Impact of Performance Scaling on Mission Analysis for Gas-Fed Pulsed Plasma Thrusters[*]

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An experimentally verified performance scaling model for gas-fed pulsed plasma thrusters (GFPPTs) is used as part of a mission study for refuelable satellites. Orbit raising, orbit phasing, and inclination change maneuvers are included in the study where payload mass fraction is evaluated as a function of specific impulse, trip time, and ΔV. For a GFPPT operating at a fixed thrust level, the power level and power supply mass is found to be nearly independent of exhaust velocity. The optimum value of specific impulse for the maximum payload mass is shown to be >5000 s for many GFPPT designs. The effects of refueling on satellite payload mass, total launch mass requirements, and total ΔV capability are also shown to favor higher specific impulse propulsion systems. The study results indicate that a GFPPT is required to have a thrust-to-power ratio >10 μN/W to produce trip times similar to other water-based electric propulsion systems with lower specific impulse values. The study results are also used to determine an appropriate path for technology development of GFPPTs.

1 Introduction

Science Research Laboratory, Inc. (SRL) has been designing and testing gas-fed pulsed plasma thrusters (GFPPTs) since 1995. In coordination with the Electric Propulsion and Plasma Dynamics Laboratory (EPPDyL) at Princeton University and the Advanced Propulsion Technology Group at JPL, nine generations of GFPPTs have been studied to date [1, 2]. The most recent designs have included modern solid-state pulsed power conditioning technology, new electrode geometries, and a new discharge circuit design to increase thruster performance [3]. These modern

GFPPT designs are well suited to spacecraft where the ΔV requirement over the mission is large, demanding a high specific impulse to reduce the necessary propellant mass. GFPPT technology also has the potential to enable some missions where a unique type of propellant is available for consumption.

The DARPA Orbital Express mission is one example where the propellant, in this case water vapor, is chosen for its ease of storage and transport with the possibility that the satellites could be refueled on orbit. Although other propulsion technologies can also use water vapor as a propellant, the GFPPT has the potential to provide the highest value of specific impulse which would allow for a wider range of maneuvering scenarios. To investigate this possibility, we have conducted a mission study using the performance scaling model for GFPPTs devel-

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oped in Ref. [4]. The goal of this research is to build upon previous mission analysis techniques for pulsed plasma thrusters [5, 6, 7] and optimize the payload mass of a spacecraft using a GFPPT.

We begin our paper with a derivation of mass scaling relations including a performance scaling model for GFPPTs. Next, the mission analysis of refuelable, earth-orbiting satellites performing a variety of maneuvers is presented. The payload mass fraction provided by a GFPPT is them compared to other propulsion technologies that could use water for propellant. Finally, the mission study is used to point out how the GFPPT technology should be developed to provide a more useful propulsion system.

2 Mass Scaling Relations

For simplicity, we begin our analysis by breaking the initial mass of the spacecraft into only four parts that could change depending on the design of the GFPPT: payload, propellant, power supply, and propulsion,

$$M_0 = M_{pay} + M_{prop} + M_{power} + M_{thrust}. \quad (1)$$

In this breakdown, the payload mass is more than simply the useful mass. It also includes all the fixed masses that do not depend on power, specific impulse or ΔV, the structural mass, and any other mass that is not contained in the last three terms of Eq. (1). We will now examine the last three terms in closer detail.

The propellant mass is based on the rocket equation,

$$M_{prop} = M_0 \left(1 - e^{-\Delta V/\bar{u}_e}\right), \quad (2)$$

where the change in velocity, ΔV, may depend on the thrust level or duration of the maneuver, and $\bar{u}_e$ is the exhaust velocity of the thruster.

The power supply mass is proportional to the power, P, the propulsion system requires to produce thrust, T,

$$M_{power} = \alpha_p P = \frac{\alpha_p T}{\eta_{pc}} \frac{E}{I_{bit}} \quad (3)$$

$$= \frac{\alpha_p}{\eta_{pc}\tau} \frac{E}{I_{bit}} I_{total} \quad (4)$$

where α_p is the specific mass of the power supply and GFPPT modulator, η_{pc} is the power conditioning efficiency for the modulator, τ is the total time

the thruster is operational, E is the energy per pulse, I_{bit} is the impulse produced by one pulse, and I_{total} is the total impulse produce by the thruster over its lifetime.

