

3D simulation of the rotating spoke in a Hall thruster

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Abstract: The 3 dimensional Particle-in-Cell code STOIC was applied to simulate the rotating spoke in a 100 W CHT thruster. In the simulation a spoke rotating with the velocity of about 1.8 km/s was observed in the thruster channel. Both the spoke and non-spoke regimes of the CHT operation were achieved in the simulation. In agreement with the experiment, the transition from spoke to non-spoke regime in the simulations was observed with an increase of the electron current from the cathode. The influence of the rotating spoke on the ion flow in the thruster was studied. The simulations have demonstrated that the presence of the spoke leads to strong decrease (factor 3.5) of the average potential drop inside the thruster channel, which results in almost twice lower average ion velocity at the thruster exit. In the spoke regime the ion flow outside of the thruster exhibits visibly higher divergence as compared with the non-spoke regime. These results agree quite well with LIF measurements of the ion velocity in CHT plume. The simulations have revealed that a rotating spoke causes the rotating asymmetry in the ion flow in the CHT thruster.

Nomenclature

B	= magnetic field
E	= electric field
$\dot{m}$	= mass flow rate
n_e	= electron density
T_e	= electron temperature
I	= electric current
Δt	= time step
v_e	= electron velocity
U	= voltage
Δx	= cell size
X, Y, Z	= coordinates

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I. Introduction

The rotating spoke instability is a phenomenon often observed in cross-field discharges like Hall thrusters, plasma columns and magnetized radio-frequency discharges. In the Hall thruster, despite decades of investigation, the origin, dynamics and nature of the spoke are still poorly understood. The earliest spoke study, carried out by Janes and Lowder¹, identified an azimuthal structure moving at tens of kilometers per second, whose presence was later attributed to the ionization type instability². In addition to the conventional annular geometry Hall thrusters, the rotating spoke was measured in a so-called cylindrical Hall thruster with a cusp type magnetic field^{3,4}. The unique feature of the spoke in CHT is that its occurrence and suppression can be controlled by the magnetic field topology and the cathode keeper discharge³. The origin of the observed spoke in annular and cylindrical Hall thrusters is still not understood. The spoke is of interest principally because it can conduct current, thereby participating in anomalous electron transport^{4,5}. It is believed that spokes in both annular and cylindrical Hall thrusters have the same physical nature, which is indeed related to the ionization instability as was predicted in preceding works. Recent Particle-in-Cell (PIC) simulations of the CHT thruster⁶ have also pointed out at the ionization nature of the spoke instability. However, this hypothesis still needs to be validated both theoretically and experimentally.

Recent studies on both annular and cylindrical thrusters have provided results indicating similarities of the spoke phenomenon in both thruster configurations, including a similar frequency range and a relation to the $\mathbf{E} \times \mathbf{B}$ velocity (much smaller than E/B). These results reinforce the necessity of understanding of the spoke. In a Hall thruster, the spoke instability is of prime importance because it has been shown to be present under any operating conditions, with the mode number varying according to the thruster size and operating conditions⁵. Cylindrical thruster measurements performed at the PPPL^{3,4} have demonstrated the $m=1$ mode is the essential mode and that a large fraction of electron current towards the anode flows through the spoke. Probe measurements of the time-evolution of the azimuthal electric field also reveal that the electric field generated by the spoke is responsible for axial electron current.

Our previous results of 3D PIC simulations of the rotating spoke in the CHT thruster⁶ suggest that the azimuthal electric field induced by the spoke can influence the ion trajectories in the thruster. In this work we apply the 3D PIC code STOIC⁶ to determine the influence of the rotating spoke on the ion flow (and thereby its role in thrust and specific impulse) as well as on the accelerating potential in a Cylindrical Hall Thruster (CHT).

The paper is organized as follows: in chapter 2 the main features of cylindrical Hall thruster are outlined. In chapter 2 the details of 3D Particle in Cell code with Monte Carlo collisions (PIC MCC) and the simulation results for CHT are presented. Chapter 4 gives a summary and discusses the outlook for future work.

II. Cylindrical Hall thruster

Figure 1 illustrates the principle of operation of the cylindrical thruster. A cylindrical Hall thruster consists of a cylindrical ceramic channel, a ring-shaped anode, which serves also as a gas distributor, a magnetic core and magnetized sources (Fig. 1). The magnetic field lines intersect the ceramic channel walls. The electron drifts are closed, with the magnetic field lines forming equipotential surfaces⁷. The radial component of the magnetic field crossed with the azimuthal electron current produces the thrust. However, the electrons are not confined to an axial position; rather they bounce over an axial region, impeded from entering the annular part of the channel because of magnetic mirroring. Two magnetized sources, electromagnetic coils with opposite currents, can produce a cusp-like magnetic field in the channel, with a strong radial component. To maintain ionizing collisions, the anode (gas inlet) is placed in the short annular part of the channel. The length of the annular part of the channel is designed to minimize the ionization mean free path, thus localizing the ionization of the working gas at the boundary of the

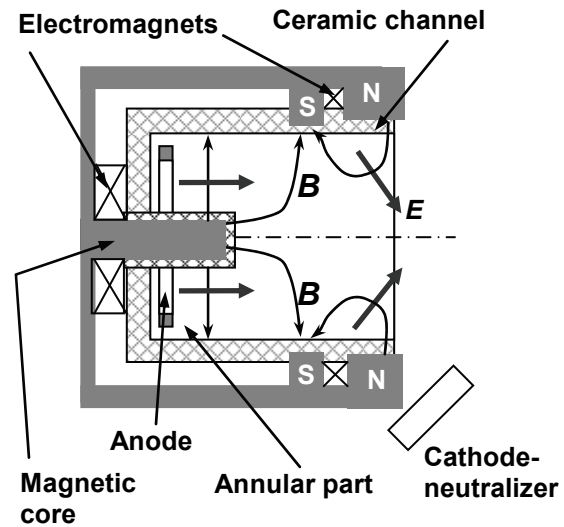


Figure 1. Schematic of a cylindrical Hall thruster

annular and cylindrical regions. Hence, most of the voltage drop occurs in the cylindrical region that has large volume-to-surface ratio. This conclusion is supported by the results of plasma measurements^{7,8} and by Monte-Carlo simulations⁹.

III. The model

The detailed description of the PIC-MCC method can be found in thorough reviews^{10,11}. Here, we just outline the main features of our model. In PIC-MCC simulations we follow the kinetics of so-called “Super Particles” (each of them representing many real particles), moving in the self consistent electric field calculated on a spatial grid from the Poisson equation. The particle collisions are handled by Monte-Carlo collision (MCC) routines, which randomly change particle velocities according to the actual collision dynamics. The simulation includes electrons, Xe^+ ions and the neutral Xenon atoms. All relevant collisional processes are included in the model: electron-neutral elastic, ionization and excitation collisions, ion-neutral momentum-transfer and charge exchange collisions. The dynamics of the background neutral gas is self-consistently resolved with direct simulation Monte Carlo.

In our model we resolve 3 spatial and 3 velocity components. The model utilizes an equidistant Cartesian grid which explicitly assures momentum conservation and zero self forces. The computational domain in the present simulations is extended beyond the discharge channel and includes the near-field region of the thruster.

