Thrust Formula for an Applied-Field MPD Thruster Derived from Energy Conservation Equation

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Abstract

Thrust formula for an applied-field MPD thruster has been derived from the energy conservation equation for the propellant plasma flow; the work done by electromagnetic forces is assumed to be converted into the kinetic energy of the exhaust plasma. Three acceleration mechanisms, i.e. generalized Hall acceleration, swirl acceleration and self-magnetic acceleration are taken into account. This formula enables one to theoretically obtain the thrust by an applied-field MPD thruster from controllable thruster operation parameters, and to estimate the contributions of the respective acceleration mechanisms.

Nomenclature

a =constant
a_r $= \bar{r}/r_a$
$\mathbf{B}$ =magnetic field
b =constant
c =constant
$\mathbf{E}$ =electric field
$\mathbf{E}'$ =effective electric field
$\mathbf{E}'_{hall}$ =effective Hall electric field
e =charge of electron
$\mathbf{j}$ =current density
J =total discharge current
l =length of acceleration region
M_A =ratio of exhaust velocity to Alfvén velocity
m_i =ion mass
$\dot{m}$ =mass flow rate
n_e =electron number density
p_e =electron pressure
R_m =magnetic Reynolds number
r, θ, z =cylindrical coordinate
T =thrust
$\mathbf{u}$ =fluid velocity
u_{ex} =exhaust velocity
V =volume
V_0 $=\pi r_a^2 l$
α =constant
β $=1/(n_e e)$
μ_0 =permeability in vacuum
ξ $=\psi/\psi_c$
ρ =density
σ =electrical conductivity
ϕ $=B_r/B_z$
ψ $=\bar{\sigma} B^2/\dot{m}$
$\omega_e \tau_e$ =electron Hall parameter
$\{\ \}$ =representative value

Subscripts
a =anode
c =cathode, critical
e =electron
ef =effective
ex =exhaust
hall =Hall effect, generalized Hall acceleration
self =self-magnetic acceleration
swirl =swirl acceleration

1. Introduction

In order to realize large-sized, long-term space missions such as a space station and a Mars mission, for example, a high-powered, high-I_{sp} propulsion device needs to be developed. For the above requirements, an MPD thruster is thought to be suitable owing to its high-power potentiality, structural simplicity and high-I_{sp} (higher than 1000 sec) operation ability. Since electrode erosion is essential to the life time of the thruster, steady-state operation is thought favorable in order to increase the input power to the thruster. In the input power range of tens kilowatts, the discharge current is usually smaller than 1 kA – the effect of self-induced magnetic field on thrust production is not high enough. In order to increase the performance of an MPD thruster in the above operation range, an external magnetic field needs to be applied in the acceleration region.

Sasoh and Arakawa[1] found that when the thruster operates under a high applied-field strength and low mass flow rate condition, the effect of azimuthal induced current on thrust production becomes dominant. They obtained the thrust formula for this acceleration mechanisms and found that the specific impulse is characterized by a characteristic parameter $B^2/\dot{m}$.

The interaction between the applied magnetic field and the discharge current results in azimuthal electromagnetic force, which causes the swirl motion of the propellant plasma. The swirl kinetic energy can be converted into the axial kinetic energy both through the solid nozzle of the thruster (the centrifugal force is to be balanced with the static pressure gradient), and the magnetic nozzle formed by the applied

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magnetic field. Fradkin et al.[2] gives the thrust formula for this acceleration mechanism. The formula is based on the conservation of the angular momentum and that of the kinetic energy of the propellant plasma flow.

When the discharge current becomes larger than 1 kA, the interaction force between the self-induced magnetic field and the discharge current cannot be neglected. The thrust formula is algebraically obtained by Jahn.[3] The formula is obtained by integrating the electromagnetic interaction force over the interelectrode region. An MPD thruster can obtain high thrust performance by increasing the discharge current through quasi-steady operation etc. (self-field MPD thruster). Moreover, Tahara et al.[4] experimentally found that an applied magnetic field at an appropriate strength enhances the thrust performance of the self-field MPD thruster.

From the above descriptions, there may seem to exist three different acceleration mechanisms which are related to electromagnetic forces. In order to design the performance of an applied-field MPD thruster, however, one needs to estimate the thrust in a wide operation range of such controllable operation parameters as applied-field strength, mass flow rate, discharge current and thruster dimensions. The purpose of this study is to obtain a general formula of the thrust produced by an applied-field MPD thruster and to make clear the interrelationships among the above acceleration mechanisms.

2. Generation of Electromagnetic Thrust in an Applied-Field MPD Thruster

Before categorizing the thrust production mechanisms, let one define the terminologies used in this paper and state the assumptions made in this study. In this paper, cylindrical coordinates, r, θ, z are used. An applied-field MPD thruster is assumed to have an axisymmetric structure. The flowfield of the acceleration region is also assumed axisymmetric. An external magnetic field is applied in the axial and radial direction ('applied magnetic field'). The radial component of the applied field is assumed much smaller than the axial one – a slowly diverging magnetic nozzle configuration is assumed. Moreover, axisymmetric discharge current in the interelectrode region induces azimuthal magnetic field, which will hereafter be referred to as 'self-induced magnetic field.'