The mass of a GFPPT propulsion system is made up of the thruster itself and the power conditioning modulator. The thruster mass is primarily made up of the discharge capacitors while the modulator mass is largely dependent on power (included in α_p above) except for capacitors that store the entire energy for a set or burst of pulses. Modern GFPPTs group pulses together in bursts to improve propellant utilization. The mass of both capacitor banks is proportional to the energy they store, yet the main discharge capacitors are built for long life and low inductance which can cause them to weigh proportionally more. The propulsion system mass that is dependent on energy can now be broken down as,

$$M_{thrust} = \alpha_e E + \alpha_{eb} N_{pb} E \quad (5)$$

$$= \frac{\alpha_e + \alpha_{eb} N_{pb}}{N_{ptot}} \frac{E}{I_{bit}} I_{total}, \quad (6)$$

where N_{pb} is the number of pulses per burst and N_{ptot} is the total number of pulses in the thruster lifetime.

The payload mass fraction, Γ, can now be evaluated,

$$\Gamma \equiv \frac{M_{pay}}{M_0} \quad (7)$$

$$= e^{-\Delta V/\bar{u}_e} - \left(1 - e^{-\Delta V/\bar{u}_e}\right) \frac{\bar{u}_e}{\mathcal{V}^2} \frac{E}{I_{bit}} \quad (8)$$

$$\mathcal{V} \equiv \left(\frac{\alpha_p}{\eta_{pc}\tau} + \frac{\alpha_e + \alpha_{eb} N_{pb}}{N_{ptot}}\right)^{-1/2}, \quad (9)$$

where $\mathcal{V}$ is solely dependent on technology and has the units of a velocity[1]. Longer lifetime (τ and N_{ptot}), higher power conditioning efficiency (η_{pc}), and smaller specific masses (α_p, α_e, and α_{eb}) lead to larger values of $\mathcal{V}$ which in turn leads to larger values of the payload mass fraction. In terms of technology development, increasing the payload mass can be accomplished by decreasing either of the specific masses or, equally, by improving lifetime. In fact, in many cases, changes in α_e can also change N_{ptot},

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[1]A similar characteristic velocity was defined in Ref. [5]

and only increases in the *ratio* of the the two parameters will lead to a larger payload mass. This will be examined more carefully in the next section.

2.1   Evaluation of $\mathcal{V}$

The GFFPT mission velocity, $\mathcal{V}$, can be evaluated by making the following assumptions:

- For the main discharge capacitor bank, a large number of pulse-type capacitors are used to increase lifetime and decrease bank impedance. For our design we have examined both WIMA FKP and Maxwell 3100 and 3700 type capacitors. Based on extrapolated trends of specific mass from catalog information [8, 9],

$$
\begin{aligned}
N_{ptot} &= 1 \times 10^{10} \; pulses, & (10) \\
\alpha_e &= 20 \times 10^{-6} \times N_{ptot}^{0.4}, & (11) \\
&= 0.2 \; kg/J, & (12)
\end{aligned}
$$

where we have used the longest lifetime possible suggested by the manufacturer. Note, as mentioned in the previous section, the specific mass of the capacitor bank increases with lifetime, but with less that a linear relationship.

- The burst energy storage capacitors will need a long lifetime, but do not need to be in a low inductance configuration, hence, have a smaller specific mass,

$$
\alpha_{eb} = 0.08 \; kg/J. \qquad (13)
$$

- The number of pulses per burst depends on the valve response and the timing of pulses within the burst. More pulses per burst equates to a more massive power modulator, while fewer pulses per burst leads to a lower propellant utilization [3]. Typically 5 pulses per burst has been used in recent studies,

$$
N_{pb} = 5 \; pulses. \qquad (14)
$$

- For this study we will use a solar power source and a reasonable efficiency for the power conditioner,

