Fundamental Design of Highly Effective Hall Thrusters

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Introduction

There are currently existing two types of Hall thrusters: the Stationary Plasma Thruster (SPT) and the Thruster with Anode Layer (TAL). These differ from each other in the presence of dielectric walls for the acceleration channel (SPT), which are absent for the TAL. This difference affects the parameters of the electron component of the plasma in the accelerating channel, i.e. the "intrinsic" characteristics of the thruster. In contrast, the "extrinsic" performance characteristics of the thruster, such as thrust, and specific impulse are determined by the ion component of the plasma. Until now, limited knowledge of the plasma mechanisms and the complicated relationship between intrinsic and extrinsic characteristics of Hall thrusters has prevented a meaningful comparison of the capabilities of TAL versus SPT. Furthermore, it is necessary to have a proper understanding of these relationships in order to design new, more efficient Hall thrusters. Analyses of these problems is the purpose and the subject of this paper.

Nomenclature

E, B - strength of electric and magnetic fields;

F, I_{sp}, η - thrust, specific impulse and efficiency;

e, c - electron charge and light velocity;

n_e, n_i, n_a - densities of electrons, ions and atoms;

m, M, m_a - electron and ion mass, rate mass flow through an anode;

Φ, U_d, U_s, U_a, U_i, U_l U_{la}- potential, voltages of discharge, the first stage, the second stage, ionization, the first critical of SEE, near anode potential;

V_i, V_e, V_{ez}, $V_{e\varphi}$, V_a - velocities of ions, electrons and its components, atoms;

l_a, l_i, R_L, λ - lengths of acceleration and ionization zones, Larmor radius and wave length;

ν, Ω, ω_p, ω - frequencies of electron collisions, Larmor, plasma and irradiation;

T_e - electron temperature ($\approx$ energy);

a, t_a - acceleration and time of motion of ions through an acceleration zone;

D, α ,σ_e, σ_i, γ - diffusion coefficient, ionization frequency, cross-section of collisions and ionization, ion-electron emission coefficient;

Q_e, Q_i, j_{ez}, $j_{e\varphi}$ - flux densities of electrons and ions, electron current components;

k, b - wave vector and width of channel.

Acceleration Zone

It is evident for both types of the thrusters, that ions pass through the acceleration zone with practically no collisions, and they are not affected by the channel walls. On the contrary, the electrons are strongly affected by the magnetic field; and the electrons move in a diffusion mode with numerous collisions, including the ones with the walls. The interaction of the electron component of the plasma with the channel walls in the acceleration zone plays a decisive role, in differentiating between the two thruster types, SPT or TAL.

The point is that for the formation of the acceleration zone, an electron must release the energy acquired by moving in the electric field. In TAL, an electron can lose energy only in non-elastic collisions with atoms and ions. For SPT there is an additional mechanism for electron energy down-flow, which is the interaction with the dielectric channel walls [1]. This is the fundamental difference between TAL and SPT. From this difference, it follows that: first, TAL must operate at higher plasma densities; second, as the density grows, SPT and TAL become less different; and third, the energy of electrons in TAL will be higher than in SPT [2].

Length of the Acceleration Zone

Let us analyze how the difference in electron temperature can affect the extrinsic characteristics of the Hall thruster, such as thrust, specific impulse, and efficiency. Neglecting collisions of ions with walls and electrons, and considering a 1-d case, the thrust can described as:

$$F = m_a V_i = \int_0^{t_a} m_a a(\tau) d\tau =$$

$$e\int_0^{t_a} n_i(z) M V_i(z) E(z) d\tau / M = e\int_0^{l_a} n_e(z) E(z) dz \qquad (1)$$

where integration is made over the acceleration zone and $n_i(z) = n_e(z)$, $V_i(z)d\tau = dz$.

In the ideal case, the acceleration zone length should be equal to the depth of the electric field penetration into plasma. Then, the ions which are formed at the acceleration zone boundary would go through the whole region where the E - field is

applied, and the specific impulse would be maximum. In a real thruster the length of the acceleration zone, l_a is smaller than the field penetration depth. Let us show this by using the equation for electron motion written in a diffusion mode:

$$eE + (e/c)[V_e B] + mV_e\nu + \nabla(n_e T_e)/n_e = 0 \quad (2)$$

From (2) it is possible to get a relationship for the electron current flowing through the acceleration zone (Axis "z" and E-field are directed from the anode). A necessary condition for the thruster operation is that electrons must move from the cathode toward the anode, i.e. (V_e<0):

$$e\nabla\Phi < T_e\nabla n_e/n_e + \nabla T_e, \quad \text{where } E = -\nabla\Phi. \quad (3)$$

Because ions move freely without collisions, and $n_e = n_i$, then in every point of space Φ and n_e are connected by the following relationship:

$$n_e \sim [2_e/M(\Phi_{max} - \Phi)]^{-1/2}. \quad (4)$$

By combining (3) and (4), condition (3) can be rewritten as follows:

$$\nabla T_e > e[1 - T_e/2e(\Phi_{max} - \Phi)]\nabla\Phi. \quad (5)$$

From which the definition for the acceleration zone boundary follows naturally:

$$T_e = 2e[\Phi_{max} - \Phi(l_a)]. \quad (6)$$

The physical meaning of (6) is clear. At the acceleration zone boundary, the relationship of (6) causes the right side of inequality (5) to change its sign. This ensures that ∇T_e will be greater than zero (in the acceleration zone, $\nabla\Phi$ is always less than zero); i.e. the loss of energy of the electron must be greater than the energy acquired. This is due to the intensive ionization of atoms (at the beginning of ionization zone, Fig.1).

