with respect to dispenser cathodes for emission current densities lower than $1.5 \times 10^5 \text{ Am}^{-2}$. Moreover, a LaB_6 emitter does not require lengthy activation procedures as needed for the diffusion of the emitting compound in the dispenser cathode. LaB_6 is thus a valuable option for long-lasting cathodes, even for low-current applications¹².

The cathode schematic adopted in this study is shown in Fig. 1. A LaB_6 emitter is placed inside the main refractory metal tube and is in contact with the orifice plate, which is considered to be made of the same material of the tube. The insert reaches the emission temperature with the aid of a heater wrapped around the tube, which is provided with thermal shields to reduce the radiation losses. The cathode tube and heater are surrounded by the keeper electrode, whose function is to help the extraction of the electrons to the discharge plasma and to protect the cathode from the ion bombardment.

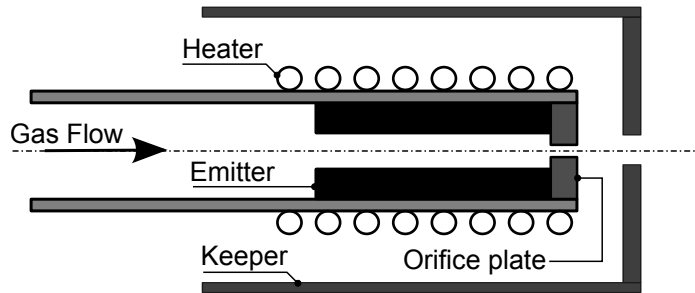


Figure 1. Schematic of an orificed hollow cathode.

II. Model Description

The model consists of three main modules: the orifice plasma model, the insert plasma model, and the cathode thermal model¹³. The models are consecutively solved by means of an iterative procedure to calculate the plasma properties, along with the temperature profile and heat exchanges in the cathode assembly. The keeper electrode is not included in the model, since the cathode performance is mainly determined by the insert and the orifice regions. The plasma properties are averaged in each control volume (Fig. 2), and steady-state conditions are assumed. The quasi-neutral plasma is considered to be a mixture of three perfect gases: thermalized electrons, singly-charged positive ions, and neutrals. Ions and neutrals are considered to be at the same temperature of the cathode wall. The model does not rely on the Saha equation, since the gas transit time is of the same order of magnitude of the characteristic time for plasma ionization, namely $\tau_i = (n_0 K_i)^{-1} \sim 10^{-6}$ s. The existence of a double-sheath at the constriction between the insert and orifice regions is assumed, as suggested in Ref. 14 and experimentally observed in Ref. 15.

A. Orifice Model

Due to its small cross-section, which must be able to carry the required discharge current, the orifice is the primary driver in determining the plasma conditions in the cathode and, ultimately, the overall performance. Following Ref. 16 and Ref. 17, the orifice model is made of three equations expressing balances of ion flux, plasma power, and pressure to compute the plasma number density, the electron temperature, and the density of the neutrals. The first equation states that the volumetric ion production rate, $\dot{n}_{ion}$, is balanced by the ion loss rate, $\dot{n}_{out}$, as follows

$$\underbrace{q\pi r_o^2 L_o n_0 n K_i}_{\dot{n}_{ion}} = \underbrace{j_i (2\pi r_o L_o + \pi r_o^2) + j_{th} \pi r_o^2}_{\dot{n}_{out}} \quad (1)$$

where j_i and j_{th} are the Bohm and the thermal current densities, respectively. The ionization rate coefficient, K_i , depends on the electron temperature considering a direct electron-impact ionization⁹. According to the plasma power balance, the resistive heating, P_R , is balanced by the energy losses due to the ionization of neutrals, P_{ion} , electron convection, P_{conv} , and ion losses across the double sheath, P_{ds} , namely

$$\underbrace{RI_d^2}_{P_R} = \underbrace{\dot{n}_{ion} \langle \varepsilon_i \rangle}_{P_{ion}} + \underbrace{2 \frac{k_B}{q} I_d (T_{e,o} - T_{e,e}) + P_{ds}}_{P_{conv}} \quad (2)$$

where $\langle \varepsilon_i \rangle$ is the average ionization energy loss. Preliminary calculations showed that the power loss across the double sheath is much smaller than the other terms in Eq. 2, thus it was neglected to relax the coupling between the insert and orifice regions. The plasma resistance, R , is evaluated on the basis of the resistivity taking into account electron-ion and electron-neutral collision events. The last balance equates the plasma pressure as computed from the kinetic theory of gases with the pressure of a continuum flow choked at the orifice exit, even though a transitional model would be more appropriate as resulting from typical Knudsen

