EFFECT OF THE PLASMA POTENTIAL OSCILLATIONS AT A THRUSTER OPERATION.

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Abstract.

A problem of ion beam density oscillations impact on an ion thrasher operation, running without additional source of electrons when an ion beam charge is compensated due to residual gas ionization. If the ion current exceeds the critical one which value depends on environmental conditions, the ion beam propagation may be broken. On the opposite case oscillations do not impact substantially on the thruster operation.

Notification.

φ - potential disturbance
φ_p - plasma equilibrium voltage with respect to wall
Õå - electron temperature
V_i - gas ionization potential
V_d - energy of fast ions (discharge voltage)
$v_s = (eT_e/M)^{1/2}$ - ion sound velocity
n_o- average density of fast ions
n_g - neutral gas density
j_w- ion flow at wall
$J_0 = n_0 v_0 S$ -flow of fast ions
$v_o = (2V_d/M)^{1/2}$ - velocity of fast ions
ω - oscillations frequency
$k = w/v_o$ - wave number
S_p - wall surface area
S - beam cross section area
e - electron charge
M - ion mass
ν_k - Coulomb collisions frequency
ν_i - frequency of ionization atoms by ions
C - plasma capacity with respect to wall

Introduction.

A plasma potential oscillations had been considered as applied to ion beams stability[1-4]. Below an impact of ion current oscillations on an electric thruster operation and life time is considered.

Practically any ion source bears its plasma density oscillations in certain range of frequencies w. As a result a ion current density of the beam has a harmonic disturbance. That is why a local perturbations of the positive volume charge take place as a rule in a plasma at a quasineutral and auto -compensated state.

The charge perturbations produce the potential perturbation for the whole plasma volume, which nevertheless remains quasineutral as a insulated metallic body. For a very little time an excessive charge is transferred to the plasma surface, at the near-wall layers, and cause the electric field growth near the wall surface. Another words, the total plasma charge variation is completed through the displacement currents over the surrounding plasma surface. So, the total excessive plasma charge **Q** variation and plasma voltage φ may be considered as being related by equation:

$$Q = C\varphi \quad ....$$

The potential of ion beam entering the plasma varies with time at these conditions up to peak value φ so that the fast ions energy is:

$$\varphi_i = V_d - (\varphi_p + \varphi)$$

and bears oscillations within limits:

$$V_p - (\varphi_p + \varphi) \le \varphi_i \le V_d - (\varphi_p + \varphi)$$

Within the same limits the energy of secondary ions varies. the last ones, fall on the walls, and sputter them. If φ is comparable with the discharge voltage $V_d \sim 300$V (for example $\varphi \sim 100$V), the ion beam to be decelerated dramatically, and secondary ions accelerating to energy range, cause a cathode sputtering of the walls.

Plasma potential.

At the ion beam injection in a bulk filled with neutral gas a transient process that directed on a positive volume beam charge compensation take place. As a result a certain potential arises in plasma to be generated under this process, that provides a steady state of the plasma / beam system. A magnitude of said potential may be derived by considering a currents balance in the plasma bulk. A number of electrons to be generated in the plasma bulk Ω for a time unite is $n_a\Omega\nu_i$
Where n_a -is an average value of the plasma density in the bulk;

$\nu_i = \sigma_m n_g v_e \exp(-V_i/T_e)$ - is a ionizing collisions frequency.

These electrons make the beam charge compensation and leave the bulk going to the walls. A chaotic electron current at the wall is $1/4 S_w n_e v_e \exp(-\varphi_p/T_e)$. Further we adopt that $n_e \approx n_a$. Equating the two currents we find:

$$\varphi_p = V_i + T_e \ln \frac{S_w}{4\Omega\sigma_m n_g}$$

So, an equilibrium plasma potential always is of the order of magnitude V_i and faintly depends on plasma and gas densities. It is noticeable that the potential is not zero even for zero electron temperature.

Plasma capacitance

It have been mentioned above that the total plasma charge variation in the plasma bulk has to be closed at the wall surface surrounding the plasma by the displacement currents. A magnitude of the currents depends on the near-wall plasma layer capacitance. and may be derived on a base of a charge and a potential balance at the layer. A value of electric field at the wall E_w is connected with a wall charge density with relation $q_w = E_w/4\pi$. To define E_w we use Poisson equation for the layer, and at its resolution obtain the relation between the wall charge and potential.

$$\frac{d^2\varphi}{dx^2} = \frac{4\pi e j_w}{v_s}\left[\exp\left(\frac{\varphi - \varphi_p}{T_e}\right) - \frac{v_s}{\sqrt{v_s^2 + \frac{2e(\varphi_p - \varphi)}{M}}}\right]$$