The computational domain represents a parallelepiped with length $Z_{\max} = 40 \text{ mm}$ and sides $X_{\max} = Y_{\max} = 30 \text{ mm}$. Z axis is directed along the thruster symmetry axis. The axial cross-section of the computational domain together with magnetic field topology is shown in Fig. 2. The thruster channel length (distance from the anode to the exit plane) is $Z_{\text{thr}} = 22 \text{ mm}$. The thruster channel walls are located at $X_{\text{ch1}} = 3 \text{ mm}$, $X_{\text{ch2}} = 27 \text{ mm}$, $Y_{\text{ch3}} = 3 \text{ mm}$, $Y_{\text{ch4}} = 27 \text{ mm}$. At $Z = 0$ the metal anode is located. At the center of the anode the annular part with the length $Z_{\text{annular}} = 6 \text{ mm}$ and sides $X_{\text{annular}} = Y_{\text{annular}} = 6 \text{ mm}$ is placed. The region outside the thruster exit: ($Z_{\text{thr}} < Z < Z_{\max}$, $X_{\text{ch1}} < X < X_{\text{ch2}}$, $Y_{\text{ch1}} < Y < Y_{\text{ch2}}$) represents the very near-field plume zone. The thruster channel and the annular part are dielectric (Boron Nitride). The lateral and axial boundaries of the computational domain are assumed to be metallic. All metal elements in the simulation, except for the anode are at ground potential. At the anode, a voltage of $U_a = 250 \text{ V}$ is applied.

In order to compute the electrostatic potential in the system, the Poisson equation is discretized on the grid, taking into account the possible change of the dielectric permittivity across the dielectric surfaces. The set of resulting finite difference equations is solved using the Watson Sparse Matrix Package¹². This approach allows us to calculate the potential inside the computational domain, self-consistently resolving the floating potential on the dielectric surfaces.

The particles' equations of motion are solved for discrete time steps with the leap-frog / Boris algorithm¹⁰. All surfaces in the simulation are assumed to be absorbing for electrons and ions. No secondary electron emission is assumed in the simulation. The neutrals are re-launched from the surfaces using a Maxwellian distribution with temperature $T_n = 1000 \text{ K}$. The neutrals are injected into the system through two coaxial ring slits at the anode with the mass flow rate $\dot{m} = 0.4 \text{ mg/s}$.

Electrons with a Maxwellian distribution and a temperature $T_e = 5 \text{ eV}$ are introduced into the system in the source region $51 \text{ mm} < X < 53 \text{ mm}$, $13 \text{ mm} < Y < 17 \text{ mm}$, $26 \text{ mm} < Z < 27 \text{ mm}$ with uniform density and the constant current $I_c = 0.25 \text{ A}$, simulating the thruster cathode. In the simulation the electrons from the source,

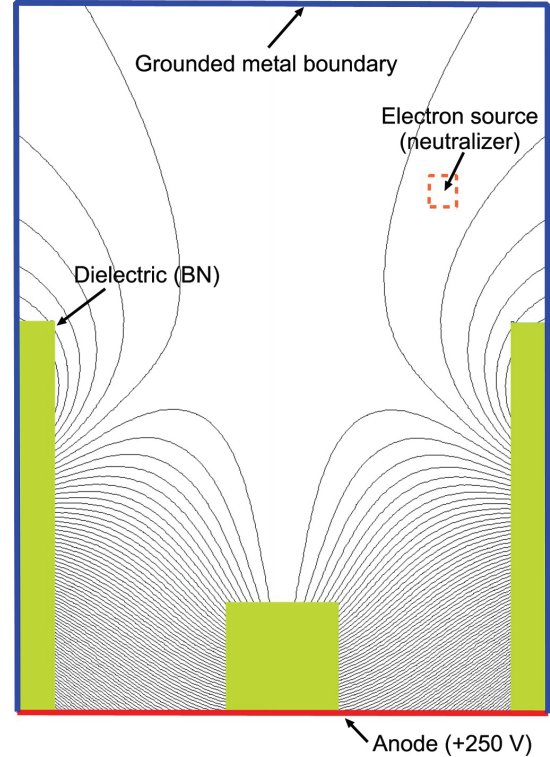


Figure 2. Computational domain with magnetic field topology.

accelerated in the thruster's electric field, are ionizing the neutral gas, creating the plasma in the thruster channel. In order to speed up the plasma ignition process, the supplementary ring source of the electrons with temperature $T_e = 20$ eV and current $I_e = 0.05$ was applied in the annular part during the first 300 ns of the simulation. After that the supplementary source was switched off and only the cathode electron source was operating.

To reduce the computational time the size of the system is scaled down by factor of 10. In order to preserve the ratio of the particles mean free paths and the gyroradii to the system length, the collisions cross-sections and the magnetic field are increased by the same factor 10.

An equidistant computational grid $60 \times 60 \times 80$ was used in the simulation. The total number of computational particles in the simulation was about 40000000. The cell size $\Delta x = \Delta y = \Delta z = 5 \cdot 10^{-2}$ mm in the simulation was chosen to ensure that it is smaller than the smallest Debye length in the system. The time step was set to $\Delta t = 5.6 \cdot 10^{-12}$ s in order to resolve the electron plasma frequency. The simulation was carried on a 4-processor Intel Xeon workstation. The duration of the run was about 20 days. About $4 \cdot 10^6$ time steps were performed which corresponds to a simulated time of 24 μ s.

The spoke and non-spoke regimes of the CHT operation were achieved in the simulations. In agreement with the experiment¹³, the transition from spoke to non-spoke regime was observed in the simulations with an increase of the electron current from the cathode (cathode overrun regime). In Fig. 3 the electron density profiles inside the annular channel for cathode currents $I_{cath} = 0.25$ A and $I_{cath} = 0.5$ A are shown. For cathode current 0.25 A, the spoke-like

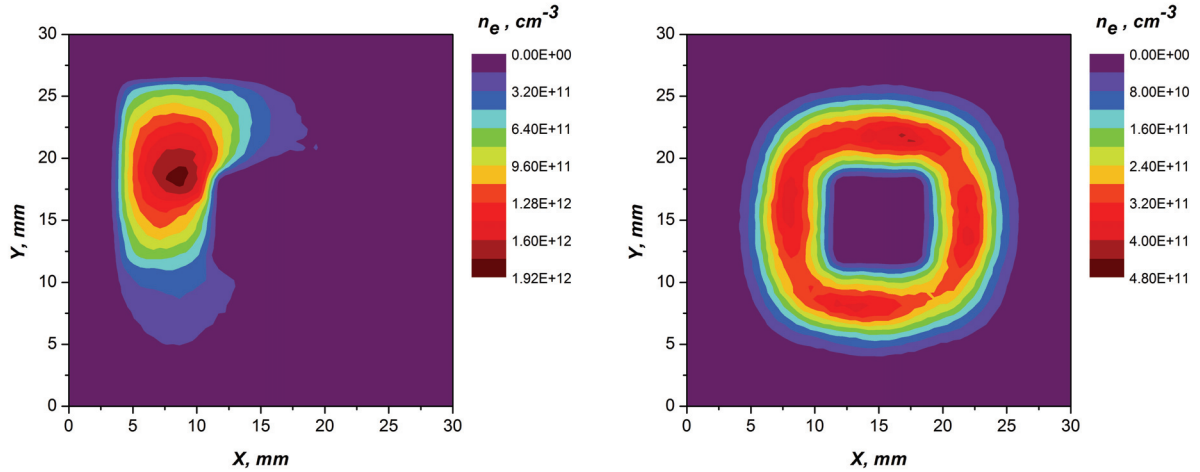


Figure 3. Electron density in the annular channel of CHT thruster: a) spoke regime; b) non-spoke regime.

structure in electron density moving with velocity 1.8 km/s was observed in the thruster annular channel. In agreement with our previous results⁶ the spoke motion is associated with the strong azimuthal depletion of the neutral gas due to ionization inside the spoke.