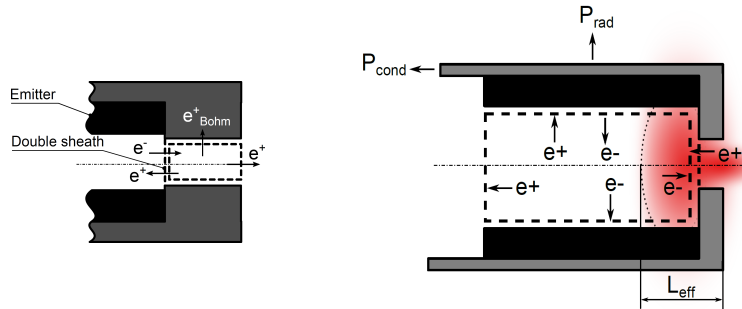


Figure 2. Illustration of the orifice (left) and emitter (right) regions.

numbers between 0.01 and 0.05¹⁸. The pressure equation writes

$$(n + n_0) k_B T_o \left(1 + \alpha \frac{T_e}{T_o} \right) = \frac{\dot{m}}{\pi r_o^2} \sqrt{\frac{R_g T_o}{\gamma} \left(1 + \alpha \frac{T_e}{T_o} \right)} \quad (3)$$

where T_o is the orifice wall temperature and $\alpha = n / (n + n_0)$ is the degree of ionization.

B. Emitter Model

The emitter model is similar to the orifice model, with an additional current density balance taking into account the thermionic emission of electrons. The unknowns are the plasma parameters and the voltage fall at the cathode sheath. Defining an effective emission area $A_{eff} = 2\pi r_e L_{eff}$, where L_{eff} is the effective emission length, the current density equation is

$$j = \frac{I_d}{A_{eff}} = j_i + j_{em} - j_{er} \quad (4)$$

where j_i is the Bohm current density, j_{em} is the thermionic current density, and j_{er} is the thermal flux of electrons recombining on the insert wall. The field-enhanced thermionic emission gives the following current density

$$j_{em} = D T_c^2 \exp \left(-\frac{q \phi_{eff}}{k_B T_c} \right) \quad (5)$$

where T_c is the emitter wall temperature, D is a material-dependent coefficient¹⁰ and ϕ_{eff} is the effective work function evaluated on the basis of the electric field at the cathode sheath¹⁹. The conservation of the ion flux states that the ion production rate due to direct electron-impact ionization, $\dot{n}_{ion}$, and the contribution due to the incoming ions from the orifice, $\dot{n}_{in}$, are balanced by the ions leaving the emitter control volume, $\dot{n}_{out}$, as follows

$$\underbrace{q \pi r_e^2 L_{eff} n_0 n K_i}_{\dot{n}_{ion}} + \underbrace{j_i \pi r_o^2}_{\dot{n}_{in}} = \underbrace{j_i (A_{eff} + \pi r_e^2 - \pi r_o^2) + j_{th} \pi r_e^2}_{\dot{n}_{out}} \quad (6)$$

The power balance takes into account the plasma heating due to the energy content of the ions coming from the orifice, $P_{i,o}$, the Joule effect heating, P_R , and the power carried by the emitted electrons, P_{em} . The plasma is cooled by the ions leaving the control volume, P_i , and the electrons recombining on the emitter surface, P_{er} , as well as by the effect of the electron current convection, P_{conv} . The power equation writes

$$\begin{aligned} & \underbrace{j_i \left(V_{ds} + 2 \frac{k_B T_c}{q} \right) \pi r_o^2}_{P_{i,o}} + \underbrace{R I_d^2}_{P_R} + \underbrace{j_{em} \left(V_c + \frac{3}{2} \frac{k_B T_c}{q} \right) A_{eff}}_{P_{em}} = \\ & = \underbrace{\dot{n}_{out} \left(\langle \varepsilon_i \rangle + 2 \frac{k_B T_c}{q} \right)}_{P_i} + \underbrace{j_{er} 2 \frac{k_B T_e}{q} A_{eff}}_{P_{er}} + \underbrace{\frac{5}{2} \frac{k_B T_e}{q} I_d}_{P_{conv}} \end{aligned} \quad (7)$$

where V_{ds} is the potential drop across the double sheath, evaluated according to Ref. 5, and V_c is the cathode voltage fall. The pressure equation is obtained by balancing the expression given by the kinetic theory and the pressure at the downstream end of the control volume, following the gasdynamic derivation found in Ref. 17. The resulting balance writes

$$(n + n_0) k_B T_c \left(1 + \alpha \frac{T_e}{T_c} \right) = \sqrt{\dot{m} \frac{16\mu}{\pi r_e^2} R_g T_o L_o + p_o^2} + \frac{1}{2} \frac{p_o}{R_g T_o} \bar{u}^2 (1 + K_l) \quad (8)$$

being μ the dynamic viscosity computed by means of the mean free path method²⁰, p_o the sonic pressure at the orifice exit, $\bar{u}$ the average velocity in the orifice, and $K_l \simeq 0.5$ a loss coefficient due to the restriction of the cross-section. The total voltage, V_t , is obtained from the sum of the potential drops from the emitter to the orifice

$$V_t = V_c + V_p + V_{ds} + V_o \quad (9)$$

where V_p and V_o are the ohmic potential drops in the emitter and orifice regions, respectively.