Generally speaking, the total discharge current for an applied-field MPD thruster is a controllable parameter; integrating the current density over the whole electrode (anode or cathode) surface yields a controllable constant discharge current. If one set a semi-infinite-long cylindrical control surface in the interelectrode region, the integration of j_r over the control surface equals the total discharge current. For this reason, j_r is, in this paper, referred to as 'supplied discharge current.' When a magnetic field is applied in the axial direction, azimuthal current is induced by the interaction between the applied magnetic field and the supplied discharge current (discussed in detail later). Therefore, j_θ is, in this paper, referred to as 'induced current.' In the absence of the applied magnetic field, the azimuthal current is not induced.

In an applied-field MPD thruster, an electromagnetic thrust is generated through the interactions between a supplied or induced current component and an applied or self-induced magnetic field component. Generally speaking, the following three components are dominant in the interaction forces:

(1) Generalized Hall acceleration. An axial electromagnetic force is produced by the interaction between the azimuthal induced current and the radial applied magnetic field ($j_\theta B_r$). As the azimuthal current is induced not only due to the Hall effect but also due to the diamagnetic effect,[1] this electromagnetic acceleration will hereafter be referred to as 'generalized Hall acceleration.'

(2) Swirl acceleration. An azimuthal electromagnetic force is produced by the radial supplied discharge current and the axial applied magnetic field ($j_r B_z$). This azimuthal force causes the swirl motion of the propellant plasma flow, resulting in swirl kinetic energy input to it. Although some of the swirl kinetic energy is once be converted into the static enthalpy of the plasma flow, it can finally be converted into an axial kinetic energy.

(3) Self-magnetic acceleration. The interaction between the radial supplied discharge current and the azimuthal self-induced magnetic field produces an axial electromagnetic force ($j_r B_\theta$). The strength of the self-induced magnetic field increases proportionally to the supplied discharge current. Therefore, this electromagnetic force scales with the square of the discharge current. When the discharge current is at low levels (smaller than $\sim$ 1 kA) this component is usually negligible.[3]

In experiments, the reaction forces of electromagnetic forces on the propellant plasma flow are exerted both on the magnetic devices (electromagnets or permanent magnets or both) which produce the applied magnetic field, and on the discharge current circuit. Also, the hydrodynamic pressure exerted on the solid surfaces of the thruster contributes a thrust component; not only the static enthalpy but also the swirl kinetic energy of the propellant plasma flow can be converted into an axial kinetic energy through the solid nozzle.

3. Derivation of Thrust Formula

The energy conversion systems are, as described above, so complicated that one cannot separate a thrust component of the total thrust. Instead of calculating the respective thrust components separately, the total thrust is obtained from the energy conservation equation.

Here, the work done by electromagnetic forces is assumed to be converted into the axial kinetic energy of the propellant plasma flow both through the magnetic nozzle and the solid nozzle. Neglecting the static enthalpy of the exhaust plasma, the energy conservation equation is

$$\int (\mathrm{j} \times \mathrm{B}) \cdot \mathrm{u} dV = \frac{1}{2} \dot{m} u_{ex}^2 \tag{1}$$

The current density is related to electric field, magnetic field, fluid velocity etc. by the generalized Ohm's law;

$$\mathrm{j} = \sigma[\mathrm{E} + \beta \nabla p_e + \mathrm{u} \times \mathrm{B} - \beta \mathrm{j} \times \mathrm{B}] \tag{2}$$

Here, one assumes that $j_z/j_\theta \ll 1$. By the order estimation of the axial component of Eq. (2), j_z/j_θ is found of the order of $\sigma \beta B_r$ which, therefore, is assumed much smaller than unity here. In this case, the azimuthal component of the current density is related to the axial one by,

$$j_\theta = \sigma(\beta B_z j_r + u_z B_r + \sigma\beta B_r^2 u_\theta). \quad (3)$$

The electromagnetic force considered here is composed of the following three components; 1. $j_\theta B_r$ (generalized Hall acceleration), 2. $j_r B_z$ (swirl acceleration), 3. $j_r B_\theta$ (self-magnetic acceleration).

Therefore, the left side of Eq. (1) is expressed by

$$\int(\mathbf{j}\times\mathbf{B})\cdot\mathbf{u}dV =$$

$$\int(-j_\theta B_r u_z)dV + \int(-j_r B_z u_\theta)dV + \int j_r B_\theta u_z dV \quad (4)$$

In the above equation, '-' signs are needed for the electromagnetic forces to be in the axially downstream direction.

