Two-Dimensional Numerical Simulation of Coaxial APPT

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The results of the numerical investigation of plasma flow in coaxial Ablative Pulsed Plasma Thrusters (APPT) connected with outer electric circuit are presented in this paper. Two-dimensional MHD model describes dynamics of constant temperature, plasma in the APPT, in axisymmetric case. Boundary conditions for plasma density at the inlet of accelerating channel use phenomenological dependencies and analytical considerations. Numerical investigation is compared with experimental results obtained at APPT – stand. The model gives a correct qualitative picture of separate stages of discharge in APPT: plasma flow formation, appearance of tubular structure at the end of the central electrode, focusing of a flow etc. The features of propellant flow rate determine character of distribution of density, magnetic field and especially axial velocity of plasma in accelerating channel. The accounted integral parameters of considered APPT have appeared close to measured in experiments. The developed numerical model of APPT is a tool to improve thruster parameters.

Introduction

Earlier for description of plasma acceleration in pulsed plasma thrusters were considered simple 0D models, in which the plasma formation was considered as a material point with constant or variable mass. These models have appeared rather useful to optimisation of integrated parameters of APPT, but did not describe plasma formation and dynamics. Presently to simulate plasma behaviour one and two non-stationary MHD models are used. One-dimensional models are useful for modelling of acceleration dynamics, connection with electric circuit, plasma formation near propellant surface. The results of numerical modelling in 1D are usually good analysed and interpreted. Numerous experiments show that the plasma flow in coaxial APPT are essentially two-dimensional; there are current loops in the accelerating channel, flow focusing towards the APPT-axis is observed etc. Therefore a 2D model is expedient to be used for an adequate representation of plasma flow geometry and focusing in such thrusters. In ideal case 2D model must obtain better correlation of calculated integral parameters of the thruster with experimental data. 2D MHD - model of plasma acceleration in APPT can additionally simulate phenomena connected with geometry of accelerating channel and flow [1,2]. The results of 2D numerical modelling are not simple for analysis and interpretation and often these needs to expand and repeat.

Experimental data suggest that the mass flow rate in APPT is proportional to the stored energy in the range of units of Joule. Ablated mass varies near 3μg/ J [3].

APPT with higher bank energy have lower ablated mass per pulse. In high power APPT value near 0.2 μg/J is realized [4]. Accordingly higher efficiencies are obtained. The development of APPT having high efficiency includes: a) reducing of propellant ablation, especially late-time ablation; b) optimization of a discharge, accelerating channel geometry and electric circuit parameters. The reducing of mass flow rate in APPT seems should be solved experimentally due to extremely complicated character of this problem for adequate modeling. Other part of work could be solved numerically. In this paper we suggest the statement and calculation of 2D MHD model of plasma acceleration in coaxial APPT to obtain numerical tool for APPT development.

Problem Statement

A rather complete model of a plasma flow in APPT should take into account the vapour molecule dissociation and neutral particle ionization; the model should represent the dynamics of molecules, atoms ions and electrons. However, even some attempts to represent the gas ionization only in a classical approach of the stationary flow encounter essential difficulties, since the gas ionization process in a thin layer is non -equilibrium one and the temperatures of ions and electrons can essentially differ from each other. In [5] the phenomenon of inlet ionisation in self field MPD thruster analysed and explained without accounting of radiation effects. The question: can radiative effects be significant, has no answer for APPT as for MPD. Moreover, one should take into account of the propellant evaporating kinetics. In this situation, it often turns out to be expedient to use a power approach to the description of an ionization process in plasma, using the idea of an electron cost, ε

$$\varepsilon = I + 3/2T_e + R,$$

where I is the ionization potential, Te is the electron temperature, R is radiation energy fraction per one act of ionization. The introduction of ε into consideration essentially simplifies the gas ionization process consideration in APPT. On the other hand, the attempts of strict and sequential ionization process consideration are often not confirmed by the experiments, since the development of ionization and radiation are mainly determined by the presence of impurities in the plasma. So one can assume that energy sink connected with energy dissipated in plasma as: $P = \varepsilon\, m^*$.

P are power losses in plasma, m*- propellant flow rate. Such assumption significantly decreases the problems with plasma description. After that assumption together with accepted evident deficiency of this simplified consideration we obtain the opportunity to concentrate the problem on plasma dynamics and energy conversion in APPT.