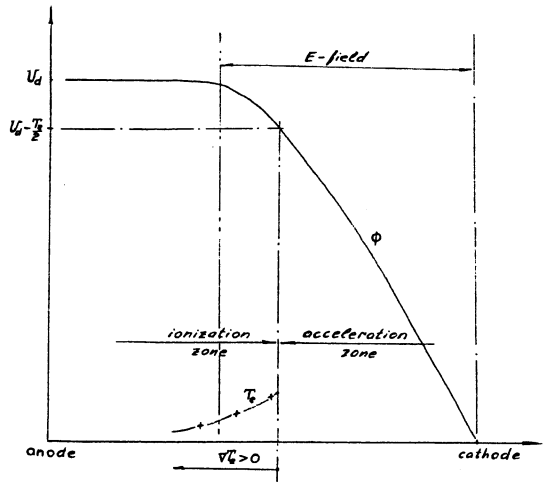


Figure 1. Distributions of the Potential Φ and Electron Temperature T_e (Only in the Ionization Zone) Along the Channel.

Therefore, in the Hall thruster, the acceleration zone length is shorter than the depth of E-field penetration into the plasma in the channel. This means that an electron will initiate intensive ionization of the atoms, even while it is still in the region of the E-field (Fig.1). Other things being equal, the acceleration zone will be shortened as the electron energy is increased.

Thrust, Specific Impulse and Efficiency as Function of Electron Energy

For simplicity let us consider an ideal thruster where: only singly charged ions are created in a narrow layer of the ionization zone. Furthermore as shown in Fig.1, we assume that $\Phi = U_d - 1/2T_e$, $\Phi_{max} = U_d$, and 100% ionization is realized. Then, according to (5), we can write

$$F = I_{sp}/m_a \sim \sqrt{U_d(1 - T_e/2U_d)}. \quad (7)$$

The relationship between the efficiency and T_e is complicated, because the efficiency depends on the electron current, which is also a function of T_e. However if $I_i/I_e = Q_i/Q_e$, then the maximum number of ions produced by one electron can be written as $(3/2T_e - U_i)/U_i$. (An electron has energy equal to T_e before entering the ionization zone, and it can also get $1/2T_e$ in the zone-see Fig.1.) Then η_{ideal} would be

$$\eta_{ideal} = \frac{1 - T_e/2U_d}{1 + U_i/(3/2T_e - U_i)} \quad (8)$$

By differentiating $\eta_{ideal} = f(T_e, U_d, U_i)$ to find the value of T_e which gives its maximum, we have the following result, which is shown in Fig.2.

$$\eta_{max} = (1 - \sqrt{U_i/3U_d})^2, \text{ when } T_e = \sqrt{(4/3)U_d U_i} \quad (9)$$

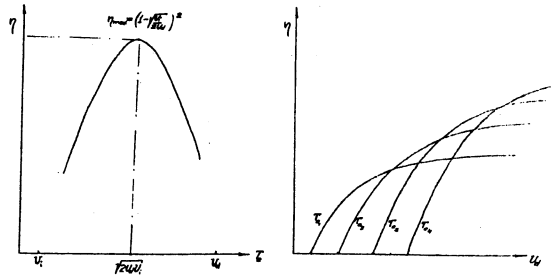


Figure 2. Dependence of Efficiency, η on Electron Temperature, T_e and Discharge Voltage, U_d: $T_{e1} < T_{e2} < T_{e3} < T_{e4}$.

The main conclusion, which follows from (8), is that a maximum of η always exists. Its existence is caused by two competing processes: the ionization stops when $T_e \rightarrow U_i$; and the thrust diminishes when $T_e \rightarrow U_d$. The relationship in (9) will be different for various ionization models, but only coefficients would be changed. Moreover, a maximum value of η would exist for any type of single-stage electric propulsion thruster, in which

the energy for ionization is provided by electrons from a single cathode.

These results permit comparison of the predicted performance of an ideal SPT with that of an ideal TAL. Considering that as a rule the electron temperature, T_e is higher in TAL than in SPT:

1) SPT should have higher I_{sp} than TAL.
2) Optimum Hall thruster efficiency depends on the inter-relationship between T_e, U_d, & U_i.
3) If m_a = constant then, thrust in an SPT should be larger than in TAL. For constant power both efficiency and thrust will be greater.

If we take into account doubly charged ions then η decreases together with an increase of F, and I_{sp}. The decrease in η is caused by the greater cathode electron current necessary for additional compensation of the doubly charged X_e^{++}. The increase of F, I_{sp} will be smaller if the doubly charged ions are produced in the acceleration zone. (Ion velocities will be less, but the additional charge compensation will the same regardless of where X_e^{++} is created.)