In order to investigate the effect of the spoke on the ion flow in the thruster, the ion flow velocity was averaged over 5 cycles of the spoke rotation. The averaged ion velocity vector map at the longitudinal axial cross-section of the simulation domain for spoke and non-spoke regimes is presented in Fig. 4. As one can see from the simulation, in the spoke regime the ion velocity in the thruster channel is distinctly lower as compared with non-spoke regime. Furthermore, the simulated ion flow velocity in the spoke regime outside of the thruster, away from centerline, exhibits visibly higher divergence as compared with the non-spoke cathode overrun regime. These results agree well with Laser-Induced-Florescence (LIF) measurements of the ion velocity in CHT plume¹⁴. It is difficult to quantify the differences between the spoke and non-spoke regimes in the vector maps. For quantitative analyses the color maps of the total ion velocity and the three velocity components with and without spoke are presented separately in Figs 5-8. Here one can see that in the spoke regime the largest part of the ion acceleration takes place outside of the thruster, in the plume. In contrast, without the spoke the largest part of acceleration happens inside the thruster channel. The maximum exhaust ion axial velocity in the simulations in both cases is about 16 km/s. The axial ion velocity at the thruster exit is 5.4 km/s with the spoke and 9.2 km/s without spoke. These results agree quite good with the measurements in Ref.14.

Analyzing the azimuthal component of the ion velocity, one can see that in both spoke and non-spoke cases its magnitude reaches the maximum of about 0.2 km/s in the region where the magnetic field is strongest - close to the thruster annular channel. Here, the azimuthal velocity component is comparable with the total ion velocity. The local magnetic field is strong enough to notably bend the ion trajectories before ions can leave this region. In the rest of the computational domain the azimuthal component of the ion velocity in both spoke and non-spoke cases is much smaller than the total ion velocity. Outside the thruster the maximum ratio of the azimuthal and total velocity in both cases is about 2 %. This corresponds to the tilt of the velocity flow of about 1 degree.

Stronger ion acceleration inside the thruster channel in the non-spoke regime points out at the larger potential drop inside the channel as compared to the spoke regime. In Fig. 9 the plasma potential map for both regimes is presented. Indeed, in the non-spoke case the potential drop inside the thruster is 140 V, whereas with the spoke it is only 40 V. These results are in agreement with probe measurements in CHT thruster¹³. Such difference in potential can be only explained by increased plasma conductivity in the spoke regime. As the plasma density in both cases is approximately equal, see Fig. 10, the higher plasma conductivity can only be caused by the presence of the additional transport mechanism - cross-field electron transport caused by the spoke⁴.

In order to see the dynamics of the ion flow due to the spoke rotation, the averaging time in the simulations was reduced by a factor of 100, such that the spoke rotation cycle was resolved. Reduction of the averaging time led to an increase of the statistical noise in the simulation results. However, the effect of the rotating spoke on the ion flow could be unambiguously identified. In the figures 11-14 the plasma parameters at four phases of the spoke rotation cycle are presented. The spoke position can be identified at the transverse cut of the ion density inside the annular channel (Figs. 11d, 12d, 13d, 14d). The rotating inhomogeneity of the plasma density leads to the rotating asymmetry in the electric potential (Figs. 11f, 12f, 13f, 14f). The potential profile defines the ion flow in the thruster. The rotating asymmetry of the potential leads to the rotating asymmetry of the ion velocity. The fluctuations up to about 13% are visible in the axial velocity at the thruster exit (Figs. 11b, 12b, 13b, 14b). The maximum of the axial velocity fluctuations is located at the periphery, at the radial position corresponding to the thruster radius, but at the axis fluctuations of about 4% are still observable. More clearly velocity fluctuations are visible in the azimuthal velocity plots (Figs. 11c, 12c, 13c, 14c). The azimuthal velocity at the thruster exit oscillates with the amplitude of 1.4 km/s, which corresponds to about 25% of the average axial velocity at this position. The rotating asymmetry in the ion velocity can be clearly observed in the vector map of the perpendicular velocity at a transverse cut at the thruster exit (Figs. 11a, 12a, 13a, 14a).

Thus, the simulations results fully support the hypothesis about the rotating spoke causing the rotating asymmetry in the ion flow in the CHT thruster.

IV. Conclusion

3D PIC MCC simulations were used to determine the influence of the rotating spoke on the ion flow as well as on the accelerating potential in a Cylindrical Hall thruster. The spoke and non-spoke regimes of the CHT operation were achieved in the simulations. A spoke rotating with the velocity of about 1.8 km/s was observed in the thruster channel, which agrees well with observations in the experiment. In agreement with the experiment, the transition from spoke to non-spoke regime in the simulations was observed with an increase of the electron current from the cathode.

The simulations have demonstrated that the presence of the spoke leads to strong decrease (factor 3.5) of the average potential drop inside the thruster channel, which results in almost twice lower average ion velocity at the thruster exit. In the spoke regime the ion flow outside of the thruster exhibits visibly higher divergence as compared with the non-spoke regime. These results agree quite well with LIF measurements of the ion velocity in CHT plume.

The simulations results fully support the hypothesis about the rotating spoke causing the rotating asymmetry in the ion flow in the CHT thruster. To our knowledge these are the first self-consistent simulation results revealing the ion flow asymmetry caused by the rotating spoke in the Hall thruster.

The deeper investigation of the spoke influence on the thruster performance as well as clarification of the phenomena underlying the spoke formation and the dynamics will be the goal of our further research.