$$
\begin{aligned}
\alpha_e &= 0.026 \; kg/W & (15) \\
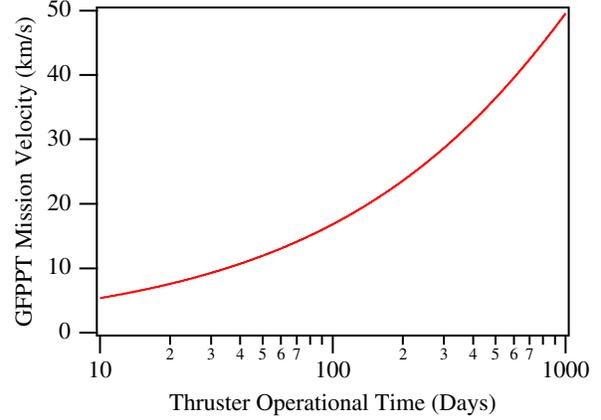\eta_{pc} &= 0.9 & (16)
\end{aligned}
$$

Figure 1: Graph of the GFPPT mission velocity, $\mathcal{V}$, versus operational time, τ.

- The operational time, τ, depends strongly on mission constraints and will help determine the power and thrust requirements for the GF-PPT. We will allow this parameter to vary in our study.

Based on the assumptions presented above, a graph of the GFPPT mission velocity, $\mathcal{V}$, versus operational time is shown in Fig. (1).

2.2   GFPPT Performance Scaling Model

A performance scaling model for GFPPTs has been experimentally verified using argon propellant in Ref. [4]. For this study, the most useful relation is,

$$
\frac{E}{I_{bit}} = \frac{\mathcal{U}}{2e^{-\sqrt{\psi}}}, \qquad (17)
$$

where ψ is a parameter between 0.3-3 that relates to the plasma resistance ($2e^{-\sqrt{\psi}} \approx 0.5$), and $\mathcal{U}$ is another technology dependent characteristic velocity that depends on the inductance gradient of the electrodes, L', the driving capacitance, C, and the parasitic inductance in the capacitor bank, L_0,

$$
\mathcal{U} \equiv \frac{3}{L'} \sqrt{\frac{L_0}{C}}. \qquad (18)
$$

Normal values for the GFPPT performance velocity, $\mathcal{U}$, range from 20-200 km/s with the smallest values giving the highest thruster performance. Note
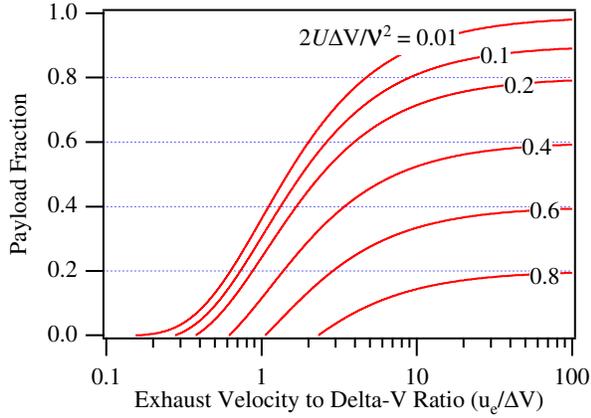
Figure 2: Graph of payload mass fraction as a function of exhaust velocity, ΔV, and technology characteristic velocities.

that the ratio of energy to impulse bit is independent of the exhaust velocity. This also indicates that the thrust-to-power ratio is independent of exhaust velocity, and that for a fixed thrust level, the power required by a GFPPT (and, therefore, a large part of the mass) is fixed based solely on mission requirements. The minimum thrust (or perhaps the maximum maneuver duration) will determine the power level, regardless of the exhaust velocity. The payload mass fraction can now be expressed entirely in terms of ratios of characteristic velocities,

$$\Gamma = e^{-\Delta V/\bar{u}_e} - \left(1 - e^{-\Delta V/\bar{u}_e}\right)\frac{2\mathcal{U}\bar{u}_e}{\mathcal{V}^2}. \quad (19)$$