Lets explore methods to obtain better extrinsic performance characteristics of a single-stage thruster. Even in an ideal single-stage thruster, the maximum ion energy is smaller than eU_d. The ion energy is lower than eU_d by 1/2Te which can typically reach ≈ 15% of eU_d. Can we increase F, I_{sp} or ion energy up to a value ≤ eU_d? In our calculations we assumed that the peak value of the electric potential, Φmax, in the plasma of the channel is equal to U_d; and the electric potential continuously decreases from the anode toward the cathode. By changing the anode's geometry and its location in a specific way, we can obtain a value of Φmax which equals U_d + (≤ T_e). In this case the distribution of the electric potential will have the maximum in the region between the anode and cathode (Fig.3). This maximum does not violate any laws of plasma physics.

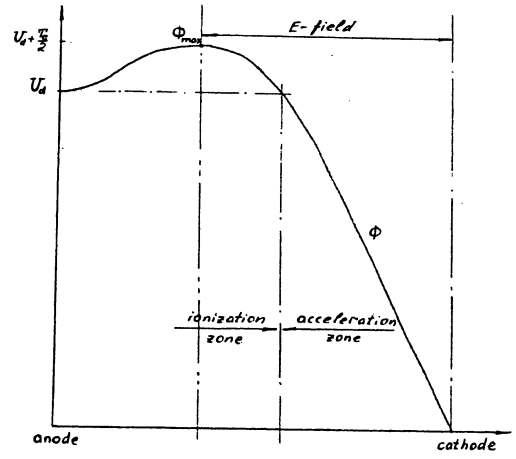


Figure 3. Distribution of Potential, Φ, Along the Channel for a Special Anode Configuration.

Selection of the wall material can increase η. In an SPT type of thruster with dielectric channel walls, the threshold energy for secondary emission (and hence the channel wall material) determines the electron temperature, i.e. T_e is determined by U_1. As shown in Equations (7). (8), and (9); there is the relationship between T_e, U_d, U_1, which gives the maximum efficiency, η. Therefore, for a particular value of U_d and type of working gas, there is the optimum type of a material for walls when η reaches the maximum. Typically the wall material is a boron nitride (BN) ceramic, which has secondary emission threshold energy that is reasonably matched to SPT operation at 300 Volts or higher. However operation at different voltage would be optimized with a different channel wall material. For example, for U_d = 100V (larger thrust) and X_e propellant, it would be better to use a ceramic with a smaller value of U_1 = 20-25V such as (Al_2O_3) instead of the usual ceramic BN.

E-Field Shape

The axial electric field profile in a single-stage thruster is constrained by a stronger inequality (V_e< const) than (3) and (5). This inequality takes into account the necessity of the full ionization of the propellant gas:

$$\nabla T_e > [1-T_e/2e(U_d-\Phi)]\nabla\Phi+\text{const}(U_d-\Phi)^{1/2}B^2/\nu \quad (10)$$

The analysis of (10) shows that, the defined E-field shape (see Fig.4) and its dependence on B, indicates similarity the shapes of the E- and B-fields; but a "narrower cone" for the E-field as compared to the shape of the B-field. This is supported by experimental facts [2].

A reasonable hypotheses is that: in the acceleration zone T_e, ν = const. In fact, strictly speaking, one inequality is only sufficient to determine one of the boundaries of the range of the variables under study. It is reasonable to apply a principle of thermodynamics: a thruster (as open system) operates in a mode with a maximum entropy production (maximum current) [4]. This requires the form of (10) must change from > into =.

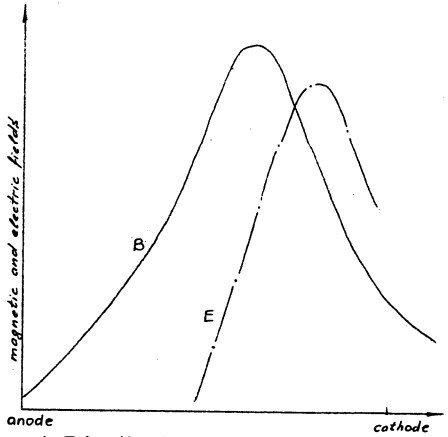


Figure 4. Distributions of the electric and magnetic fields in the case of the classical conductivity of electrons.

The inequality (10) provides an explanation of experimental data, which showed an increase of I_{sp} ~ 10% when m_a increased ~ 40%; although B and U_d were constant [5]. Under these conditions, the last number in (10) will only depend on ν; which, to first approximation, will be ~ m_a. For simplicity let us also suppose that $\nabla\Phi$= - A (const). A solution is found when the side of (10) changes its sign (the beginning of an acceleration zone, Fig.1).

Thus for the constant voltage and magnetic field the maximum ion velocity is increased with increasing mass flow rate through an anode. The physical meaning of this conclusion is that: Increasing collisions (plasma density or m_a) facilitates the motion of electrons through the acceleration zone.

The new result from (10) is the non-coincidence of the maxima of the E- and B- fields. The reason of this non-coincidence is contained in the requirement for j_{ez} = const >> 0 in each cross-section of the channel; and in the assumption of classical collisions. Besides, this non-coincidence can be a cause of additional plasma instabilities.