Rather small plasma temperature changes in accelerating channel are a specific feature of the electric discharge in APPT. This fact is related with a strong plasma radiation intensity growth with an increase in temperature. Weak temperature changes in the accelerating channel allow one to use an isothermal model of the acceleration.

Thus, the plasma dynamics in APPT can be represented by the set of magnetic hydrodynamics equation, which has the following form:

$$\frac{\partial \rho}{\partial t} + div\rho\mathbf{v} = 0 \qquad (1)$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{\nabla P}{\rho} + \frac{1}{4\pi\rho}\left[\mathrm{rot}\mathbf{H}, \mathbf{H}\right] \qquad (2)$$

$$\frac{\partial \mathbf{H}}{\partial t} = \frac{c^2}{4\pi\sigma}\Delta\mathbf{H} + rot\left[\mathbf{v}, \mathbf{H}\right] + \frac{M_i c}{4\pi e} rot \frac{\left[rot\mathbf{H}, \mathbf{H}\right]}{\rho} \qquad (3)$$

$$P = \frac{2\rho kT}{M_i} \qquad (4)$$

$$T = T_i = T_e = const \qquad (5)$$

The generally- adopted designations of the values are used in Eqations. (1-5): ρ is the plasma density, v is the velocity, P is the pressure, H is the magnetic field, T is the temperature, σ is the conductivity, M_i is the ion mass, c is the velocity of light, e is the electron charge. In the equation of motion we neglect the viscosity forces; in the equation of magnetic field induction, the electron pressure gradient. The last term in the equation of induction (3) takes account of the Hall effect in plasma. Equations, representing the plasma acceleration within the framework of a two-dimensional model, can be obtained from Equations (1-5) and from the relationships for the initial and boundary conditions considered above.

APPT is schematically presented in Figure 1. The performance of this thruster has been described in [4].

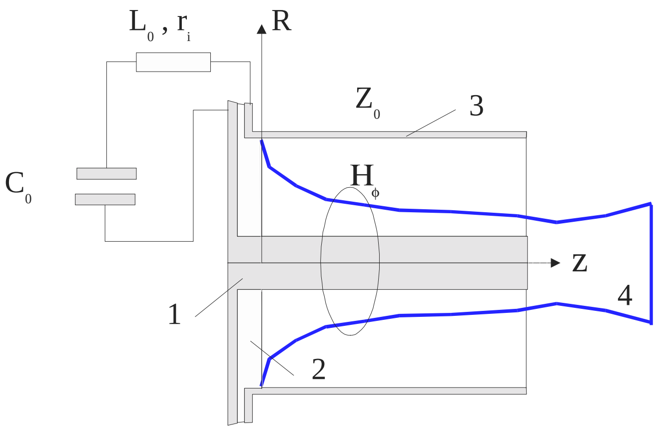

Figure 1. APPT and the electric circuit diagram; 1-internal electrode, 2 – solid propellant, 3 - external electrode, 4 -plasma.

In this case, the electron pressure gradient, the thermal force, and the Hall term are dropped in Ohm's law for the plasma. Moreover, it assumed that the degree of plasma ionization is equal 1, and $T=T_e=T_i$. The plasma conductivity is calculated from the relation $\sigma = 1.5\ 10^{17}\ T_e^{3/2}$ (the Coulomb logarithm =6). It is also assumed that the plasma temperature is constant. This last assumption distorts the real distribution of values in the accelerating channel, especially at the leading edge of the plasma flow and at the dielectric surface.

Dimensionless Equations

The choice of main units for dimensionless equations is: $t_0 = 1/c\sqrt{L_0 C_0}$, - characteristic time of electric circuit; $R_o=R_1$- radius of central electrode of accelerating channel; V_o is the initial voltage on the capacitor bank. Other units are:
Velocity- $v_o=R_o/t_o$, magnetic field is $H_o=cV_o t_o/R_o^2$, plasma density is $\rho_o=H_o^2/4\pi v_o^2$, energy unit is $E_o=C_o V_o^2/2$, current is $cR_oH_o/2$, resistance unit - $r_{go}=2v_o/c^2$.
Taking the assumption made above into account, we write a system of dimensionless equation with the same notation for the quantities used in the problem.
Then dimensionless equations can be written as:

$$\rho\left(\frac{\partial v_z}{\partial t} + v_z\frac{\partial v_z}{\partial z} + v_z\frac{\partial v_z}{\partial R}\right) = -A_3\frac{\partial \rho}{\partial z} - H\frac{\partial H}{\partial z},$$