2.3 Maximum Payload Mass Fraction

From Eq. (19) we see that the payload mass fraction depends on mission requirements and the GFPPT technology level with the exhaust velocity and ΔV as a free parameters. Figure 2 shows the payload mass fraction as a function of $\bar{u}_e/\Delta V$ with $2\mathcal{U}\Delta V/\mathcal{V}^2$ as a parameter. For a fixed ΔV, the following conclusions can be drawn from this graph:

- Payload mass fraction increases monotonically with exhaust velocity. In this case, there is no true "optimal" exhaust velocity. It is best to simply use the largest value possible, although for $\bar{u}_e > 10\Delta V$ the payload fraction

increases only slightly more, and thruster lifetime may begin to decrease.

- The payload fraction depends strongly on $2\mathcal{U}\Delta V/\mathcal{V}^2$ with the smallest values leading to the largest payload fractions.

- There is a minimum value of exhaust velocity for each ΔV below which the mission simply is not possible.

By expanding Eq. (19) for large values of $\bar{u}_e/\Delta V$, some interesting trends can be identified,

$$\Gamma \approx 1 - \frac{\Delta V}{\bar{u}_e} - \frac{2\mathcal{U}\Delta V}{\mathcal{V}^2}. \quad (20)$$

From this relation, small values of ΔV and $\mathcal{U}$ as well as large values of $\bar{u}_e$ and $\mathcal{V}$ lead to large payload fractions. For payload fractions greater than zero, the following relation must also be true for exhaust velocity,

$$\bar{u}_e > \frac{\Delta V}{1 - \frac{2\mathcal{U}\Delta V}{\mathcal{V}^2}}, \quad (21)$$

with another condition that,

$$\frac{\mathcal{V}^2}{2\mathcal{U}} > \Delta V. \quad (22)$$

This last relation gives a design criteria for the technology. If $\mathcal{V}^2/2\mathcal{U}$ is not larger than the mission ΔV requirement, then the mission is not possible for that technology. Conversely, evaluating $\mathcal{V}^2/2\mathcal{U}$ for a given technology or design also yields the maximum ΔV it can provide, *regardless of spacecraft mass or power*. Returning to Eq. (17), in terms of performance, the impulse bit to energy ratio must meet the following requirement,

$$\frac{I_{bit}}{E} > \frac{\Delta V}{\mathcal{V}^2}. \quad (23)$$

Again, this relation provides motivation for increasing $\mathcal{V}$. A larger value of the GFPPT mission velocity allows a larger ΔV or a smaller $\frac{I_{bit}}{E}$ requirement.

3 Mission Analysis for GFPPTs with Refueling Options

We will consider three different types of earth orbit maneuvers: orbit raising, orbit phasing, and inclination changing. Because the GFPPT is a low-thrust

device with long operation times, we will use the optimal low-thrust trajectory equations derived by Edelbaum [10] to evaluate the ΔV requirements for each maneuver.

For orbit raising and inclination changes, the Edelbaum equation is,

$$\Delta V = \sqrt{V_0^2 + V_f^2 - 2V_0V_f \cos\left(\frac{\pi}{2}\Delta i\right)}, \quad (24)$$

where V_0 and V_f are the initial and final orbit velocities, respectively, and Δi is the inclination angle change in radians. For orbit phasing, the ΔV requirement depends on the orbit radius, r, and the amount of time allowed for the maneuver, τ_m,

$$\Delta V = \frac{4}{3}\frac{r\Delta\theta}{\tau_m}, \quad (25)$$

where $\Delta\theta$ is the orbit phase angle change in radians. Note that shorter maneuver times require larger ΔV increments. These equations for ΔV are plotted in Fig. (3).

In many cases, missions will be made up of a variety of maneuvers in combination or separately, distributed over the mission. Based on Fig. (3), inclination change maneuvers are the most costly in terms of ΔV. If a mission requires such a maneuver, the total ΔV can easily reach above 1-2 km/s. Comparatively, many short-duration orbit phasing maneuvers can be conducted for a similar ΔV. Furthermore, if there is a chance for the satellite to be refueled, then the total change in velocity over the lifetime of the GFPPT, ΔV_{tot}, can be quite large. This concept will be examined in more detail in the next section, but for now, from examining Fig. (2), it is clear that a large exhaust velocity and impulse-to-energy ratio is required for the GFPPT to be useful.