(1)

On the right side the difference of electron and ion densities in the layer are given. As for as electrons to be repelled from the layer, the Boltzmann equilibrium law for density description has to be used; once ions being accelerated to the wall by the layer potential, the ion density varies due to their velocity growth. At the layer boundary both densities are similar and can be expressed via a secondary ions current density j_w at the wall, and ion sound velocity: $n = j_w/v_s$. We assume, as usual, that ions flow at a high voltage side of the layer with ion sound velocity. The boundary conditions

After executing the integrating procedure and implementation the boundary condition $d\varphi/dx = 0$ at $\varphi = \varphi_p$ (below we use for definitiness sake an equilibrium potential φ_p as a steady state potential since the last one can be quite different from φ_p if a certain voltage between a thruster and a wall is set) we obtain for electric field at the wall ($\varphi = 0$), or that is the same, for the charge of the wall of S_p area:

$$Q_w = A(\sqrt{1+2\eta_p} - 2)^{1/2} \quad A = S_p\sqrt{\frac{2ej_wT_e}{v_s}} \qquad (2)$$

It is taken here that $\exp(-\eta_p) \ll 1$.
If the plasma potential bears a small oscillations of a magnitude $\eta = \varphi/T_e$ in the vicinity of its steady state magnitude η_p the displacement current at the layer is:

$$\frac{dQ_w}{dt} = C\frac{d\varphi}{dt}; \quad C = \frac{A}{T_e}F(\eta_p)$$
$$F(\eta_p) = \frac{1}{2\sqrt{1+2\eta_p}(\sqrt{1+2\eta_p} - 2)^{1/2}} \qquad (3)$$

Here C may be treated as a plasma capacity. It depends on the steady state potential and for $\eta_p \gg 1$ (if a rather high voltage between a thruster and walls occurred to be) we have $C \sim (\eta_p)^{-3/4}$.

The electron entrainment length.

An existence of a volume charge is a subject such situation that the electron gas and the ion beam are initially independent, being subject to only to the electric field s that arise in the bulk. A difference of their directional velocities cause a longitudinal potential waves that move along the beam with the ion velocity. As a result of collisions the electron gas is entarinmented by ions and acquire at a certain distance an ion velocity. As a result a ion beam becomes as though a beam of neutral particles and a charge vanishes. To estimate the distance it is necessary to consider the effect of electron entrainment under the Coulomb collisions with the beam ions. An electron motion equation can be written as:

$dv_e/dt = -\nu_k(v_e - v_o)$; $dx/dt = v_e$;

Where $\nu_k = 2 10^{-5} n_i T_e^{3/2}$ -the Coulomb collisions frequency [5];

The solution is:

$v_e = v_o[1 - \exp(-\nu_k t)]$; $x = v_o[t + \exp(-\nu_k t)/\nu_k]$;

after the time $t \sim 1/\nu_k$
we have $v_e \sim v_o$ and $x = 2v_o/\nu_k$; (4)

That is the distance along the ion beam where the uncompensated charge exists. It is necessary to notice that the entrainment length above is derived on condition that an electron motion is not limited, as it may be in presence of a magnetic field, when electrons to be tied to a magnetic lines of force are not entrained at all.

The magnitude of the potential oscillations

Let assume that the density of the ion beam escaped from an thruster bears a slight oscillation so far its Boltsmann function can be written as

$$f_i = n_0[1 + \xi \sin(kx - kvt)]\delta(v - v_0), \qquad (5)$$

and oscillations peak value $\xi \ll 1$.

Arriving into the ambient plasma, the beam changes a plasma total charge, and consequently a plasma potential. A potential growing, an electron current at the wall decreases, a total negative charge in the plasma bulk grows that compensates charge to be inserted. A charge that has not been compensated under this process is drifted out at the walls.

An equation for a currents balance can be written as:

$dQ_i/dt - dQ_e/dt = C dj/dt$ (6)

Where:

- **dQ_i/dt** - is a charge affected by the ion beam;
- **dQ_e/dt** - is a charge variation;

The last one can be written as[2]:

$$\frac{dQ_e}{dt} = \nu_i Q_e[1 - \exp(-\frac{\varphi}{T_e})] \qquad (7)$$

Where Q_e on the order of magnitude is the total electron charge in plasma. At $\varphi = \varphi_p$ we have $dQ_i/dt=0$ and the plasma potential equals to the equilibrium one.