$$\rho\left(\frac{\partial v_R}{\partial t} + v_z\frac{\partial v_R}{\partial z} + v_R\frac{\partial v_R}{\partial R}\right) = -A_3\frac{\partial \rho}{\partial R} - \frac{H}{R}\frac{\partial}{\partial R}(RH),$$

$$\frac{\partial H}{\partial t} = \frac{\partial}{\partial R}(v_R H) - \frac{\partial}{\partial z}(v_z H) - A_2\left(\frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial H}{\partial R} - \frac{H}{R^2} + \frac{\partial^2 H}{\partial z^2}\right) -$$
$$\xi\left[\frac{\partial}{\partial R}\left(\frac{H}{\rho R}\right)\frac{\partial}{\partial z}(RH) - \frac{\partial}{\partial z}\left(\frac{H}{\rho R}\right)\frac{\partial}{\partial R}(RH)\right],$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R}(R\rho v_R) + \frac{\partial}{\partial z}(\rho v_z) = 0, \quad P = A_3\,\rho. \quad (6\text{-}11)$$

Here $A_2 = \dfrac{c^2}{4\pi\sigma R_0 v_0}$, $\xi = \dfrac{cM_i}{eR_0\sqrt{4\pi\rho_0}}$ - Morozov parameter, $A_3 = \dfrac{2kT}{M_i v_0^2}$.

Boundary and Initial Conditions

For APPT, the plasma mass flow rate at the accelerating channel inlet (z = 0) depends on the discharge parameters. Ideally, this dependence should be accounted from the consideration of a gas ionization process in the near- propellant surface and energy flux transfer from main discharge. This problem is very complicated for theoretical consideration and present separate question. More simple way is to use experimental data analysis. Experiments show that propellant flow rate is approximately proportional to the electric discharge power. Moreover experimental results indicate that with energy flux densities to the propellant greater than 10^5 W/cm^2 and with an aperiodic discharge, the local value of plasma density near the dielectric is proportional to the product of the current density j and the electric field near the APPT propellant bar (Teflon insulator), i.e.,

$$\rho(t,r,0) = \alpha\, j(t,r,0)E(t,r,0)\,, \qquad (12)$$

where α is a coefficient that depends on the propellant material, channel geometry etc. Equation (12) is one of boundary conditions of the problem. Use of such a condition deletes the number of questions connected with inlet description of APPT. This is not right for the very beginning of a discharge, when plasma is generated with an igniter plug, and begin to work when the quantity of ignition plasma become relatively small. The remaining boundary conditions for the inlet are determined by the equations of electric circuit of the APPT:

$$V - \ln(\lambda)\left[v_z H(1,0,t) + A_2 \frac{\partial}{\partial z} H(1,z,t)\Big|_{z=0}\right]$$

$$- A_1 \frac{\partial}{\partial t} H(1,0,t) - H(1,0,t) r_g = 0$$

$$\frac{dV}{dt} = A_1 H(1,0,t) \quad (13\text{-}14)$$

A_1= L/2R_o- ratio of initial inductance to diameter of central electrode, $\lambda=R_2/R_1$.

At the inlet of the accelerating channel (z=0), the following boundary conditions are used :

$$v_z = v_0,\ v_R = 0,\ T = T_0,\ H(R,0,t) = \frac{H(1,0,t)}{R} \quad (15)$$

The magnetic field at the point (1,0) is determined from the electric circuit equation.

At the initial stage of the discharge, the gap between the APPT-electrodes is filled with some amount of plasma. One can assume that the this plasma occupies a cylindrical layer, z* -thick, at t=0:

$0<z<z^{**}$, $R_1<R<R_2$,

and it is characterized by the following parameters

$$v_z = v_0,\quad v_r = 0,\ H = 0,\quad T = T_0,\ \rho = \rho^*$$

Some complicated processes occur at the plasma-metal interface. Heat releases at the electrodes under Ohmic heating of their surface layer and under absorption of radiation from discharge, as well as a result of electron and ion plasma heat conduction can result in the electrode melting and evaporation. Nevertheless, these phenomena, in the microsecond range of the discharge duration even in high power APPT, weakly affect APPT parameters. Indeed, it has experimentally been shown that the losses at the electrodes do not exceed 10-15 % in the total energy balance. The boundary layer thickness in the plasma should be of the order of a few particle mean free paths and the consideration of this range within the framework of the MHD- model does not seen to be possible. Therefore it is expedient to formulate the boundary conditions of the problem to exclude the boundary layer consideration near electrodes. Within the framework of the MHD -model it can be done, assuming that:

$$\frac{\partial v_z}{\partial R} = 0,\ \frac{\partial \rho}{\partial R} = 0,\ \frac{\partial}{\partial R}(RH) = 0,\ V_R=0 \quad (16)$$

at the electrode surfaces (when $R=R_1$, $R=R_2$); and

$$\frac{\partial v_R}{\partial z} = 0,\ \frac{\partial \rho}{\partial z} = 0,\ \frac{\partial H}{\partial z} = 0 \quad (17)$$

at the central electrode face (when $z = z_o$, $R<R_1$).
As for the velocity v_z at $z=z_o$, $R<R_1$, i.e. at the inner electrode cut, two versions of boundary conditions are possible:
$v_z = 0$, the electrode cut is impenetrable for the plasma;

$$\frac{\partial v_z}{\partial z} = 0, \quad (18)$$

if the cathode cut is absolutely penetrable (divertor).
At the thruster-axis, when R=0, $z>z_o$, the boundary conditions, follow from the condition of symmetry for all the values

$$H=0,\ v_R=0,\ \frac{\partial v_z}{\partial R} = 0,\ \frac{\partial \rho}{\partial R} = 0. \quad (19)$$

Since the plasma conductivity is a finite in our case, one should formulate the boundary condition for the magnetic field. At the output from the APPT- channel ($z=z_o$), the condition has been used.

$$\frac{\partial H}{\partial z} = 0.$$

It is often used in the MHD- calculations. It corresponds to the neglecting of a radial current density component at the accelerating channel output.

The equations (6-11), are calculated by using a finite difference scheme. A structured rectangular grid with dual structure and different cells is used. To proceed calculations in 2D-geometry the alternating directions method is used. Successive factorisation is use to account magnetic field diffusion. In future studies, the advanced numerical schemes will be implemented for better satisfaction of energy conservation.

Results of Simulation

The parameters of the APPT under modeling are shown in Table 1.

Table1. APPT parameters.

Inner electrode radius, R_1	1 cm
Outer electrode radius, R_2	5 cm
Length of inner electrode, z_o	5-10 cm
Length of outer electrode, z*	17 cm
Initial voltage, V_o	3 kV
Bank capacitance, C_o	200 μF
Inductance, L_o	$2\ 10^{-8}$ H
Av. Mass of ion, M_i	$2.9\ 10^{-23}$g
Temperature, T	1.5 –3 eV
Input plasma density, ρ	10^{-7}–10^{-6}g/cm^3
Input plasma velocity, v_o	$5\ 10^5$ cm/s

Accordingly dimensionless parameters are : A_1= 8, A_2=2, A_3=5. Plasma density at the inlet of APPT is proportional $j(t,r,0)E(t,r,0)$. Maximal density is near 10^{-3}. (~ 10^{18}cm^{-3}).

The discharge in APPT is initiated with arriving in the accelerating channel of some initial plasma. Quantity of initial plasma must exceed some mass necessary for low impedance current propagation, as usually realized in experiments. From the very beginning of current propagation, energy flux from this initial plasma exposes the propellant surface. Propellant material are heated and ablated due to this flux. Plasma, produced at the destruction, dissociation and ionization of propellant material comes to the accelerating channel. So the quantity and area of initial plasma must provide necessary conductivity of inter-electrode gap. Simulation produced in a frame of constant temperature model is applicable from the noticeable ionization in plasma flow. According experiments such ionization arises at a distance less than 1 mm from propellant surface for 300 J of stored energy.

As it was often marked, the discharge in APPT is accepted conditionally to divide into following stages:

- formation of the discharge, which includes a temporary interval from the moment of initiation of the discharge up to plasma occurrence at the outlet of the accelerating channel;
- quasi- steady plasma flow, which in our case covers an interval ≈ $2t_o$. In this stage the distribution of parameters of plasma in the accelerating channel varies poorly;
- damping, when a current and power of the discharge decreased, and are small in comparison with their maximum values.

To find the parameters of numerical model close to experimental model, firstly the current - voltage diagram of the model was obtained and analyzed. It is produced similar to experimental one. The accounted current-voltage diagram of APPT discharge is shown in Figure 2.