The effective emission length is a function of the cathode geometry, the mass flow rate and, to a lesser extent, the discharge current^{2,21}. An analysis of the available experimental data suggested an inverse proportionality of the effective emission length to the internal pressure, p , thus the semi-empirical parameter K was introduced

$$K = p \cdot L_{eff} \quad (10)$$

and a K value equal to 15 m-Pa was chosen after a review of the empirical data^{4,22}. Nevertheless, K was also used as a free parameter to fit the experimental data, as shown in Section III-E.

C. Thermal Model

The temperature profile along the cathode axis is evaluated with the aid of a lumped-parameter thermal model, where the cathode is divided in elements to draw an equivalent thermal network²³. The model takes into account the heat exchanges by conduction and radiation, assuming a background temperature of 0 K, along with gas convection at the inner surface. Temperature-dependent expressions for the thermal conductivity of the cathode materials are included as found in the literature. The results of the plasma models are required to give the following input to the thermal model: the power deposition on the emitter due to the recombining ions and the backstreaming electrons, and the power removed by the emitted electrons. In addition, the evaporation of the emitter material is taken into account with a proper power output. The power balances are imposed considering two kind of nodes, respectively identified by the temperature and the radiosity of each surface. Combining the various heat-transfer terms, the power balance for the first type of node writes

$$-P_{in} + (\sigma_{SB}T_i^4 - J_i) \frac{A_i \varepsilon_i}{1 - \varepsilon_i} + \sum_j (T_i - T_j) \frac{kS_{ij}}{b_{ij}} = 0 \quad (11)$$

where P_{in} stands for the external power input to node i and the material-dependent emissivity, ε_i , is a function of temperature. For the second kind of node the power balance is expressed as

$$\sum_j (J_i - J_j) A_i F_{ij} - (\sigma_{SB}T_i^4 - J_i) \frac{A_i \varepsilon_i}{1 - \varepsilon_i} = 0 \quad (12)$$

where J stands for the surface radiosity and F_{ij} is the geometric view factor from surface i to surface j . The temperatures of the nodes are obtained by solving the set of equations having fixed a cathode base temperature of 1000 K.

The cathode lifetime, L_t , is calculated as the time needed for the evaporation to halve the initial mass of the insert

$$L_t = \frac{1}{2} \frac{\rho s}{W}, \quad (13)$$

where ρ is the density, s is the initial thickness of the emitter, and W is the temperature-dependent evaporation rate¹⁰. Even though this definition does not consider many important life-limiting mechanisms, such as poisoning due to impurities in the gas and sputtering of various cathode components due to energetic ions, the estimations are expected to give the order of magnitude of the actual lifetime. As a matter of fact, the deposition of evaporated material on the emitter surface, as well as the reduction of the work function due to preferential sputtering could lead to an increase in the lifetime^{9,24}.

D. Erosion Evaluation

As a step forward in the computation of the cathode lifetime, an additional iterative cycle was added to the original code to evaluate the emitter erosion and the change of the operational parameters over time²⁵. First, the time-step for the lifetime discretization is defined, along with the operating conditions and the initial cathode geometry. The whole cathode model, comprising both the plasma and the thermal models, is then solved to yield the plasma properties and the wall temperatures, which are held constant over the time-step dt . The evaporation rate is evaluated with the value of the emitter temperature, and the erosion depth is found as $d = Wdt/\rho$. The insert inner radius is updated by adding the evaporated thickness, and the computation is repeated with the new emitter geometry. The iteration stops when the insert mass drops below the fifty percent of its initial value. The evaporation of the orifice was also checked and, given the lower evaporation rate of refractory metals with respect to LaB₆, no significant effect is expected on the cathode lifetime.

III. Model Results

The results of the theoretical model are presented in the following sections, along with a preliminary validation of the model. The baseline cathode geometry features a 0.3 mm-thick tantalum tube 7.5 mm in external diameter and about 20 mm in length. The LaB₆ emitter is a hollow cylinder 3 mm in internal diameter and 6 mm in length. The orifice is shaped as a cylindrical channel 0.4 mm in diameter and 0.36 mm in length. The cathode operates at a discharge current ranging from 1 to 3 A and at mass flow rates between 0.05 and 1 mg/s of xenon. The figures are provided with dashed lines obtained by means of a shape-preserving piecewise cubic interpolation to guide the eyes.