Oscillations

Let us take only one example: a two-stream instability, and show that in any case in the acceleration zone such oscillations will appear. To avoid operation with oscillations (in axial direction), it is necessary to fulfill the condition: $j_{ez} = n_e V_{ez}$ = const >> 0, in each cross-section of the channel. Because, in each cross- section, V_{ez} and $V_{e\varphi}$ are connected by the condition $V_{ez} = V_{e\varphi}\,\nu/\Omega$; then $V_{e\varphi}$ will be $V_{e\varphi} = \text{const}, \ \Omega/\nu n_e = \text{const}\bullet B\sqrt{(U_d - \Phi)}/\nu$. In order to prevent two-stream instability (in azimuthal direction), it is necessary to fulfill the condition $V_{e\varphi} \neq f(z)$, i.e. $B\sqrt{(U_d - \Phi)}/\nu$ = const. For the classical collisions this equality can be written as $B\sqrt{(U_d - \Phi)}$ =const. It is impossible to meet this requirement (see Fig.4). On the contrary, if we want to avoid the azimuthal oscillations (coincidence of the maxima of E and B fields), then we do not fulfill the condition j_{ez} =const, i.e. longitudinal oscillations will occur.

Thus, in an one stage thruster, the requirement that j_{ez} >> 0 in an acceleration zone, necessary for its effective operation, is also a reason for much of the losses.

Ionization Zone

Ionization Zone Length

The electron temperature in the acceleration zone of SPT is approximately equal to the first critical potential of SEE. When the electron enters the ionization zone, its energy is decreased and its interaction with the channel walls stops. Therefore, there is no basic difference in the ionization zone, between SPT and TAL. Because the electron moves in a diffusion mode, it is convenient to introduce a new spatial scale - free electron path in the axial direction. It will approximately be equal to the Larmor radius of an electron (one collision). In this scale, the length of ionization zone in SPT will be shorter than in TAL; because the latter has a higher electron temperature.

In modern thrusters one electron ionizes about three atoms on the average, but then its energy gets lower than the ionization potential and it is no longer useful. Hence, a minimum, and consequently optimum (extra collisions with no ionization mean extra energy loss), ionization zone length in the modern thruster is $\approx 3R_L$; regardless of the thruster type. That is why: first, the traditional view point [2] that all processes in TAL including ionization take place in the layer as thick R_L is erroneous; second, in the experimental measurements the TAL ionization zone can be shorter because it starts earlier, i.e. in stronger magnetic fields where the Larmor radius is shorter (see "Acceleration Zone").

Influence of Ionization Oscillations on Electron Mobility

Let us show that in ionization zone (E = 0) of a single stage Hall thruster there must be a region of intensive ionization oscillations. Let us consider the system of partial equations by writing a plasma balance and continuity.

$$\begin{cases} \partial n_e/\partial t = -\partial(n_e V_e)/\partial z + \alpha n_e n_a \\ \partial n_a/\partial t = -V_a \partial n_a/\partial z - \alpha n_e n_a \\ V_e = -(T_e/m)(\sigma_e n_e + \sigma_a n_a)/n_e\Omega \times \partial n_e/\partial z \end{cases} \quad (11)$$

To solve this system we consider a solution of type n, V_e = n, V_e + n′, V'_e where n′, V'_e are very small. At the first approximation for small values we obtain the following system

$$\begin{cases} \partial n'_e/\partial_t = -V_e \partial n'_e/\partial z - n_e \partial V'_e/\partial z + \alpha n_a n'_e + \alpha n_e n'_a \\ \partial n'_a/\partial_t = -V_a \partial n'_a/\partial z - \alpha n_a n'_e - \alpha n_e n'_a \\ V'_e = -A(\sigma_e + \sigma_a n_a)\partial n'_e/\partial z + A\sigma_a n_a/n_e \ \partial n_e/\partial z \times n'_e - A\sigma_a/n_e \times \partial n_e/\partial z \times n'_a \end{cases} \quad (12)$$

Further, using n′, V′ ~ exp{-i(ω$_t$ - kz)}

$$\begin{cases} -i\omega n'_e = -i_k V_e n'_e - i_k n_e V'_e + \alpha n_a n'_e + \alpha n_e n'_a \\ -i\omega n'_a = -V_a n'_a - \alpha n_a n'_e - \alpha n_e n'_a \\ V'_e = -ikCn'_e + n_a/n_e \times \Gamma n'_e - \Gamma/n_e \times n'_a \end{cases} \quad (13)$$

where $A = (T_e/m)^{3/2}/\Omega^2$, $C = A(\sigma_e + \sigma_a n_a/n_e)$, and $\Gamma = A\sigma_a \partial n_e/\partial z$. From (13), by substituting $\omega = f(k)$ and solving, we obtain the condition of excitation of oscillations as

$$\alpha(n_a - n_e) > k^2 n_e C \quad (14)$$

Or, for the ionization zone ($n_a >> n_e$)

$$\lambda > 2\pi R_L(\sigma_e/\sigma_i) \approx 10R_L \quad (15)$$

For B ≅ 10G λ equal several cm., i.e. one ionization oscillation will occupy all of the

ionization zone. Maybe, therefore, they look like relaxation oscillations.