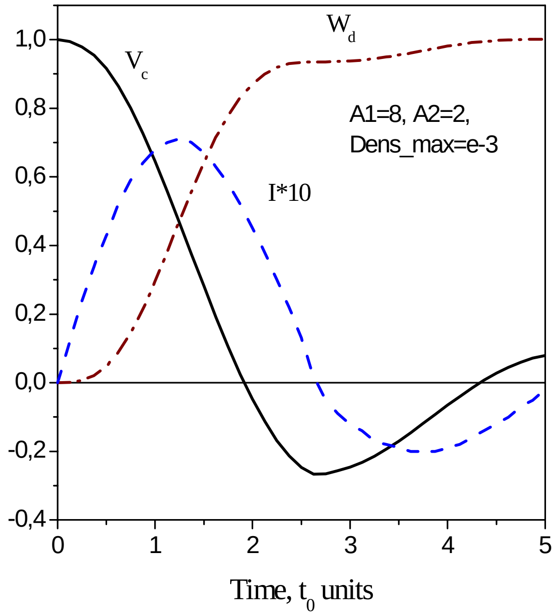

Figure 2. Time dependencies of a current and bank voltage in APPT; t_o= 2 μs.

The maximal current and form of current and voltage have a good correlation with experiment.

Figures 3, 4 give the distribution of plasma density and magnetic field in the accelerating channel for moments of time 1.75 (3.5μs). These distributions are changed in time with duration as current and voltage. Other words at the time (1 -2) quasi-steady plasma flow takes place (see Figures 3 - 7). Due to special condition for plasma density in the inlet, outlet axial velocity of plasma has weak radial dependence. Other picture arises for radial velocity (Figure 8). Due to boundary condition for plasma density that dramatically depends on radius, significant radial density gradients arise in the accelerating channel.

Radial acceleration to outer electrode produce radial velocities at the level (1-2) 10^6 cm/s. Accordingly relatively high density area arises near outer electrode.

In a quasi-steady part of discharge one can notice that plasma is magnetized. Electromagnetic mode of operation is realized.

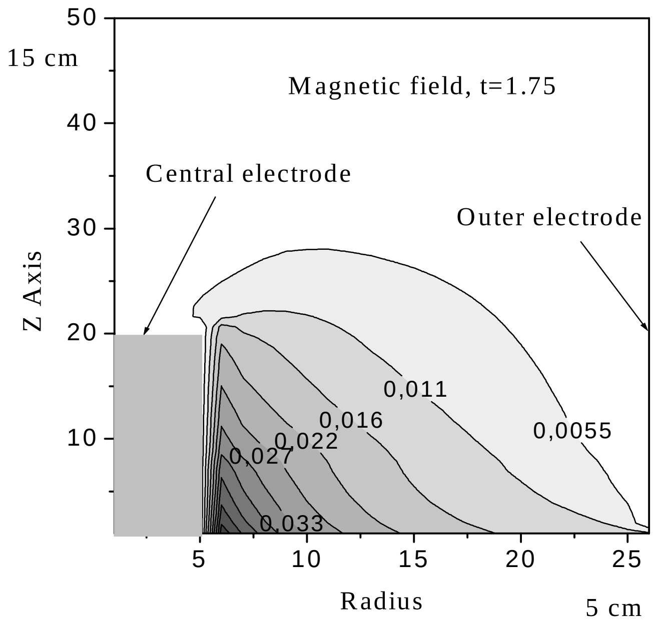

Figure 3. Magnetic field distribution in APPT accelerating channel (t= 3.5 μs).

When the front of plasma flow reaches the end of the central electrode, the flow with characteristic tubular structure is formed in this area. The similar structure of a flow exists at the end of the central electrode and further the distribution of plasma parameters varies poorly. The tubular distribution of density received in the simulation, has good quantitative agreement with interferometric measurements of plasma flow.

In the axis, part of plasma flow is focused and produces plasma focus (the area of increased plasma density).