A. Plasma Parameters

The effect of the operating conditions on the plasma parameters is presented for the baseline cathode geometry. The average plasma densities in the insert and the orifice regions are reported in Fig. 3 as a function of discharge current at 0.3 mg/s Xe mass flow rate. The nearly linear dependence of the plasma densities on discharge current is consistent with both theoretical and experimental trends found in the literature^{26,27}. The electron temperatures in the two cathode regions are shown as a function of mass flow rate in Fig. 4, displaying a decreasing trend as also found in Ref. 17 with a negligible dependence on the considered range of discharge current. The voltage fall at the cathode sheath is found to decrease with increasing discharge

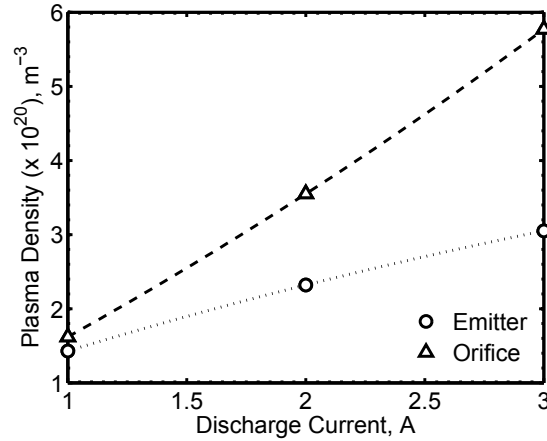


Figure 3. Plasma density as a function of discharge current at 0.3 mg/s Xe.

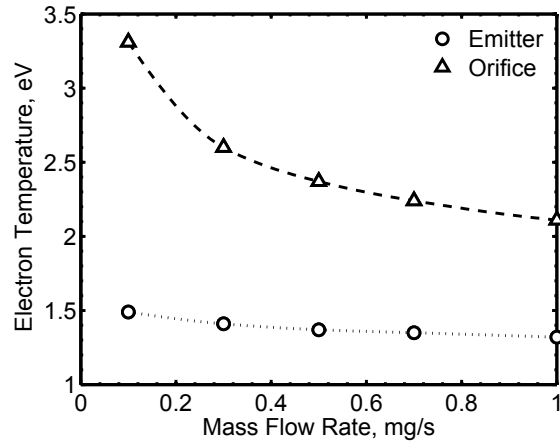


Figure 4. Electron temperature as a function of mass flow rate at 3 A.

current, as seen in Fig. 5 and consistently with the estimations of Goebel⁹. The dependence of the effective emission length on mass flow rate is reported in Fig. 6 at 3 A, showing a decreasing trend as also theoretically and experimentally found in Ref. 26. The plasma penetration depth was nearly insensitive to variations in the discharge current from 1 to 3 A, as a consequence of the trend in the insert pressure as follows from Eq. 10.

B. Power Distribution

The power consumption in the cathode assembly is divided in the contributions accounting for the voltage drop at the emitter sheath, P_e , the plasma Ohmic heating in the insert region, P_p , the potential drop across the double sheath, P_{ds} , and the plasma Ohmic heating in the orifice region, P_o . All of the power contributions increase with increasing discharge current, as seen in Fig. 7 at 0.3 mg/s Xe mass flow rate. In addition, the power fraction to the insert region decreases with increasing discharge current, due to a higher resistive heating in the orifice region. The effect of geometrical variations on the power consumption is evaluated at the reference conditions of 3 A and 0.3 mg/s. As shown in Fig. 8, the model predicted an increase in power consumption of about 30% while halving the orifice diameter with respect to the baseline value at the cost of a lifetime reduction from 6×10^5 to 2×10^4 hours. The local power deposition in the orifice region increases with increasing the orifice length, with minor effects on the overall power consumption as seen in Fig. 9.

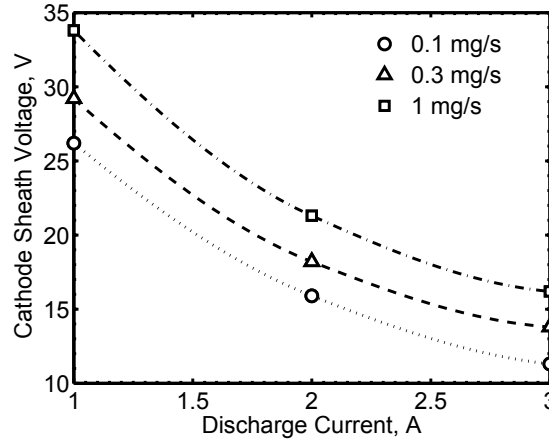


Figure 5. Cathode sheath voltage as a function of discharge current.

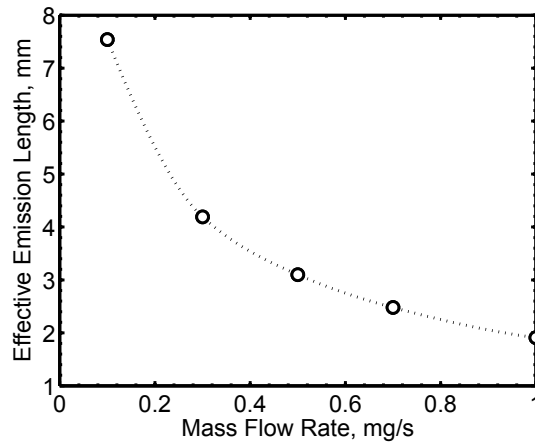


Figure 6. Effective emission length as a function of mass flow rate at 3 A.