Acknowledgments

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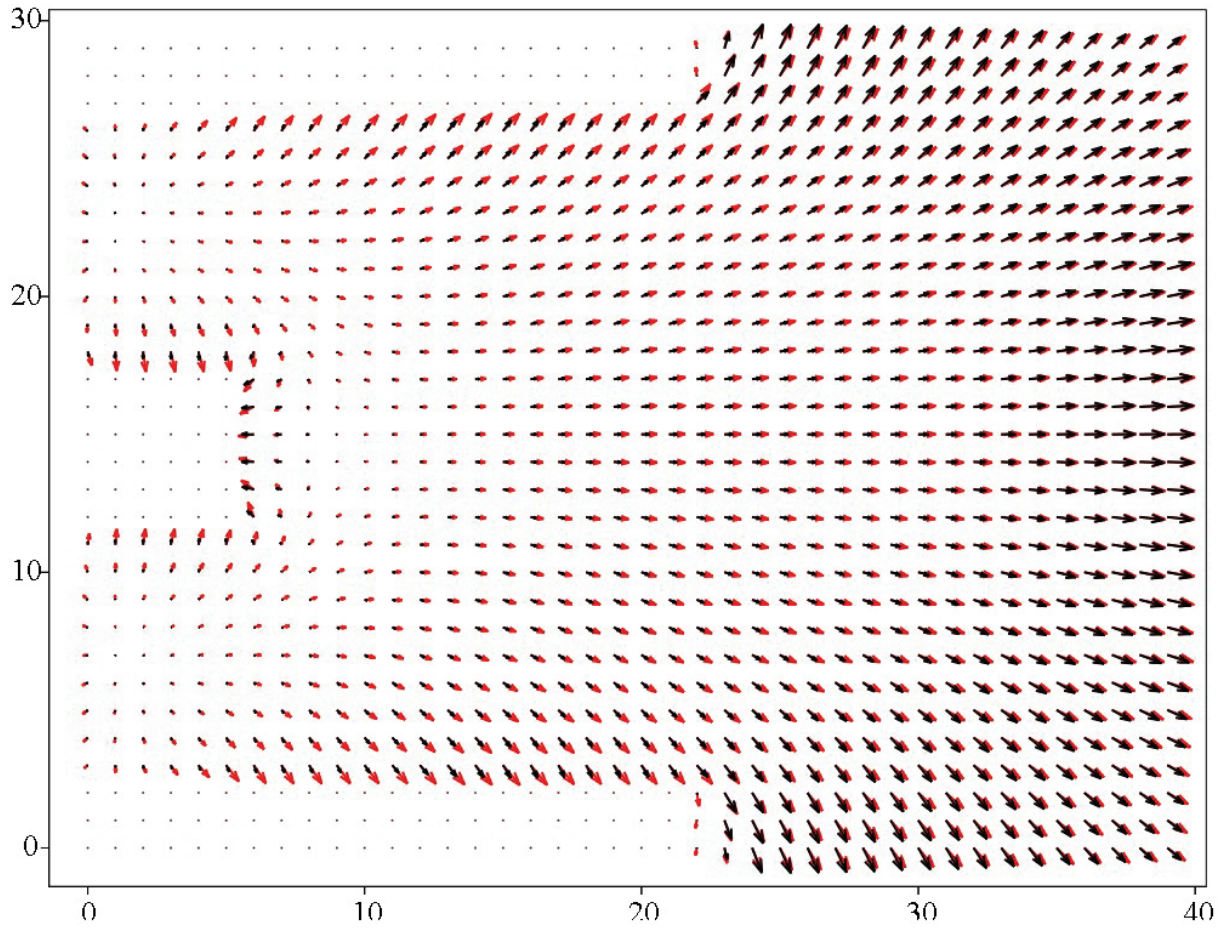


Figure 4. Averaged ion flow map for spoke (black) and non spoke regimes (red).

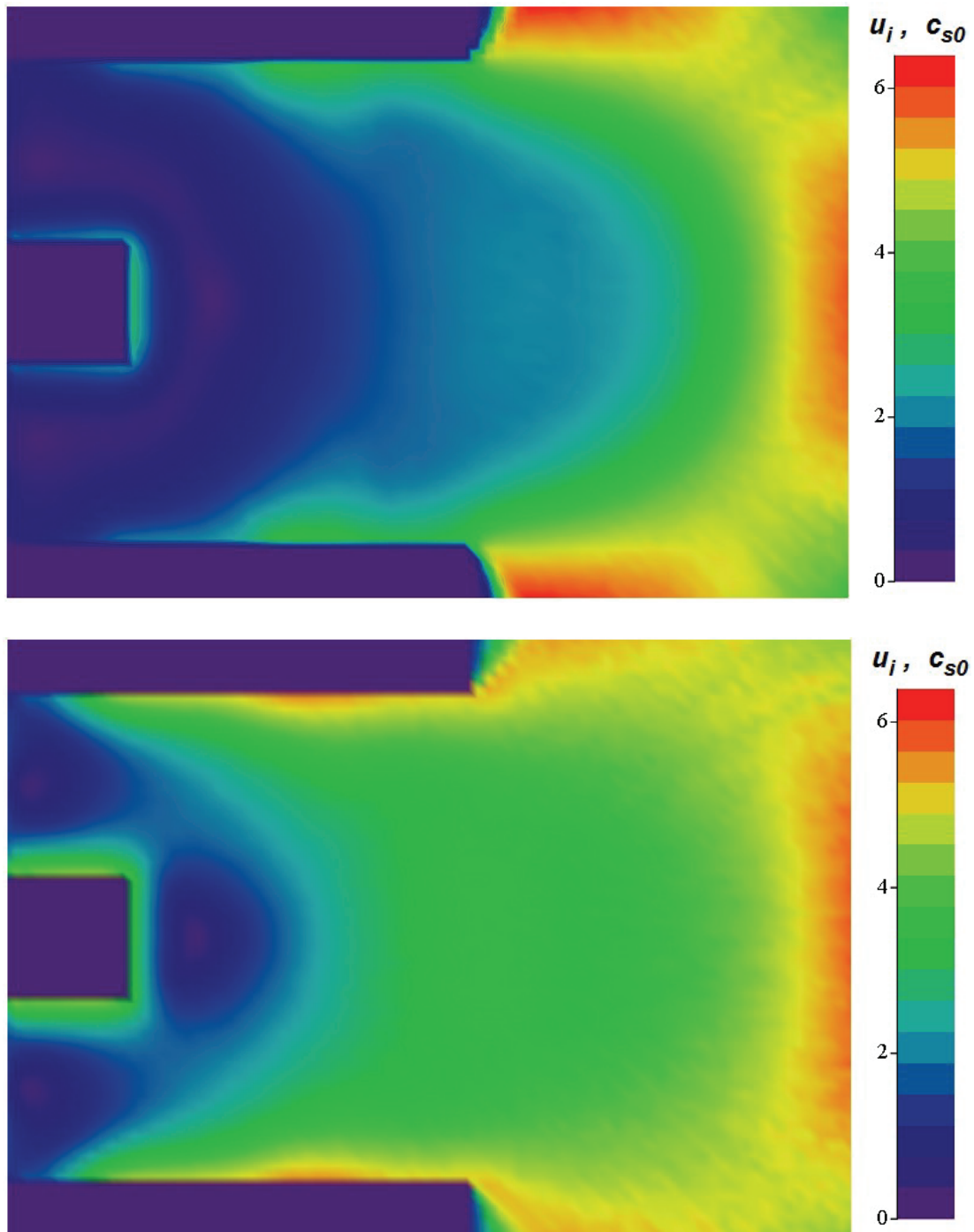


Figure 5. Map of the total ion velocity. Spoke (top) and non-spoke regime (bottom). $c_{s0} = 2.7$ km/s.