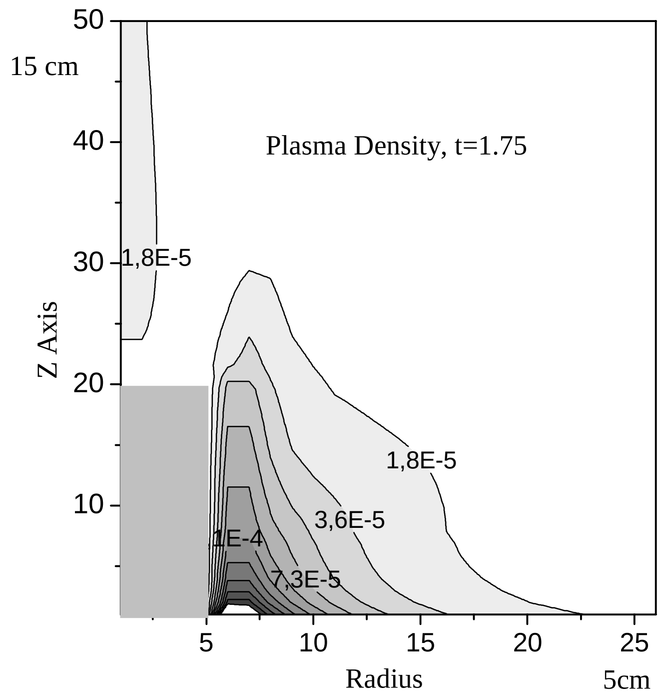

Figure 4. Plasma density distribution in APPT accelerating channel (t= 3.5 μs).

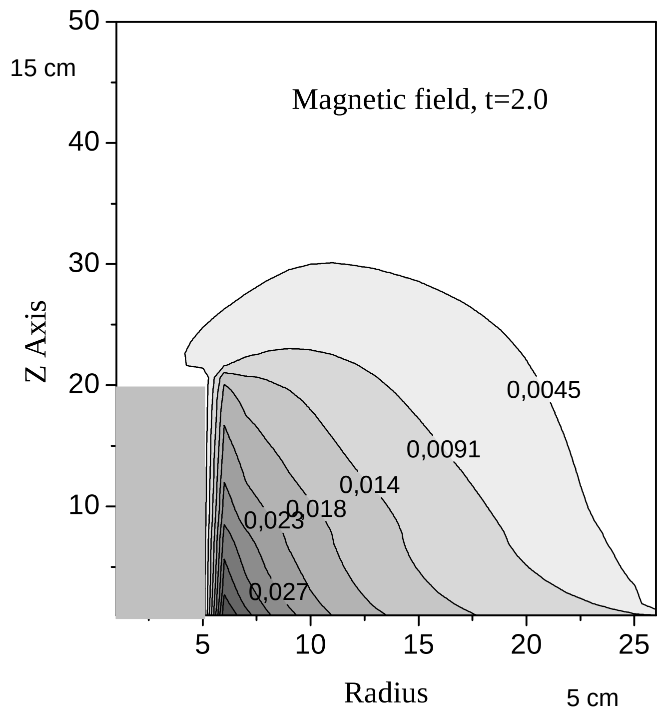

Figure 5. Magnetic field distribution in APPT accelerating channel (t= 4 μs).

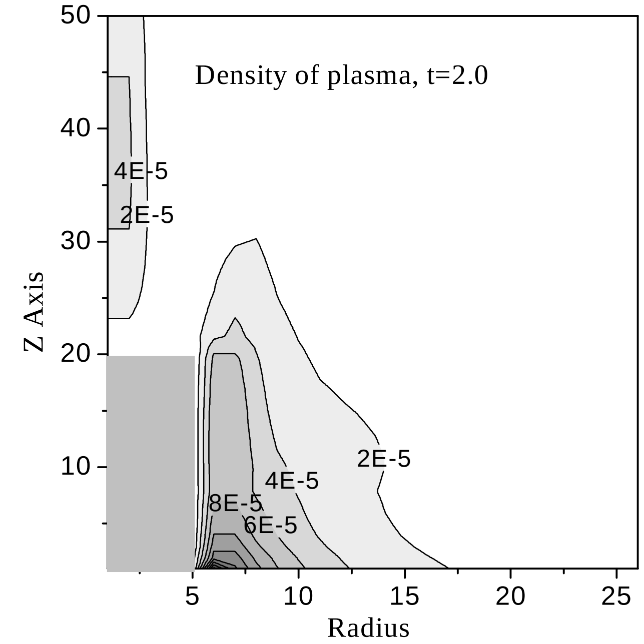

Figure 6. Plasma density distribution in APPT accelerating channel (t= 4 μs).