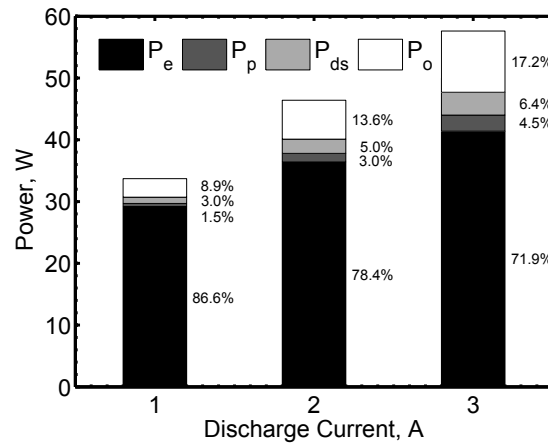


Figure 7. Power distribution as a function of discharge current at 0.3 mg/s.

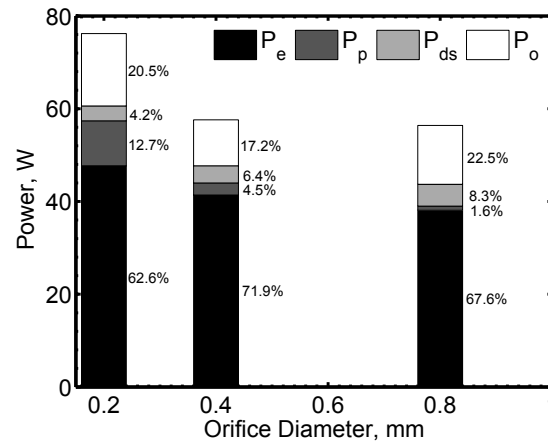


Figure 8. Power distribution as a function of orifice diameter.

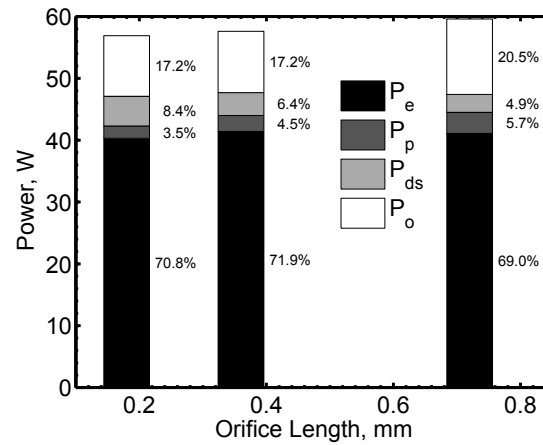


Figure 9. Power distribution as a function of orifice length.

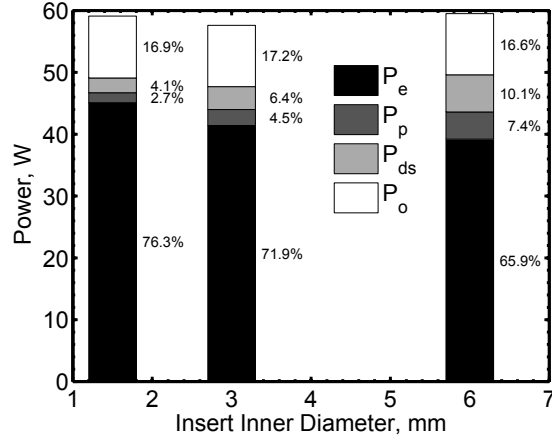


Figure 10. Power distribution as a function of insert inner diameter.

The predicted lifetime slightly decreases with increasing the orifice length from 0.18 to 0.72 mm, still remaining in the order of 10^5 hours, as a consequence of an increase of about 50 K in the emitter temperature. While enlarging the insert, the power contribution coming from the sheath voltage decreases, as shown in Fig. 10. The overall consumption is not significantly affected by an increase in the insert inner diameter, whereas the lifetime is slightly higher due to a reduction in the emitter temperature of about 100 K while enlarging the insert from 1.5 to 6 mm diameter.

The thermal model which is coupled to the plasma models allows for the evaluation of the power exchanges among the various elements of the cathode assembly. The heat transfer at the reference conditions is shown in Fig. 11, where the various terms (expressed in watts) include conduction and radiation, along with the power input from the plasma. By inspection of the latter figure, it is apparent the predominance of conduction over radiation in the heat transfer along the cathode as also reported in Ref. 17.

C. Temperature Profile

The temperature at the emitter surface is shown in Fig. 12 for the baseline cathode as a function of current and at 0.3 mg/s Xe. This temperature directly impacts the cathode lifetime, and it is found to increase with discharge current and mass flow rate, consistently with both theoretical and experimental trends found in the literature^{17,19,27}. The trend in the orifice wall temperature was found to be similar to the emitter temperature, even though with higher values in the investigated range of operating conditions and showing a maximum difference of about 50 K at 3 A and 0.1 mg/s Xe. The temperature profile along the cathode axis is illustrated in Fig. 13 considering the reference operating conditions. As already stated by Kaufman²⁸,