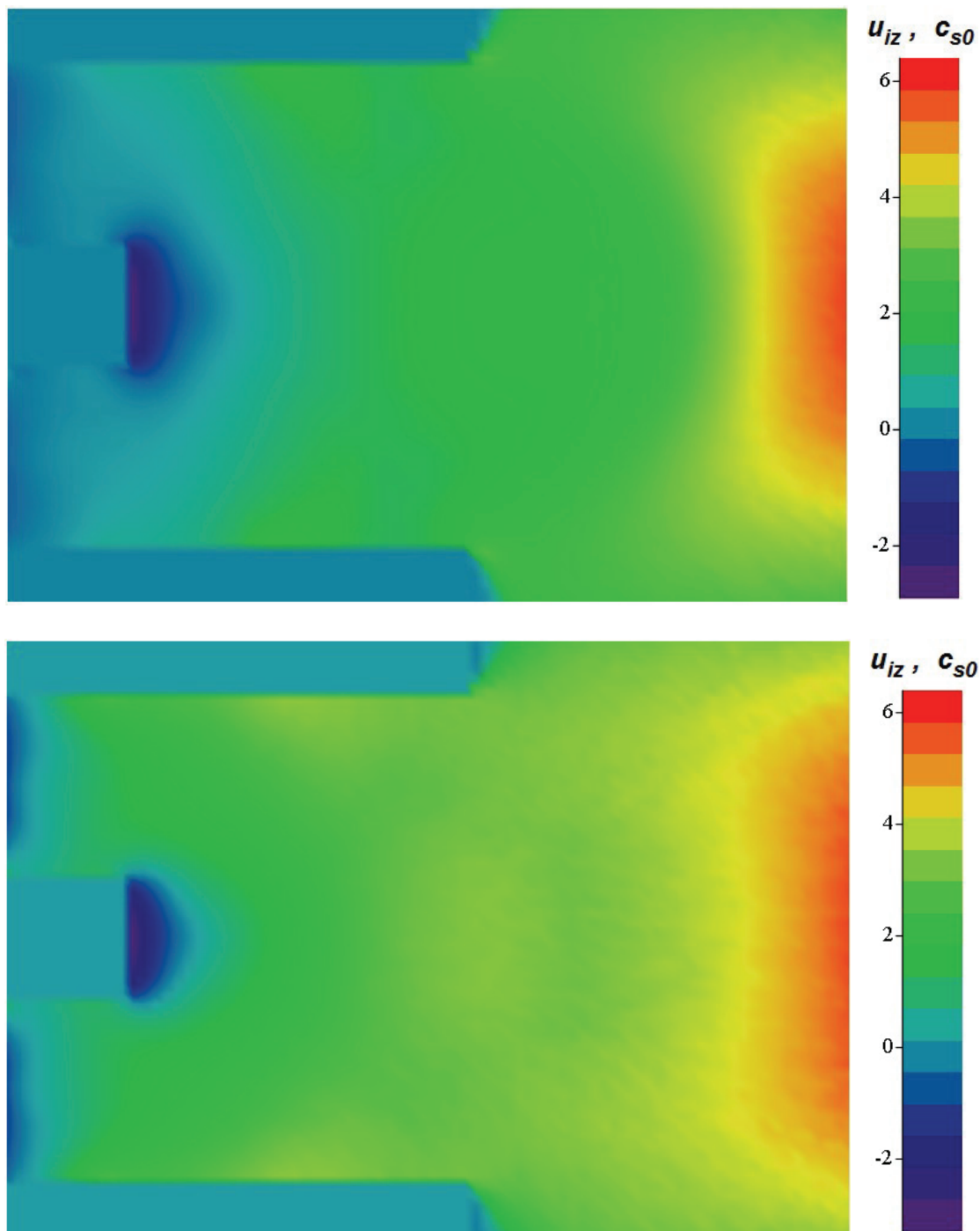


Figure 6. Map of the axial component of ion velocity. Spoke (top) and non-spoke regime (bottom). $c_{s0} = 2.7$ km/s.

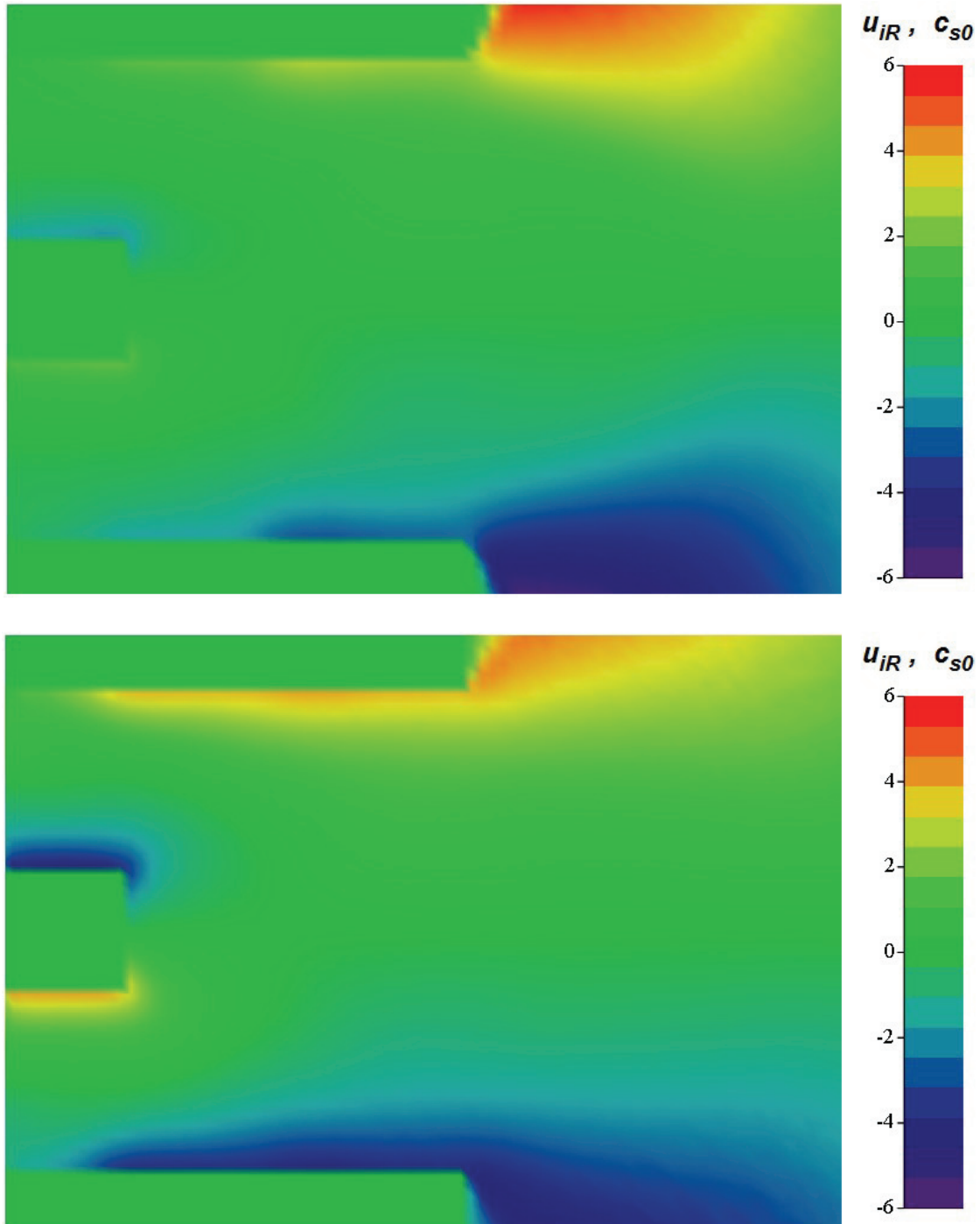


Figure 7. Map of the radial component of ion velocity. Spoke (top) and non-spoke regime (bottom). $c_{s0} = 2.7$ km/s.