3.1 Effects of Refueling

For missions such as DARPA Orbital Express, there is a possibility that the spacecraft will be refueled on-orbit. This potentially enables a smaller satellite mass initially, yet a large amount of propellant will be launched to keep the satellite running. If the total velocity change over the entire span of the satellite lifetime (including propulsion system) is known,
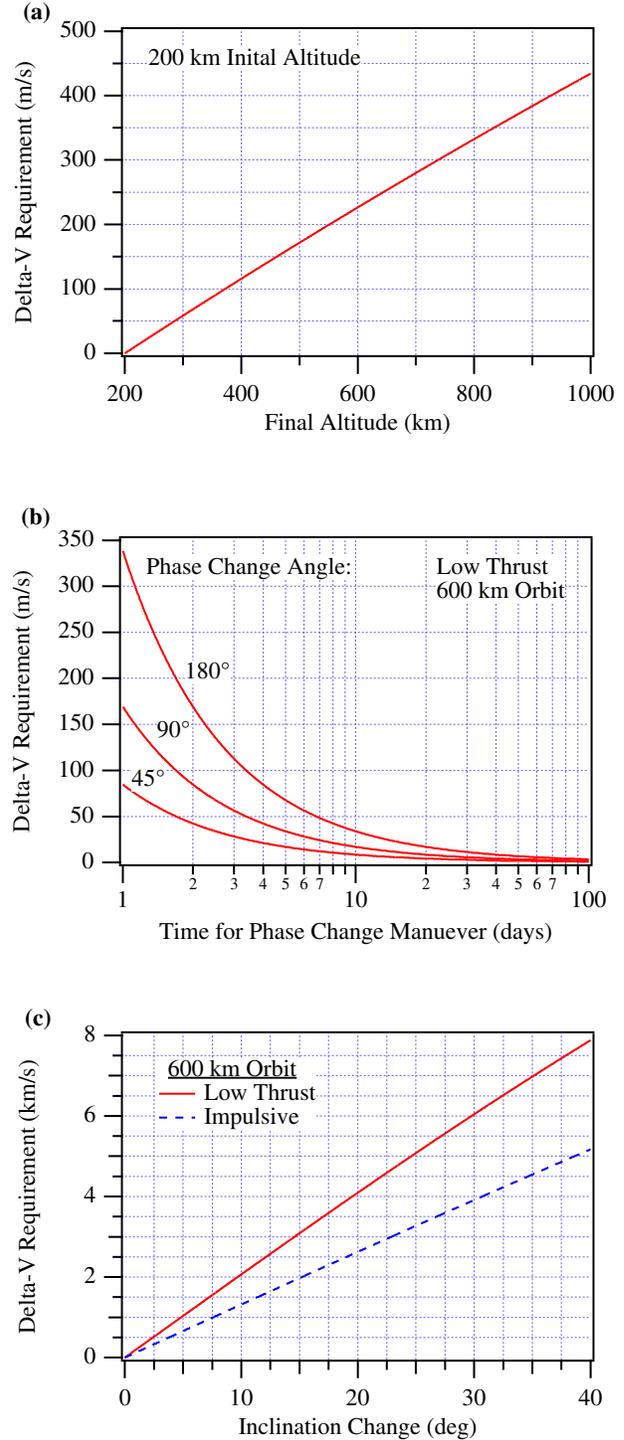

Figure 3: Three graphs of ΔV requirements for orbit raising, orbit phasing, and inclination changing using low-thrust trajectory equations.
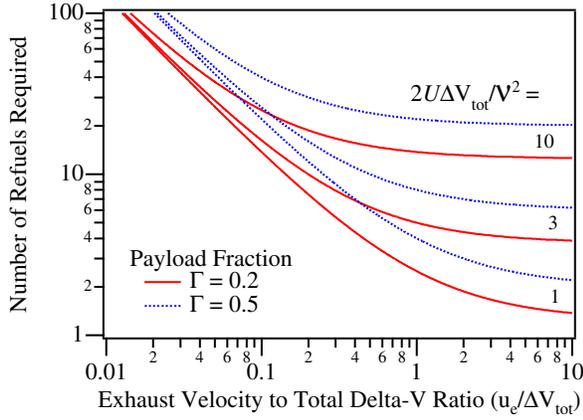
Figure 4: Graph of the number of refueling trips required to achieve a total ΔV with the desired payload fraction and characteristic velocities ratio as parameters.