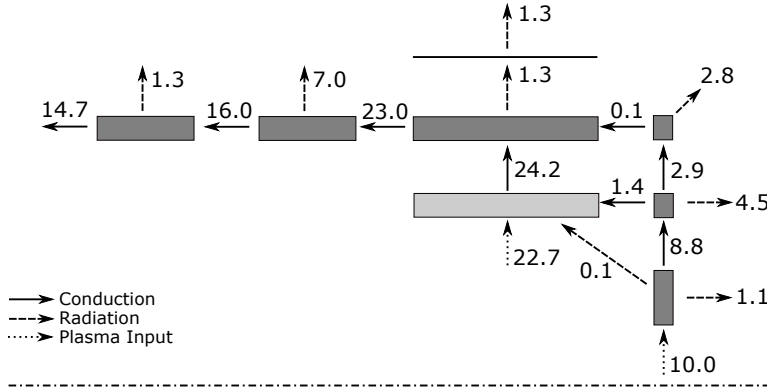


Figure 11. Heat exchanges (in watts) among the cathode elements.

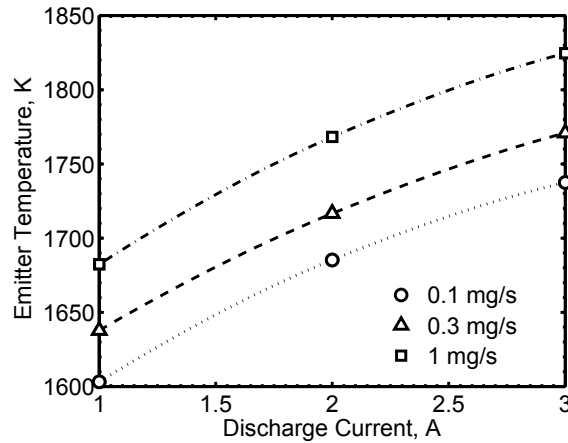


Figure 12. Emitter temperature as a function of discharge current.

thermal shielding has no significant effect on the temperature profile along the cathode. The model correctly predicts this finding, since no substantial difference exists in the maximum temperature or in the temperature distribution along the cathode between the two cases. The temperature of the shield was computed to be about 300 K lower with respect to the facing surface of the cathode tube.

D. Cathode Lifetime

The cathode lifetime as computed from the evaporation of the emitter is shown for the baseline geometry as a function of mass flow rate in Fig. 14. The lifetime decreases with both discharge current and mass flow rate, as a direct consequence of the trends in the emitter temperature. In particular, an increase in mass flow rate reduces the emission length due to a higher pressure, leading in turn to a higher operating temperature.

The erosion computation is presented in Fig. 15 for the baseline cathode operated at the reference conditions, in terms of the increase in the insert inner diameter and the decrease in percentage insert mass with respect to the initial value. The temperature at the emitter surface drops from about 1755 K to about 1716 K over a time span of 6×10^5 hours, and the predicted average erosion rate is about 0.02 ng/C. The decreasing trend of the emitter temperature slightly increases the lifetime prediction with respect to the definition given in Section II-C.

E. Comparison with Experimental Data

A hollow cathode with the geometry described in Section III was tested by Alta with two different orifice diameters (0.4 mm and 0.3 mm). For both cases the cathode-to-keeper voltage is compared with the total voltage drop predicted by the model as expressed in Eq. 9, at two mass flow rates and assuming a K parameter equal to 15 m-Pa. The results are shown in Fig. 16 and in Fig. 17 for the 0.4 mm diameter and for the 0.3 mm diameter orifice, respectively. Although the general trend seems to be captured by the model, it overestimates the experimental data from 15 % to 40 %, depending on the mass flow rate. However,

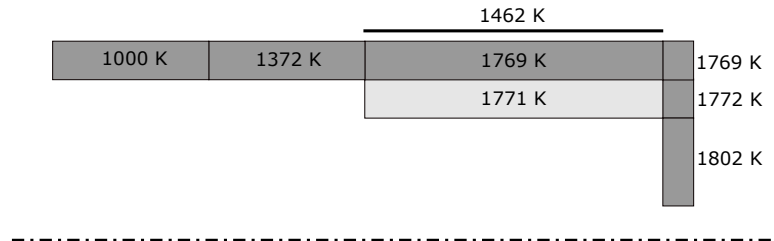


Figure 13. Temperature profile at 3 A and 0.3 mg/s.

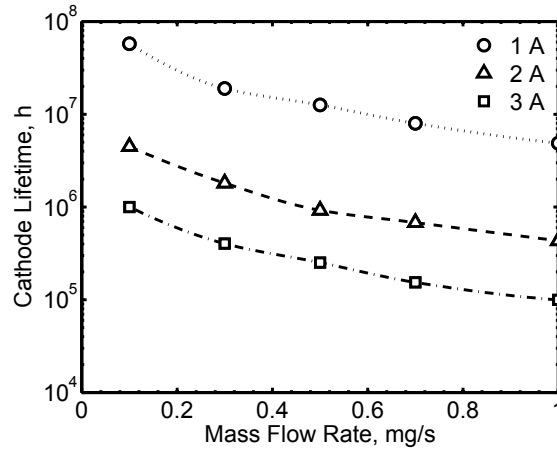


Figure 14. Cathode lifetime as a function of mass flow rate.