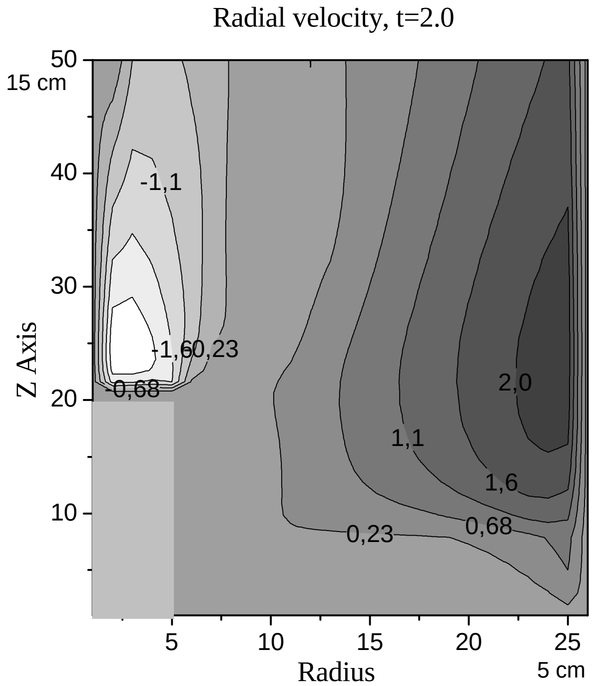

Figure 8. Radial velocity distribution in APPT accelerating channel (t= 4 μs).

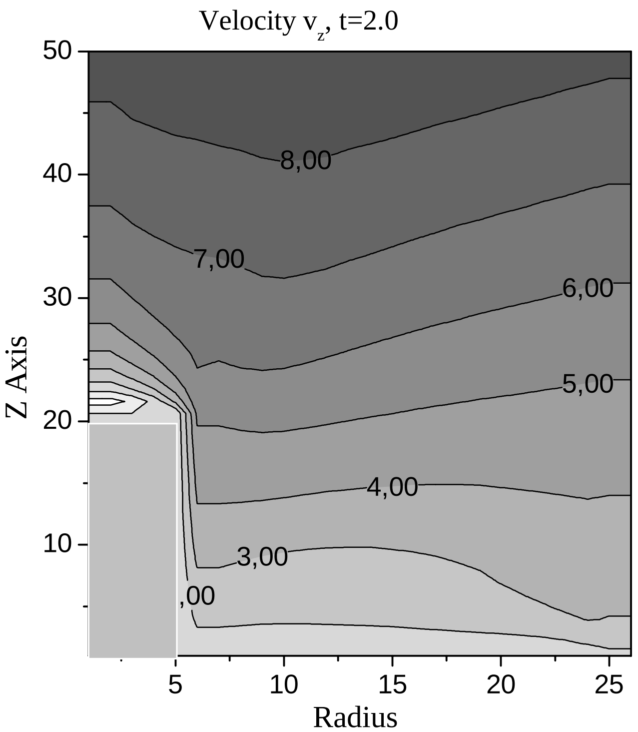

Figure 7. Axial velocity distribution in APPT accelerating channel (t= 4 μs).

There are not effectible current loops in volume of the accelerating channel. It is due to rather weak dependence of plasma axial velocity, from radius and large value of average plasma axial velocity (Figure 7). Thus the skin-effect, causing occurrence of current loops, play a smaller role. Energy conservation law in used numerical code was fulfilled with accuracy near 10%.

Conclusion

The developed two-dimensional model of APPT describes formation and acceleration of a plasma flow in coaxial accelerating channel. The calculated integral APPT-parameters (plasma impulse bit, evaporated mass, efficiency) are close to the experimentally measured ones. The model gives a correct qualitative picture of separate stages of discharge in APPT: plasma flow formation, appearance of tubular structure at the end of the central electrode, focusing of a flow etc.

The accounted distributions of the main values, characterizing the acceleration process (density, magnetic field, plasma velocity), qualitatively agree with those observed in the experiments.

The features of propellant flow rate determine character of distribution of density and especially axial velocity of plasma, and magnetic field in accelerating channel.

The developed two–dimensional model allows one to find the optimal operating conditions for APPT under which the maximal efficiency of energy conversion from the power source to the kinetic plasma energy is obtained.

The developed numerical model of APPT is a tool to improve thruster parameters. In future studies, the advanced numerical schemes will be implemented to satisfy energy conservation. The results obtained can be seen as a first step for the development of a adequate numerical 2D code for optimising APPT.

Acknowledgements

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