Figure 5: Payload mass fraction as a function of the operational time per refueling with the number of refueling trips as a parameter.

then the number of required refueling trips, N_{refuel}, can be evaluated based on the desired payload fraction,

$$N_{refuel} = \frac{\frac{\Delta V_{tot}}{\bar{u}_e} + \frac{2\mathcal{U}\Delta V_{tot}}{\mathcal{V}^2}}{1 - \Gamma}. \tag{26}$$

This relation is plotted in Fig. (4).

First, it should be noted that the ΔV_{tot} for a refuelable spacecraft will be larger than the ΔV for a single-trip satellite. It follows that the ratio $\bar{u}_e/\Delta V_{tot}$ and $2\mathcal{U}\Delta V_{tot}/\mathcal{V}^2$ will also be smaller and larger, respectively, than the analogous parameters we used in the previous section. Second (and also related), we can see that a large value of exhaust velocity is necessary to reduce the number of required refueling trips. This is true because when the exhaust velocity is higher than the total ΔV, the terms related to the power supply, thruster, and modulator masses dominate over the propellant mass.

One important concern with refueling is that the capacitors and electrode set of the GFPPT are not replaced at the same time as the propellant. The lifetime of these components must be spread out over the entire mission and every refueling. With increasing ΔV_{tot}, the total impulse, I_{total}, also increases but the total number of pulses must stay fixed. This requires a larger impulse bit, which in turn requires a larger discharge energy and capacitor bank. Therefore, with each refueling and increase of ΔV_{tot}, the
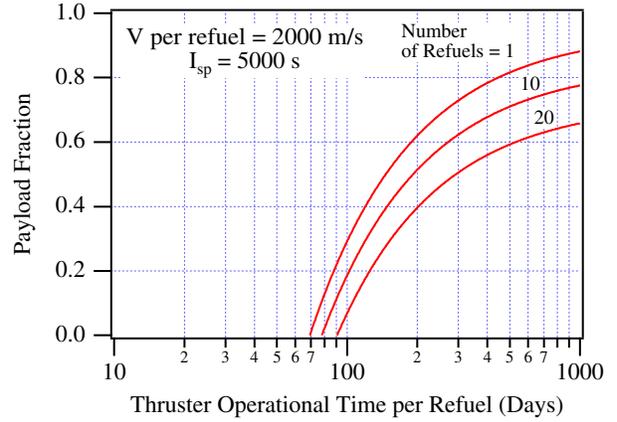
payload mass fraction will decrease due to the increase in thruster mass. This is shown in Fig. (5) with a fixed ΔV per refueling of 2000 m/s. At the longer operation times, it can be seen that the payload fraction is reduced by nearly a percent for each refueling trip. Conversely, if quick maneuvers are required so the operational time of the thruster is short, then the payload fraction is reduced even more by each refueling trip.

Combining the data presented in Fig. (4) and Fig. (5) suggests that a GFPPT will yield the highest payload fraction with a high exhaust velocity, a limited number of refueling trips, and longer operational times.

3.2 Influence of ΔV and I_{bit}/E on Payload Mass Fraction

To guide the development of GFPPT technology and compare it to other potential electric propulsion systems, we will now examine the payload mass in terms of the impulse-to-energy (thrust-to-power) ratio and the specific impulse of the propulsion system. We will also fix the number of refueling trips at 10 and eventually fix the specific impulse of the GFPPT at 5000 s with a ΔV of 2000 m/s per refueling ($\Delta V_{tot} = 20$ km/s).