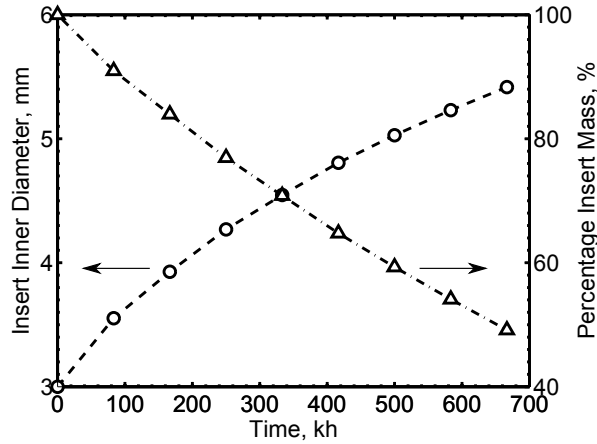


Figure 15. Cathode erosion as a function of time at 3 A and 0.3 mg/s.

since a negative keeper voltage drop estimated in the order of 5-10 V is expected, the overestimation should be actually lower. The experimental voltage decreases by increasing mass flow rate, as also found in the literature^{5,17,22}, whereas the model predicts an opposite trend. This is likely due to the assumption of an effective emission length decreasing while increasing the internal pressure, leading to an excessively high sheath voltage at the emitter surface. The electrical characteristic comparison for the 1.5-cm-diameter LaB₆ cathode tested by Goebel and Chu²² is shown in Fig. 18 at a mass flow rate of 0.98 mg/s. The K parameter was set equal to 10 m-Pa to approximately match the experimental emission length. Since the orifice length was not specified, the length-to-diameter aspect-ratio (AR) was used as a parameter and varied over a range of typical values found in the literature. The difference between the theoretical and the experimental data reaches a maximum value of about 20% at 10 A for the AR6 case. It is noted that the empirical data include the anode voltage drop, which is not considered in the theoretical model. The predicted electron temperature of about 1.7 eV is close to the measured values, which are in the order of 2 eV. The average plasma density in the insert region was computed to be in the order of 10^{20} m^{-3} , consistently with the measured values. The last comparison refers to the experimental data collected at the MIT during the testing of a 0.5-cm-diameter LaB₆ cathode²⁹. The electrical characteristics are reported in Fig. 19 at 0.29 mg/s with K equal to 4 m-Pa, showing a maximum discrepancy of about 25%.

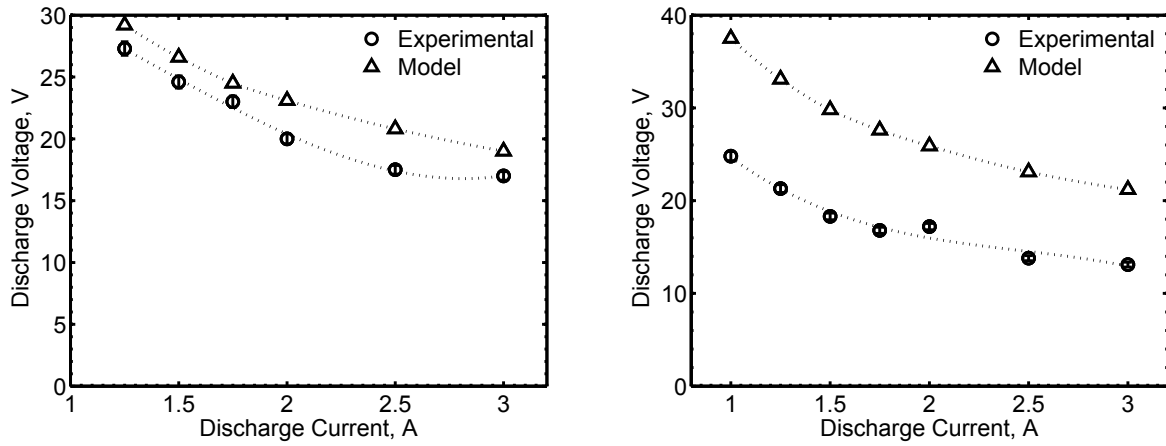


Figure 16. Electrical characteristic of the Alta cathode with 0.4-mm-dia orifice at 0.2 mg/s (left) and 1 mg/s (right), $K = 15$ m-Pa.