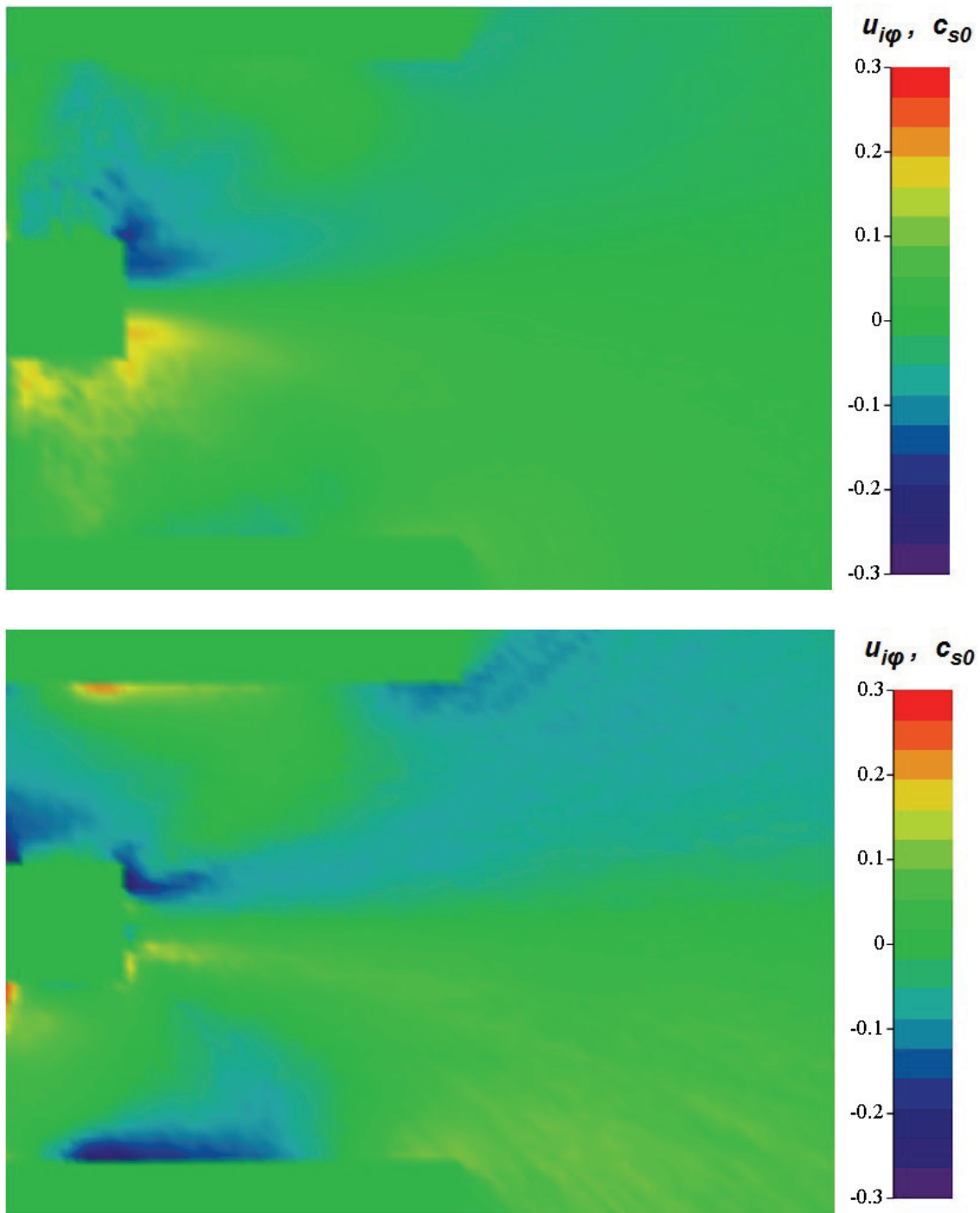


Figure 8. Map of the azimuthal component of ion velocity. Spoke (top) and non-spoke regime (bottom). $c_{s0} = 2.7$ km/s.

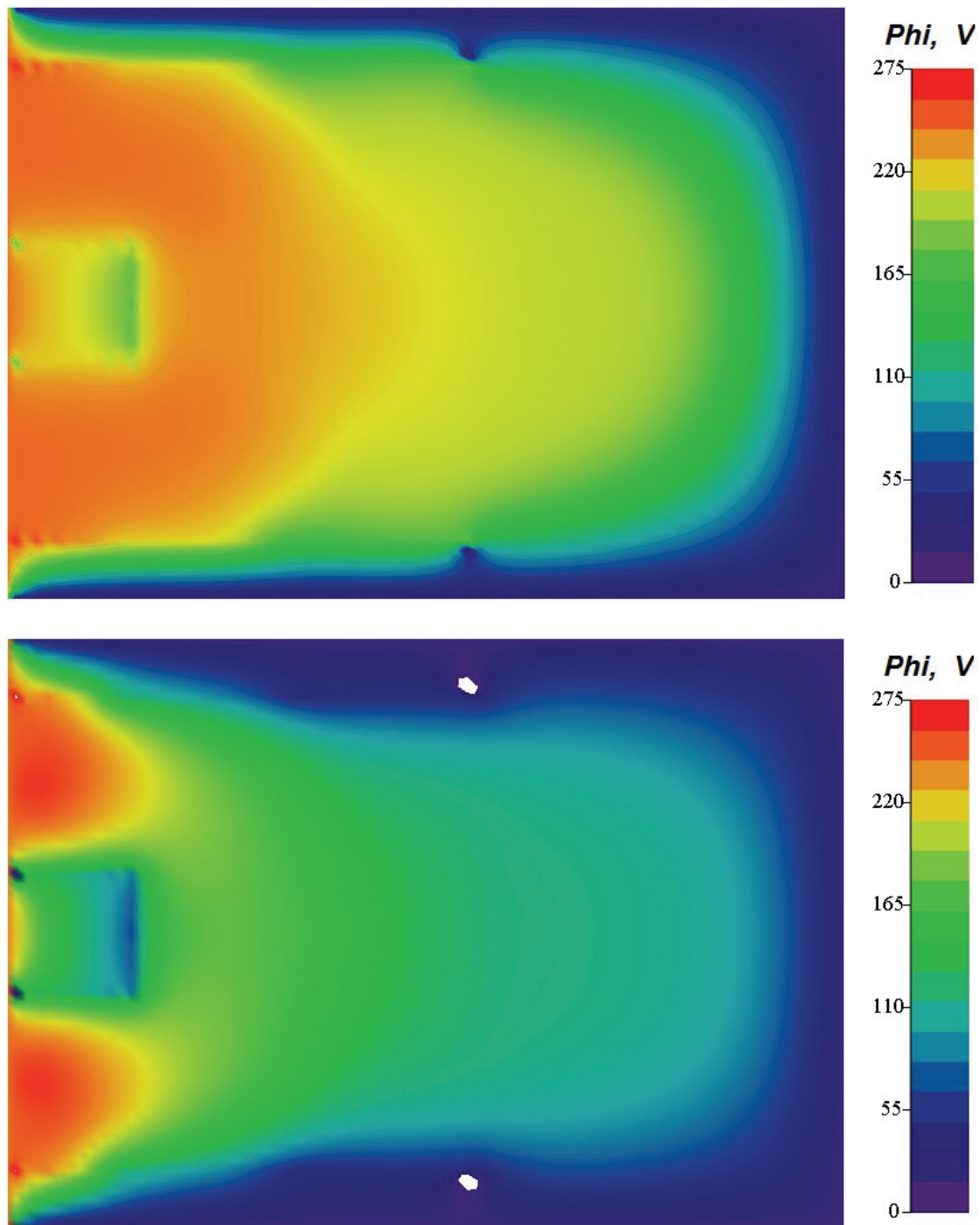


Figure 9. Map of the plasma potential. Spoke (top) and non-spoke regime (bottom).