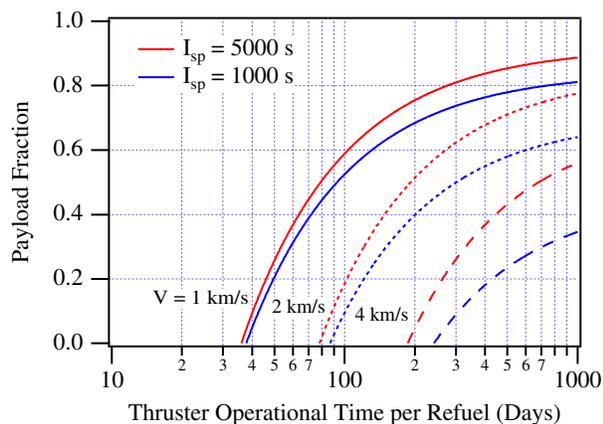First, we examine the payload fraction using two specific impulse values and three values of the ΔV

Figure 6: Graph of payload fraction as function of the operational time of the thruster between refueling trips. Specific Impulse (I_{sp}) and ΔV are parameters.
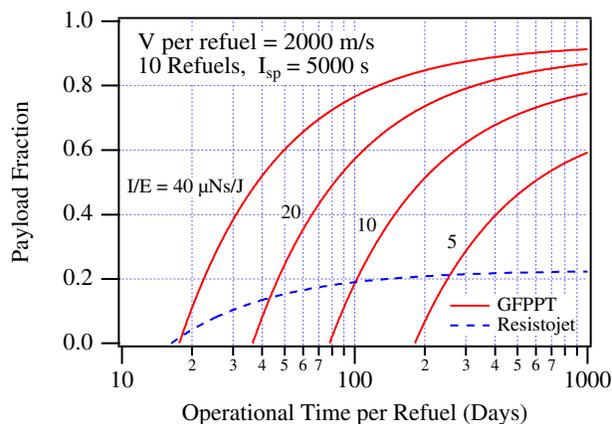

Figure 7: Payload fraction as a function of operational time for both the GFPPT and water resistojet. The ΔV for this phase is 2000 m/s with the I_{bit}/E for the GFPPT as a parameter.

per refueling. As seen in Fig. (6), the payload fraction increases with increasing operation time. This relates to using less power and thrust over a longer time while providing the same total impulse. If the maneuvers need to be quick, the payload fraction will be reduced with some short operational times simply not available as there would be no payload. The payload fraction also increases with increasing specific impulse with a more noticeable influence at the highest ΔV value.

Second, we will now compare the payload fraction that can be obtained with a GFPPT and a water-based resistojet for two different types of maneuvers. The performance of the resistojet is taken from Ref. [11] as I_{sp} = 150 s and T/P = 100 μN/W. Again, we fix the ΔV per refueling trip at 2000 m/s with 10 refueling trips total. The results, shown in Fig. (7), indicate that there are conditions at which one or the other thruster would be more beneficial. For many situations the GFPPT has a higher payload fraction, especially when the operation time is long. The resistojet, however, can preform almost any duration mission while the GFPPT has a cut-off that depends on the impulse-to-energy ratio. For example, if the GFPPT has an impulse-to-energy ratio of 10 μNs/J, then the trade-off operation time between the two technologies is 100 days. If the entire 2000 m/s velocity change needs to occur in less that 100 days,

then the resistojet will have a higher payload fraction. If there are more than 100 days available for the maneuver(s), then the payload fraction provided by a GFPPT quickly increases beyond the capabilities of the resistojet. The situation favors the GFPPT more as the impulse-to-energy ratio increases.

Finally, the last maneuver studied consists of using ten phase change maneuvers during one of ten refueling trips. Figure 8 shows how the payload fraction varies as a function of the time for one of the ten phase change maneuvers. Again, there are conditions at which one or the other thruster would be more beneficial. In fact, the resistojet is better in many cases because of its high thrust-to-power ratio. The GFPPT only starts to look better than the resistojet when its impulse-to-energy ratio is > 10 μNs/J.

These results point to a technology development path for the GFPPT. To compete with other water-based propulsion systems, the impulse-to-energy ratio of the GFPPT must be increased to > 10 μNs/J while maintaining lifetime and specific impulse. These efforts are on-going at Science Research Laboratory, Inc. and JPL.