An ion beam emerging from thruster, arrives an area, occupied with plasma under an equal **potential** $\varphi_s = \varphi_p + \varphi$, in the whole bulk, jumping at the boundary. As a result the beam acquires there an another density and velocity. Assuming that at the boundary the potential is linear along a little distance Δx as $\varphi = \varphi_s x/\Delta x$ let us define an ion Boltzmann function in the plasma after their traveling through the distance Δx. for this purpose let us write the Boltzmann equation for ions:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} - \frac{e\varphi_p}{M\Delta x}\frac{\partial f}{\partial v} = 0 \qquad (8)$$

It has two evident motion integrals:

$$c_1 = \frac{v^2}{2} + \frac{e\varphi_p}{M\Delta x}x \qquad c_2 = v + \frac{e\varphi_p}{M\Delta x}t$$

Then a general solution can be written as $f = F(c_1, c_2)$, where F is an arbitrary function. to find the is There is well known the procedure for the solution foundation that meets the boundary conditions [5].

Using (5) as a boundary condition at $x=0$ we can find the Boltzmann function at $x=\Delta x$. Letting after that Δx as zero, we find the Boltzmann function for ions that pass the potential jump φ_s.

A Boltzmann function for ions running trough a plasma can be found resolving an equation similar to (8) where however the last term to be missed since the plasma potential is constant in any point of the bulk and hence any forces acting on the ions to be absent. As a result we have:

$$f = n_0\left[1 + \xi\sin\left(k\frac{x - vt}{v}\sqrt{v^2 + \frac{2e\varphi_p}{M}}\right)\right]\times \left(\sqrt{v^2 + \frac{2e\varphi_p}{M}} - v_0\right)$$

A total charge of a beam with length **L** and the cross section area **S** can be found after integrating that expression by the velocity and beam length (according to (4) we take $L=2v_0/v_k$).

$$Q_i = S\int_0^L dx\int dv f(v,x) = \frac{Q_0}{\sqrt{1 - \frac{\varphi}{V_d}}} + \frac{\xi Q_0}{kL}\left[\cos\left(\frac{kL}{\sqrt{1 - \frac{\varphi}{V_d}}} - \omega t\right) - \cos\omega t\right]$$

After differentiation with respect to time and taking into account that $\varphi_s/V_d \ll 1$ we find in the first approximation by ξ:

$$Q = -\frac{Q_0}{2}\frac{\varphi}{V_d} + \frac{\xi Q_0}{kL}\sin\frac{kL}{2}\sin\omega t \qquad (9)$$

After substitution (3), (7) and (9) in(6) we obtain equation for a potential perturbation.

$$\frac{Q_0}{2V_d}\frac{d\varphi}{dt} + \frac{\xi Q_0}{kL}\sin\frac{kL}{2}\sin\omega t - \nu_i Q_e[1 - \exp(\varphi / T_e)] = C\frac{d\varphi}{dt} \qquad (10)$$

Inserting dimentionless variables $\eta = \varphi / T_e$ and $\tau = \omega t$ the equation can be put in the form:

$$\frac{d\eta}{d\tau} = p[\xi_L \cos\tau - q(1-e^{-\eta})]$$

$$\xi_L = \frac{2\xi}{kL}\sin(kL/2) \qquad q = \frac{\nu_i Q_e v_0}{J_0\omega} \qquad (11)$$

$$p = \frac{2V_d}{T_e}\frac{J_0/J_c}{1-J_0/J_c} \qquad J_c = 2v_0 C V_d / L$$

Equation (11) allows full integration, but the solution is too complicated for analysis and we do not show it here. We note, only, that the type of solution depends mainly on a sign of parameter **p**. For **p >0** the solution becomes periodical at large **t** and for **p<0** we have aperiodical, indefinitely growing solution. It is easy to see that the sign of **p** depends on the magnitude of the beam current. Notice that the problem determines its own current scale J_c . The beam is capable to run through the plasma if its current is less than the scale J_c. At that case a charge to be inserted is compensated due to driving an electron charge out of near-wall layer. At the opposite case a compensation can not be provides because $dQ_i/dt > dQ_e/dt$.

For little oscillations $\eta \ll 1$ expanding into a series the exponential term in the right side of (11), we obtain:

$$\frac{d\eta}{d\tau} = p[\xi_L \cos\tau - q(\eta - \frac{\eta^2}{2})] \qquad (12)$$

If the square term were removed, the solution of deduced linear equation can be presented as:

$$\eta = Be^{-pq\tau} + \frac{p\xi_L}{1+(pq)^2}(\sin\tau + pq\cdot\cos\tau) \qquad (13)$$

For **p>0** and at large τ the first term vanishes and solution becomes periodical, according to mentioned above, for the opposite case potential grows dramatically. From (13) it follows that even for large **p** an oscillation peak value may be small: $\eta_m \sim \xi_L/q$ at $p\to\infty$. The most peak magnitude corresponds to a low frequency oscillations at $kL\to 0$, for this case $\eta_m \sim \xi/q$.