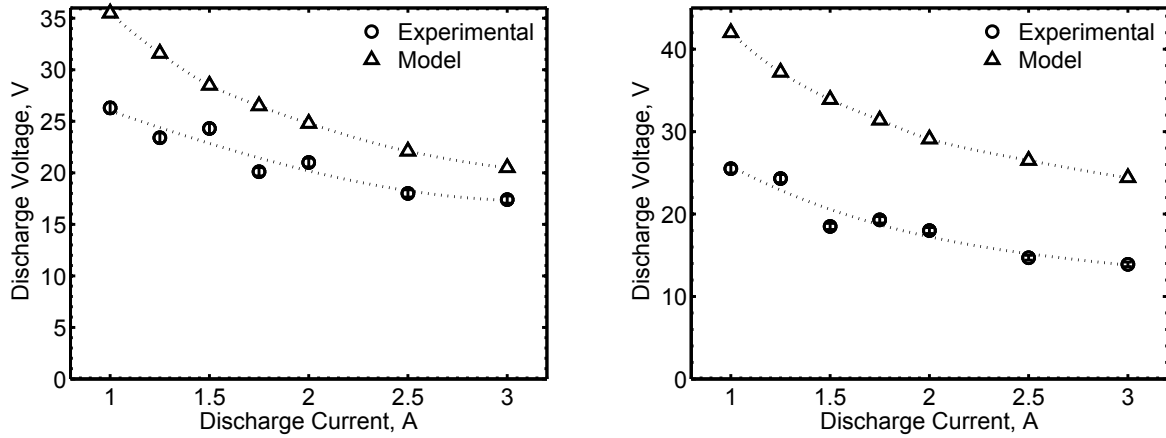


Figure 17. Electrical characteristic of the Alta cathode with 0.3-mm-dia orifice at 0.2 mg/s (left) and 1 mg/s (right), $K = 15$ m-Pa.

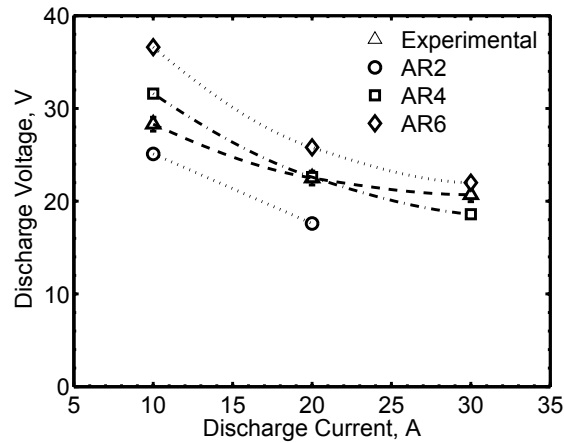


Figure 18. Electrical characteristic of the Goebel-Chu cathode²² at 0.98 mg/s, $K = 10$ m-Pa.

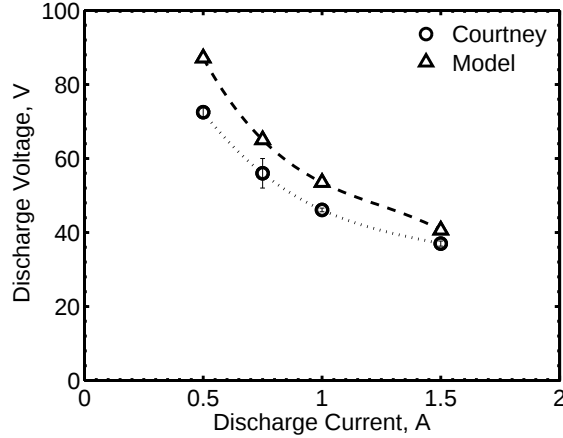


Figure 19. Electrical characteristic of the MIT cathode²⁹ at 0.29 mg/s, $K = 4$ m-Pa.

IV. Cathode Design

Several recommendations can be derived from the results of the theoretical investigation, with the purpose of helping the design of an efficient hollow cathode. Since the model indicated heat conduction as the predominant heat transfer mechanism in the cathode assembly, it is essential to minimize the wall thickness of the cathode tube and to choose a material with a low thermal conductivity at the predicted operating temperature (e.g. tantalum). The cathode tube should also be sufficiently long to prevent overheating of the base plate. The numerical results also suggest not to reduce the orifice diameter, since this would significantly increase the power consumption. This consideration is consistent with the cathode design from Goebel³⁰, where the orifice diameter is close to the insert inner diameter. Moreover, a larger orifice would ease the discharge initiation, with a deeper penetration of the positive potential from the keeper in the narrow orifice region. A low mass flow rate is shown to increase the cathode lifetime, given the deeper plasma penetration in the interior of the cathode which increases the emission area. However, the transition from spot to plume mode has to be taken into account while operating at low mass flow rate, given the appearance of large ionization instability oscillations³¹. While variations in the orifice length and in the insert inner diameter are not expected to affect the power consumption, both shortening the orifice and enlarging the insert are beneficial for the cathode lifetime.

V. Conclusion

A reduced-order model was suggested to gain a deeper insight into the operation of orificed hollow cathodes. Despite the assumptions made to develop a flexible and time-saving numerical tool, the qualitative comparison with the available experimental data is encouraging and suggests the use of the model as a quick tool for the design of new devices. Future developments will include a broader comparison with experimental data and a deeper investigation into the effective emission length, to avoid the estimation of this parameter by means of semi-empirical solutions.

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