The main parameter, which determines existence of ionization oscillations in a Hall thruster, is the value of the magnetic field. If B is zero (or small value) then ionization oscillations are absent. The physical explanation of this is very simple: The magnetic field prohibits electrons from leaving the ionization region, and the ionization rapidly grows in the zone.

The oscillations of the electron density and velocity [the last equation in (13)] differ in phase by an angle smaller than π. Therefore, ionization oscillations will increase the electron current to the anode. Moreover, until now, specialists spoke only about density oscillations; and they did not speak about velocity oscillations. The present results show that velocity oscillations play an important role in the current oscillations. The increasing of n_e leads to increase ν, i.e. V_e. Thus, it can be said that the magnetic field acts to change the flow of the electron current from "laminar" into "turbulent".

The main reason for increasing of the electron current (decreasing of efficiency of Hall thruster) is the spatial inhomogeneity of electron density ($\partial_{ne}/\partial z \neq 0$). It is an indefeasible property of the ionization zone in a single stage Hall thruster; because it is necessary in order to guarantee the requirement: $j_{ez} >> 0$.

Near Anode Processes as Function of Magnetic Field

It is well known that there is a near-electrode layer between a neutral plasma and any electrode placed in it. The condition of neutrality inside this layer is not fulfilled, and the electric field appears in it. A value of the potential (U_{la}) between the electrode and neutral plasma depends on the electrode material, ionization potential, and electron mobility.

Let us find the dependence of U_{la} on other parameters of a Hall-thruster. For this, we write the condition of conservation of the current for both boundaries (plasma-"p", anode-"a") as $j(a) = j_e(a) = j(p) = j_e(p) + j_i(p) = \text{constant}$. Moreover, for the electron current, the relationship $j_e(a) = \beta j_e(p)$ must be fulfilled; where β is an ionization parameter, i.e. in near-anode layer, ions must form.

Combining both relations we can obtain an expression for the velocities of particles under the condition $\{n_e(p)=n_i(p)\}$

$$(\beta - 1)V_e(p) - V_i(p) = \text{const} \qquad (16)$$

In the ideal thruster, ionization of atoms occurs near the beginning of an ionization zone; and electrons move towards an anode, having $T_e \leq U_i$. Therefore, for an ionization of atoms, electrons must get energy, i.e. $\nabla U_{la} < 0$ (accelerating field for electrons). Remembering that $\beta \sim \exp(T_e) \sim \exp(U_{la})$, $V_e \sim 1/B$, $V_i \sim \sqrt{U_{la}}$, to first approximation we can write

$$U_{la} \sim B \qquad (17)$$

It is obvious that $U_{la} \approx T_e$.

Because a large part of ions, are in the near-anode layer, they can not go away from the channel (they can go to walls in an ionization zone), it is desirable to decrease U_{la}. This permits an increase F and I_{sp}. Therefore it is necessary to place the anode in a region where $B \rightarrow 0$.

There is another important result. For a typical case, $\nabla U_{la} < 0$, it will be better to place a gas distributor between the anode and cathode (or to combine the gas distributor and anode into one element). Then the value of U_{la} will be smaller. On the contrary, if we want to increase U_{la}, then it will be better to separate these components.

Interconnection of Zones

As follows from the above, the main fundamental limitation to the efficiency increase of currently used Hall thrusters is the firm interconnection in electron current between the acceleration and ionization zones. In particular $j_{ez} >> 0$. Inequality (10) gives a quantitative representation of this constraint. When moving from the cathode through the acceleration zone, an electron has to lose energy. When the loss is small, the electron acquires a higher energy (T_e) in the E-field; and it must start ionization earlier, which means that the efficient use of U_d for accelerating ions is not possible. Conversely, in order to efficiently use U_d, then the ionization usefulness of one electron is decreased.

In order to get out of this "vicious" circle it is necessary to release, or if possible to break, the interconnection of the zones in electron current ($j_{ez} \geq 0$ in the acceleration zone); i.e. to build a double-stage thruster. It can be done in two ways:

- Transfer the ionization energy to electrons in the ionization zone, thus reducing the need for the electron current passing through the acceleration zone;
- Make two flows of electrons with an independent power supply: the first is to compensate for ions in the acceleration zone and in the jet, the second is for the ionization

Now let us discuss how to implement these ideas.

Double-Stage Hall Thruster with a Criterion for Introducing the Second Stage

As it was shown earlier, the maximum energy that an electron can bring into the ionization zone of a single stage thruster and use for formation of ions does not exceed $\leq T_e$; the remaining $U_d - \frac{1}{2} T_e$ will be lost in the acceleration zone. For a quantitative assessment of the efficiency of the ionization process, it is convenient to introduce a new parameter "p" representing an ion cost (price). It can be defined as the energy spent by one electron for formation of one ion. Then the minimal ion price for an one stage Hall thruster is

$$p_{min} = U_d U_i/(3/2T_e - U_i). \quad (18)$$

Note, that as U_d grows, "p" grows proportionally as shown in Fig.(5) below.