4   Conclusions

A performance scaling model for GFPPTs has been integrated into mass scaling relations for a refuelable
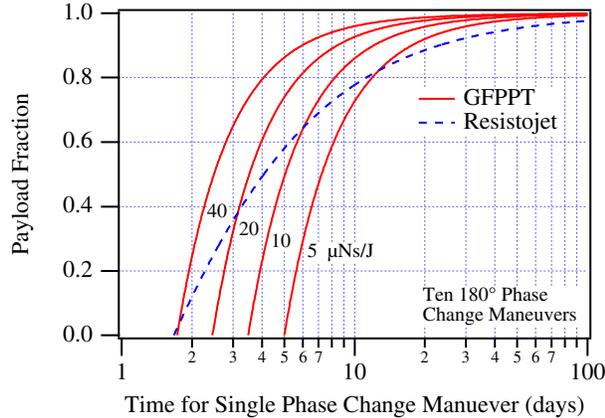
Figure 8: Payload fraction for a GFPPT and resistojet providing orbit phasing maneuvers as a function of the time for a single maneuver.

satellite. It was shown that an "optimal" exhaust velocity does not exist for the GFPPT and the highest exhaust velocity or specific impulse operating point should be used while maximum lifetime is maintained. In the limit of a large exhaust velocity to ΔV ratio, the power supply, thruster, and modulator mass dominate over the propellant mass. These power and energy related masses have been compressed into a GFPPT mission characteristic velocity that is solely dependent on the state of the technology. The results from the mission studies show that the impulse-to-energy ratio and the lifetime of the GFPPT need to be greater than 10 μNs/J and 1×10^{10} pulses, respectively, to reach payload fraction values and maneuver durations of interest to DARPA Orbital Express and other refuelable missions.

Acknowledgements

References

[1] J.K. Ziemer. *Performance Scaling of Gas-Fed Pulsed Plasma Thrusters*. PhD thesis, Princeton University, 2001.

[2] J. Blandino, D. Birx, J.K. Ziemer, and E.Y. Choueiri. Performance and Erosion Measurements of the PT8 Gas-Fed Pulsed Plasma Thruster. Technical Report EPPDyL-JPL99b, NASA Jet Propulsion Laboratory, August 1999.

[3] J.K. Ziemer, E.Y. Choueiri, and D. Birx. Comparing the Performance of Co-Axial and Parallel-Plate Gas-Fed PPTs. In 26th *International Electric Propulsion Conference*, Kitakyushu, JAPAN, October 17-21 1999. IEPC 99-209.

[4] J.K. Ziemer and E.Y. Choueiri. Scaling laws for electromagnetic, pulsed plasma thrusters. *PLASMA SOURCES SCIENCE & TECHNOLOGY*, 10(3):395–405, 2001.

[5] J.K. Ziemer, E.Y. Choueiri, and R.G. Jahn. Scaling Laws for Pulsed Electric Propulsion with Application to the Pluto Mission. In 24th *International Electric Propulsion Conference*, Moscow, Russia, September 1995. IEPC 95-147.

[6] E.Y. Choueiri. Optimization of Ablative Pulsed Plasma Thrusters for Stationkeeping Missions. *Journal of Spacecraft and Rockets*, 33(1):96–100, 1996.

[7] R.L. Burton and P.J. Turchi. Pulsed Plasma Thruster. *Journal of Propulsion and Power*, 14(5):716–735, 1998.

[8] *WIMA Capacitor Catalog*, 2001.

[9] *Maxwell Capacitor Catalog*, 2001.

[10] T.N. Edelbaum. Propulsion Requirements for Controllable Satellites. *ARS Journal*, pages 1079–1089, August 1961.

[11] M.N. Sweeting, T. Lawrence, and J. Leduc. Low-cost orbit manoeuvres for minisatellites using novel resistojet thrusters. *PROCEEDINGS OF THE INSTITUTION OF MECHANICAL ENGINEERS*, 213(G4):223–231, 1999.