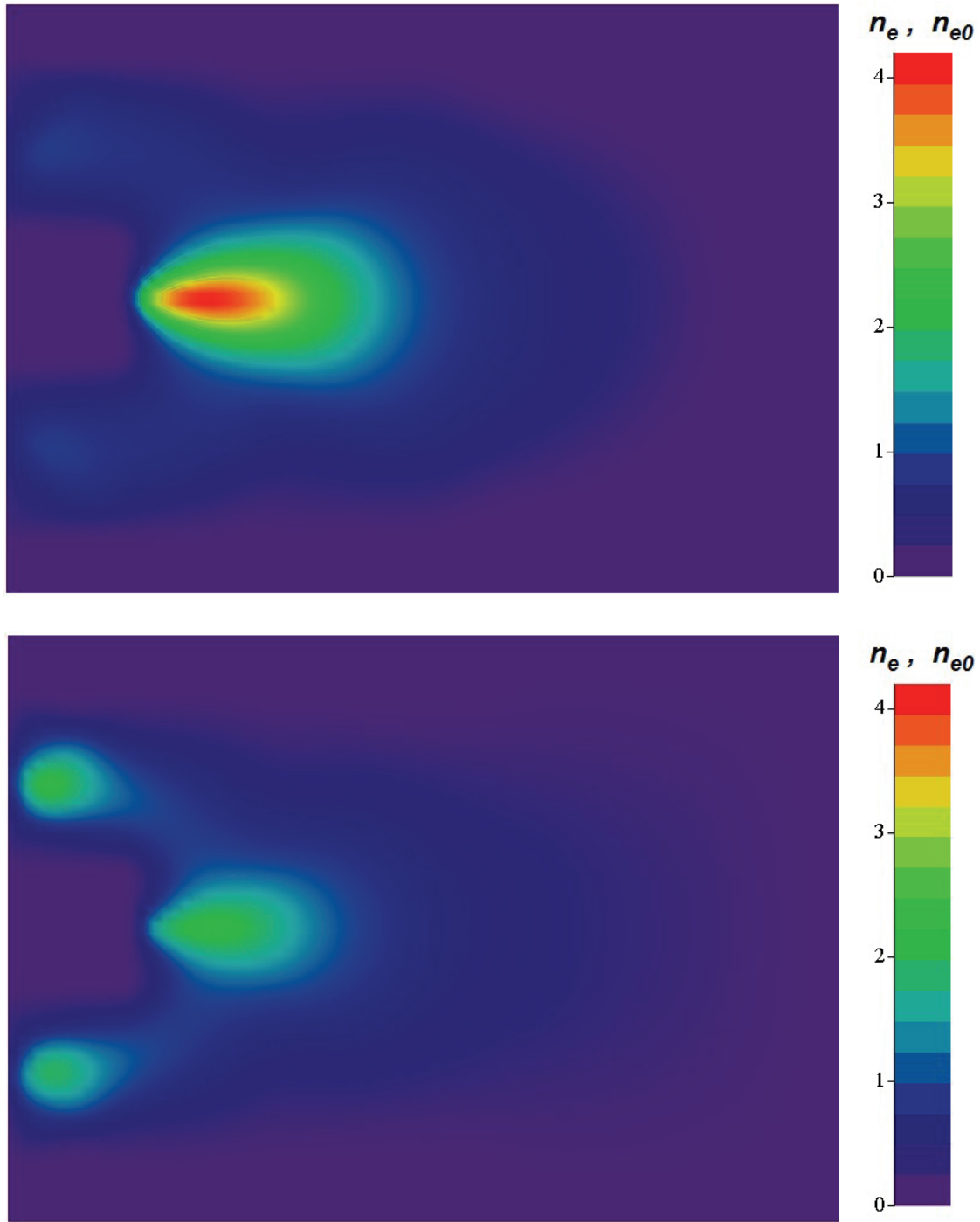


Figure 10. Averaged map of the electron density. Spoke (top) and non-spoke regime (bottom). $n_{e0}=4 \cdot 10^{11} \text{ cm}^{-3}$.

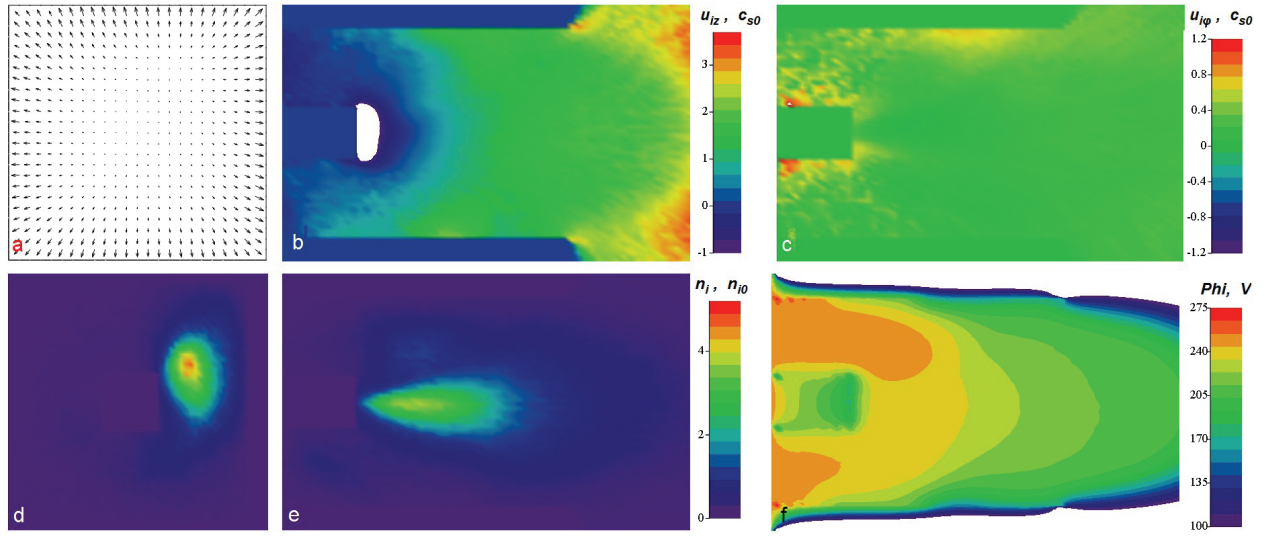


Figure 11. Vector map of the perpendicular ion velocity at transverse cut at the thruster exit (a). Ion axial velocity (b) and ion azimuthal velocity (c) at the longitudinal axial cut. Ion density at transverse cut at the annular channel (d) and at the longitudinal axial cut (e). Plasma potential at the longitudinal axial cut. All parameters are taken at phase $\varphi = 0$.