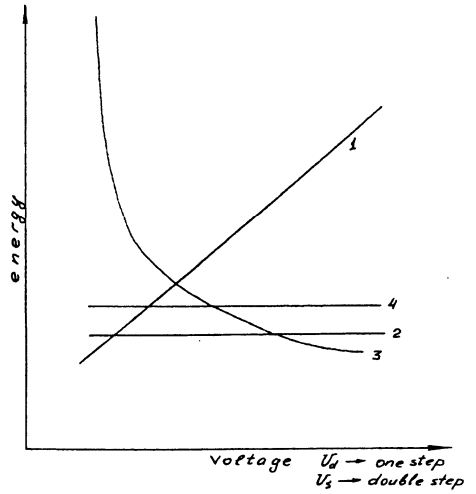


Figure 5. Qualitative dependence of ion prices for various designs: 1-one stage thruster, 2-double stage thruster (hot cathode), 3-double stage thruster (ion-electron emission cathode), 4-double stage thruster (microwave irradiation).

In modern thrusters ($U_d \approx 300V$, $U_i=12V$, $T_e=60V$), the ion price is $\cong 75V$; i.e. the electron spends for ionization only about 25% of its energy and 75% is lost in the acceleration zone. Using a double stage design, we can avoid this loss and increase the thruster efficiency.

Transition from a single stage to a double stage design is worthwhile only if the ion price is decreased by the change. Conversely, if it is increased, then the double stage design is not beneficial, from the point of view of energy.

Reducing Electron Current through the Second Stage

To provide for efficient operation of a double stage thruster it is necessary to minimize the value of the electron current through the second stage (acceleration zone). Analyzing (3) and (10) shows that it can be done in the following ways:

- Increase the magnetic field in order to decrease an electron mobility because the strong interconnection between the E and B fields, in a single one stage thruster, does not exist in a double stage thruster;
- Operate at the lower plasma density, i.e. lower m_a, and/or use a larger channel width that will accommodate the decrease in electron mobility.

The first of these is more obvious and simpler for realization. Therefore only it will be considered in this article.

Ionization Zone of Double-Stage Hall Thruster

Ionization via Micro Wave Irradiation

In a usual gas discharge a condition for efficient transfer of an irradiation energy to plasma electrons is the correlation of $\omega \cong \nu$. This can not be realized in modern Hall thrusters, because of the low plasma density. However, because of presence of the magnetic field, it is possible to use resonances. Along with this, one should remember that there are a lot of resonances in the magnetized plasma, but there are also the closed wave paths. Therefore it is necessary to know the correlation between them in order to produce an effective interaction between micro wave irradiation and plasma.

An analysis of those equations is difficult, and their solution strongly depends on a specific design of a waveguide system. Therefore we shall investigate a simpler case. Our purpose is to show the method itself, and that a solution of this problem exists.

Let us consider the propagation of an electromagnetic wave in a continuous waveguide containing magnetized cold plasma with E⊥B (as in the ionization zone with magnetic shields). For the dominant mode of the transverse magnetic wave (TM11); obtained from Maxwell equations, for E_φ - component in the azimuthal direction will have the following type

$$[\nabla^2\perp + (\omega^2/c^2 - k^2)(1 - \omega_p^2/(\omega^2 - \Omega^2)]E_\varphi = 0. \quad (19)$$

From (19) for a rectangular waveguide (a×b), we can write the dispersion equation as

$$K^2(a^2+b^2)=\omega^2(a^2+b^2)/c^2-\pi^2/\{1-\omega_p^2/(\omega^2-\Omega^2)\}. \quad (20)$$

The waveguide changes the dispersion characteristics of the magnetized plasma in a region of small wave vectors only, and permits operation at ω a little smaller than $\sqrt{\omega_p^2 + \Omega^2}$. This is near the main resonance (upper hybrid frequency) where there is very strong absorption (Fig.6). In this case, if the loss of irradiation to external space is zero, then up to 50% of the irradiation power can be expended to increasing of electron energy (half electrons are in phase with oscillations). Theoretically, p_{min} can be nearly equal to $2U_i$ (Fig.5). The addition of T_e in equations does not change the frequency of the main resonance, but it increases the number of resonances with near-by frequencies $n\Omega$ (Bernstein modes, the case $\omega_p > \Omega$), which only reinforce the absorption.

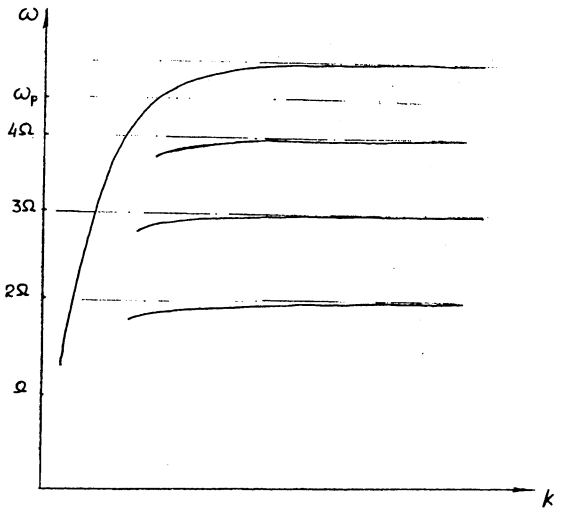


Figure 6. Dispersion characteristics of the continued waveguide with the magnetized plasma.