A charge oscillations in a plasma cause to an average plasma bulk potential shift, that leads to the beam deceleration, or, another words, to a loss of the thruster efficiency. To estimate this magnitude we use equation (12) where the square term to be replaced by solution (13). The equation type is not changed at that, but its solution includes a secondary harmonics of oscillatory component and a stationary term, the last one corresponds to The average component. It is as follows:

$$\bar{\eta} = \frac{1}{4}\frac{(p\xi_L)^2}{1+(pq)^2}$$

If ξ is not little so that decomposition of the equation (12) does not provide a necessary accuracy but still $\xi_L/q<1$ and $p\to\infty$, it is reasonable to get a solution assuming zero the expression in brackets of the right side of (11). Indeed, since the solution to be limited, and its derivative as well, the right part of (11) must be limited also. Since **p** to be large, the brackets must be little. Hence:

$$\eta = -\ln\left[1 - \frac{\xi_L}{q}\cos\tau\right] \qquad (14)$$

and for the average magnitude of the potential we have:

$$\bar{\eta} = -\frac{1}{2\pi}\int_0^{2\pi}\ln\left[1-\frac{\xi_L}{q}\cos\tau\right]d\tau = -\ln\left[\frac{1+\sqrt{1-(\xi_L/q)^2}}{2}\right]$$

It follows from these relations that for large peak magnitudes of the beam density oscillations when $\xi_L/q\sim 1$, the potential oscillations become high while the average value tends to the limited magnitude **ln2**.

According to said above a conditions of a thruster operation must be set so that its current did not exceed the critical one $J_o < J_c$. If it is assumed that fast ions bear the full charge change along their path in plasma then the chaotic current of the secondary ions at the walls is equal to the ion current $J_o = j_w S_p$. After substitution into inequality above expressions (11) and (3) for critical current and capacity we obtain:

$$1 < \frac{8v_s}{S_p L^2}\frac{V_d^3}{J_0}F^2(\eta_p)$$

This relation shows the linkage between the beam parameters and the test conditions, at which a proper thruster operation being provided at a test under an autocompensation conditions.

The secondary ions energy.

The potential oscillations cause a variation of a energy distribution of a slow, ions which to be generated under the ionization and charge change processes. The last ones, arriving to the near-wall potential fall with energies of the order of magnitude of a electron temperature, undergo the acceleration by this voltage and falling at the wall, sputter it.

It is possible to determine a secondary ions energy distribution resolving the equation (8) at **x=0** , and defining now the initial function at the right boundary, for $x = \Delta x$. Let us define it as

$$f_m(v) = n_m \delta(v + v_s)$$

Where v_s and n_e are the secondary ions velocity and density (note, that $j_w = n_e v_m$).

After implementation the same procedure of finding the solution as it is described above, we get the Boltzmann function for secondary ions:

$$f_m = n_e \delta(v_s - \sqrt{v^2 - \frac{2e\varphi_s}{M}})$$

As it was stated before $\varphi_s = \varphi_p + \varphi$, where for the oscillatory potential φ the determined above solutions (13) and (14) has to be used. Using (14) foe example and making an averaging operation over the time period we get the ions energy distribution in the form:

$$f_m(w) = \frac{q}{\xi_L}\frac{j_w}{v_s^2}\frac{\exp(-w)}{\sqrt{1-\left(\frac{q}{\xi_L}\right)^2(1-\exp(-w))^2}} \qquad (15)$$

where $w = (v^2 - v_s^2 - 2e\varphi_p/M) / v_s^2$.

The secondary ions occurred to be distributed in the energy range

$$-\ln(1+\xi_L/q) < w < -\ln(1-\xi_L/q)$$

and at the boundary points of the range the number grows indefinitely. Taking into account a little heat deviation of ions velocity we would get a sharp peaks at that points.

Conclusions

A conditions of a thruster test cause a certain impact on its operation. In particular, for the case where an additional electron source to be absent and a mode of autocmpensation of an ion beam charge occurred to be in use. If the beam current at so conditions exceeds the critical one, as it stated by (11), even little non density oscillations are capable to cause strong plasma potential perturbations that break the beam propagation. At the opposite case, ion density oscillations even of a high peak magnitude are not dangerous, and average plasma potential grows negligibly.

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