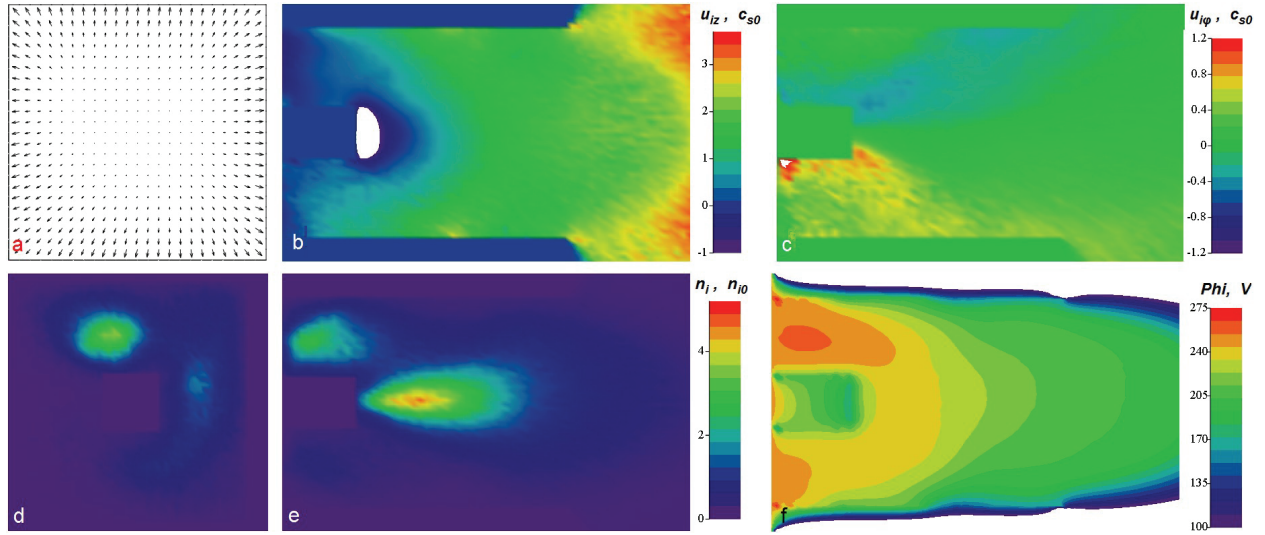


Figure 12. Vector map of the perpendicular ion velocity at transverse cut at the thruster exit (a). Ion axial velocity (b) and ion azimuthal velocity (c) at the longitudinal axial cut. Ion density at transverse cut at the annular channel (d) and at the longitudinal axial cut (e). Plasma potential at the longitudinal axial cut. All

parameters are taken at phase $\varphi = \frac{\pi}{2}$.

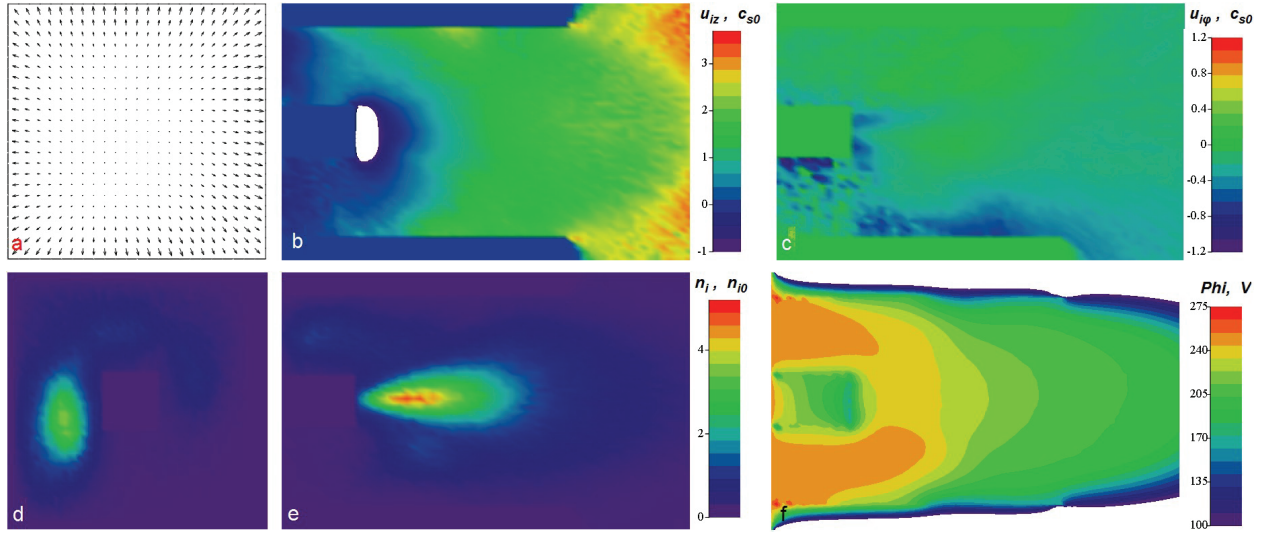


Figure 13. Vector map of the perpendicular ion velocity at transverse cut at the thruster exit (a). Ion axial velocity (b) and ion azimuthal velocity (c) at the longitudinal axial cut. Ion density at transverse cut at the annular channel (d) and at the longitudinal axial cut (e). Plasma potential at the longitudinal axial cut. All parameters are taken at phase $\varphi = \pi$.

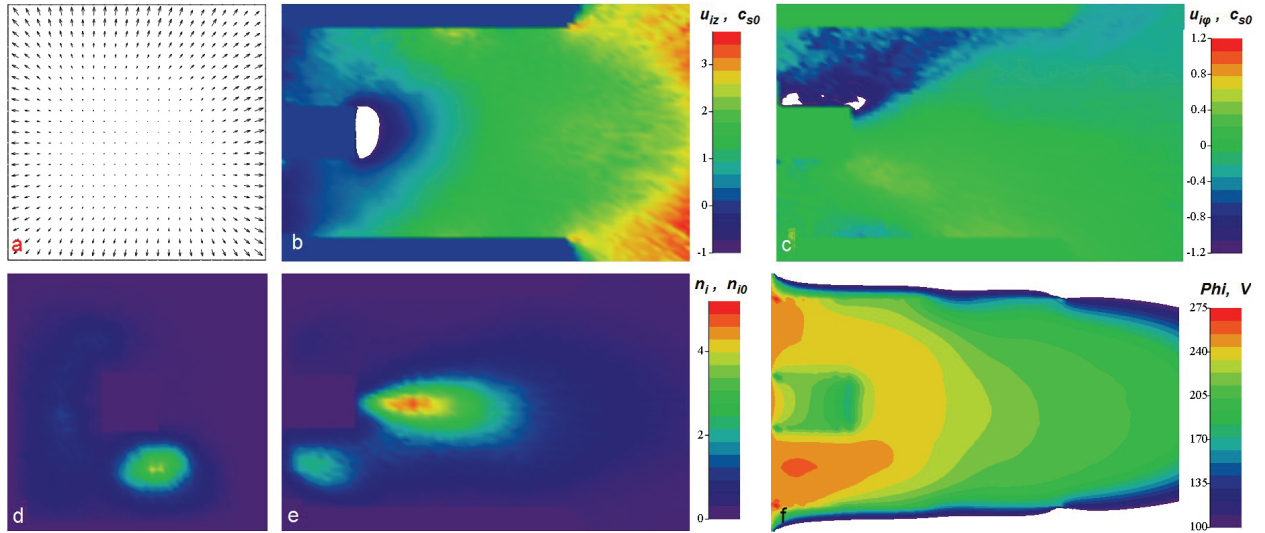


Figure 14. Vector map of the perpendicular ion velocity at transverse cut at the thruster exit (a). Ion axial velocity (b) and ion azimuthal velocity (c) at the longitudinal axial cut. Ion density at transverse cut at the annular channel (d) and at the longitudinal axial cut (e). Plasma potential at the longitudinal axial cut. All parameters are taken at phase $\varphi = \frac{3\pi}{2}$.