Note, that the main difference of the obtained result from the known result [6] is in the resonant frequency. In that experimental work of [6] the irradiation frequency equals Ω. As is known, this accommodates the condition Ω < ωp. Therefore authors got little positive effect from the interaction of the irradiation with plasma of their Hall thruster.

Additional cathode

Using an additional cathode and power source it is possible to ignite an independent discharge in the first stage (ionization zone). The type of discharge will depend on a cathode emission mechanism.

Ion-Electron Emission Mechanism. The calculations of ignition conditions in a gas discharge with the magnetic field were made in [7], and they are described the following inequality:

$$\int_0^{l_i} 3Ed_x/(4mcE/eB + U_i) > \ln(1+1/\gamma) \qquad (21)$$

In order to effectively ionize atoms it is necessary that, after each an ionizing collision, an electron must gain the energy ≡ $2U_i$, i.e. $mcE/eB = 4U_i$. To take into account this relationship we can rewrite (21) in the following form:

$$U_s > 6U_i \ln(1 + 1/\gamma). \qquad (22)$$

For example, for Xe (U_i=12V) and the cathode material Pt (γ=0.1) U_s will be 175V and the ion price will be very large value: p=1440V. But the ion price decreases rapidly with the increase of U_s; and, for U_s=175V+U_i, the ion price will be p=100V; for U_s=175V+2U_i, p=60V, etc. (see Fig.5). Thus by means of small increase of U_s we can build an effective first stage.

There is the additional way to decrease the ion price. It is the use of metastable atoms of Xe for initiation of electron emission since their coefficient for ion-electron emission is more than γ.

Because of complexity of the model (γ large depends on the state of the cathode surface) perhaps it would be more correct and simpler to use experimental data for a determination of U_s.

The parameters of the E-and B-fields should be selected on the basis of the condition $U_i < T_e < U_1$ that prohibits the energy loss on walls. Moreover, using this cathode type, it is necessary to remember two things: the first is cathode lifetime (sputtering); and the second is a reflux of Xe from the cathode into the first stage for the repeated ionization.

Hot Electron Emission Mechanism. It is possible to operate at voltages lower than U_1, i.e. without the energy loss on walls. The value of p_{min} will only depend on the energy used by the cathode heater (Fig.5). Let us consider concrete examples of realization of these ideas.

Design Variants of Double Stage Hall Thrusters

Hall Thruster with Micro Wave Ionization

Figures 7 and 8 show the conceptual designs of Hall thrusters with the different systems for introducing irradiation into the ionization zone. As a rule $\omega_p >> \Omega$ in the ionization zone of modern thrusters. Then the main resonant frequency will equal ~ω_p, approximately which corresponds with the wavelength of irradiation $\lambda \approx$ 1-5cm. Thus we should use the microwave technique.

For the first case (Fig.7) the microwave system is a resonator with the widely separated parts: two-inductive parts and one capacitive part. The capacitive part is placed in the ionization zone for reinforcement of the interaction of irradiation with electrons, where the electric field is concentrated. The control of the operational frequency of the resonator is realized by changing the parameters of the external inductive part only, which is rather far from the ionization zone. The resonator is made mainly from a ferromagnetic material and is covered by the skin-layer of Cu or Ag. It operates as the magnetic screen of the total magnetic scheme. Such a microwave system can be applied only in SPT in which the magnetic scheme is made according to the patent [8]. The elements of the resonator placed in the ionization zone are made with no ferromagnetic material and they can be covered by a dielectric film.

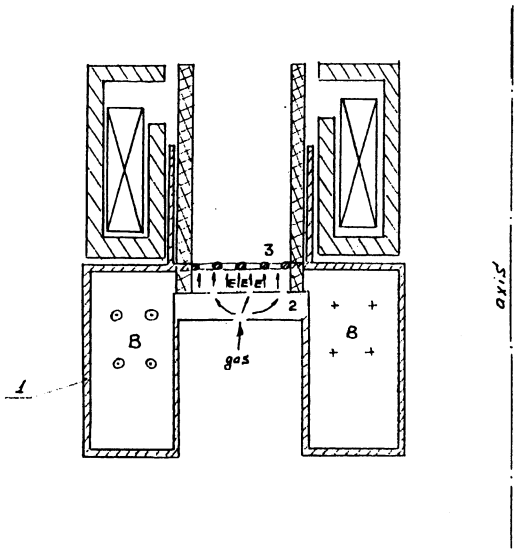


Figure 7. SPT with microwave irradiation: 1-magnetic screen and resonator (magnetic screen covered by Cu-film), 2 anode and gas distributor 3-grid of resonator, E and B wave field.

The second case (Fig.8) is an attempt to join the Hall thruster and microwave system as two independent devices in one design but without strongly adapting one under the other. For this we must have a magnetic scheme [8] without a ferromagnetic path connecting the inner and outer sections. The microwave system will be operated with its magnetic field directed oppositely with respect to the main magnetic field. Such a design will permit us to use each part of the device more effectively.

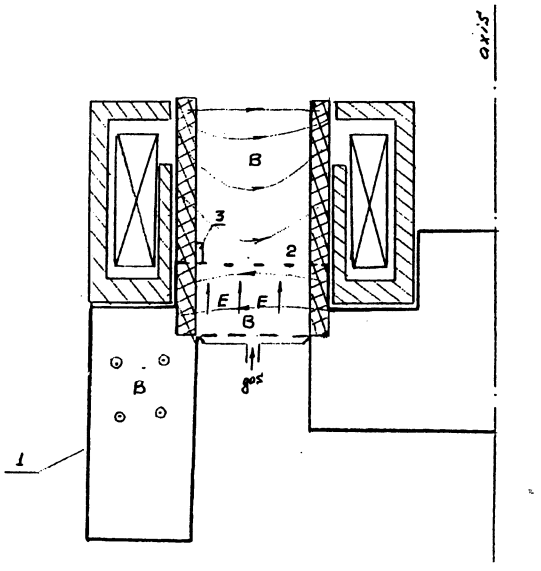
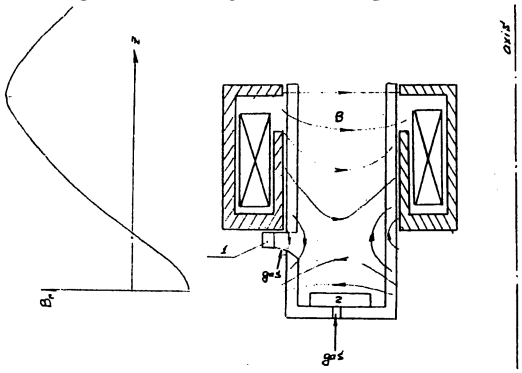


Figure 8. SPT with microwave irradiation: 1-resonator, 2-grid covered by a dielectric film, 3-anode of the second stage, E and B wave field.

Double Stage Hall Thruster

In order to arrange two flows of electrons an additional cathode (negatively charged with respect to the anode) should be put into the thruster's channel. Thrusters of this type have been tested over twenty years ago [2]; but they did not find application because their parameters did not surpass those of single stage thrusters. In recent years this has been attempted again [9]; but with the same outcome. The observed increase of η was the result of increasing of U_1 up to 1200V; but not due to the additional cathode operation. Modern single stage thrusters can achieve the same increase of η. The previous efforts lacked a good design for the shape of the magnetic field specific to a two-



stage thruster

Figures 9 and 10 give conceptual design schematics of thrusters and shapes of magnetic fields in their channels.

In the first case there is a cathode with ion-electron emission (Fig.9). For this type of discharge, the ignition voltage of the independent discharge is more than 150V> U. To avoid electron energy loss on walls, the walls must be conducting, and be at cathode potential. Alternatively, the magnetic field could be such as to fulfill the condition $U_i > T_e > U_i$. In order that the change of U_a would be used more efficiently, it is desirable to independently adjust B. The magnetic fields in the two stages must be as independent as possible.

Figure 9. Double stage thruster and the distribution of B_r along the middle of the channel: 1-cathode of the first stage, 2-anode, B-magnetic lines.

In the second case the use of a thermal cathode allows operating with ceramic channel walls in the first stage and also allows lower U_s values. The thermal cathode does not depend upon ion bombardment for electron emission. It only requires ions for space charge compensation. This allows a more efficient magnetic field shape and makes it possible to shield the first stage cathode from the second stage electrons (Fig.10). These electrons can be used for an additional ionization.

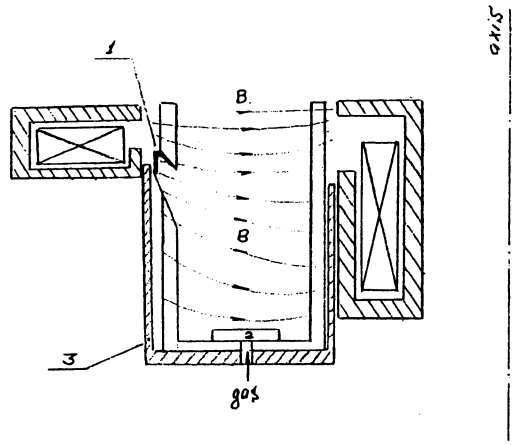


Figure 10. Double stage thruster: 1-cathode of the first stage, 2-anode, 3-magnetic screen, B-magnetic lines.

Conclusions

The interconnection between the main intrinsic parameter of plasma - the electron temperature - in a one stage thruster and its extrinsic performance parameters such as thrust, specific impulse, efficiency has been described. Several methods of improvement in single-stage Hall-thruster parameters have been discussed.

The primary limit of efficiency in single stage Hall-thrusters is the interconnection of the electron current between the acceleration and ionization zones; because the electrons and energy for ionization come from the cathode compensator.

Methods to overcome this limitation have been proposed. Hybrid combinations TAL and SPT may be beneficial in a double-stage thruster. They can be realized in Hall-thrusters of the second and third generations. Conceptual designs of these were shown and discussed.

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