The first term of Eq. (4) is expressed such as[1]

$$\int(-j_\theta B_r u_z)dV = -\bar{j}_\theta \bar{B}_r \bar{u}_z a_{hall} V_0 \quad (5)$$

$$V_0 \equiv \pi r_a^2 l \quad (6)$$

In Eq. (5), the value of a_{hall}, which is the ratio of the equivalent volume for the generalized Hall acceleration to V_0, is of the order of 0.1 to 1.

Neglecting the radial and azimuthal velocity components of the exhaust plasma, the representative value of the axial fluid velocity in Eq. (5) is assumed to be

$$\bar{u}_z = \alpha_1 u_{ex} \quad (7)$$

Here, α_1 is a constant, the value of which will be determined later.

The second term of the right side of Eq. (4) is calculated under the assumption that the plasma rotates as a rigid rotor.

$$\int(-j_r B_z u_\theta)dV = \int_{r_c}^{r_a}[-j_r B_z \bar{u}_\theta(r)]\cdot 2\pi r l dr \quad (8)$$

The representative value of the azimuthal fluid velocity is assumed, such that,

$$\bar{u}_\theta(r) = \frac{\alpha_2 J B_z r}{\dot{m}}\frac{1-(r_c/r_a)^2}{1+(r_c/r_a)^2}$$

$$\equiv \frac{\alpha_2' J B_z r}{\dot{m}} \quad (9)$$

Therefore, Eq. (8) is transformed into

$$\int(-j_r B_z u_\theta)dV = \frac{\alpha_2'' J^2 B_z^2 r_a^2}{\dot{m}} \quad (10)$$

$$\alpha_2'' = \frac{[1-(r_c/r_a)^2]^2}{2[1+(r_c/r_a)^2]}\alpha_2 \quad (11)$$

The last term of the right side of Eq. (4) is transformed into the product of the self-magnetic thrust[3] and the representative value of the exhaust velocity.

$$\int j_r B_\theta u_z dV = b\frac{\mu_0 J^2}{4\pi}\alpha_1 u_{ex} \quad (12)$$

where

$$b = ln(\frac{r_a}{r_c}) + b' \quad (13)$$

In Eq. (13), b' varies from 1/4 to 3/4, depending on the current distribution on the cathode.

Here, one changes the notations B_z and $\bar{B}_r$ into B and $\bar{\phi}B$, respectively. Substituting Eqs. (5), (10) and (12) for Eq. (1) yields the following quadratic equation;

$$(1+c_1\psi)u_{ex}^2 - 2(c_2\psi - c_3\psi^2 + c_4)Ju_{ex} - c_5\psi J^2 = 0. \quad (14)$$

Here,

$$\psi = \frac{\bar{\sigma}B^2}{\dot{m}} \quad (15)$$

$$c_1 = 2\alpha_1^2\bar{\phi}^2 a_{hall}V_0 \quad (16)$$

$$c_2 = \frac{\alpha_1\bar{\beta}\bar{\phi}a_{hall}V_0}{2\pi\bar{r}l} \quad (17)$$

$$c_3 = \alpha_1\alpha_2'\bar{\beta}\bar{\phi}^3\bar{r}a_{hall}V_0 \quad (18)$$

$$c_4 = \frac{\alpha_1 b\mu_0 J}{4\pi\dot{m}} \quad (19)$$

$$c_5 = \frac{2\alpha_2''r_a^2}{\bar{\sigma}\dot{m}} \quad (20)$$

Solving Eq. (14), the thrust is calculated by

$$T = \dot{m}u_{ex}$$

$$= \frac{c_2\psi - c_3\psi^2 + c_4 + [(c_2\psi - c_3\psi^2 + c_4)^2 + (1+c_1\psi)c_5\psi]^{1/2}}{1+c_1\psi}\dot{m}J \quad (21)$$

Equation (21) is transformed as follows:

$$T = \frac{T_{hall}+T_{self}}{2} + [(\frac{T_{hall}+T_{self}}{2})^2 + T_{swirl}^2]^{\frac{1}{2}}. \quad (22)$$

Here,

$$T_{hall} \equiv \frac{\psi(1-\psi/\psi_c')}{\psi+\psi_c}T_{hall,max} \quad (23)$$

$$T_{swirl} \equiv [\frac{\psi}{\psi+\psi_c}]^{\frac{1}{2}}T_{swirl,max} \quad (24)$$

$$T_{self} \equiv \frac{\psi_c}{\psi+\psi_c}T_{self,max} \quad (25)$$

The parameters which appear in the above equations are given by

$$\psi_c = \frac{1}{2\alpha_1^2\bar{\phi}^2 a_{hall}V_0} \quad (26)$$

$$\psi_c' = \frac{1}{2\alpha_2'\bar{\phi}^2 a_f^2 V_0}, \quad a_f \equiv \frac{\bar{r}}{r_a} \quad (27)$$

$$T_{hall,max} = \frac{\bar{\beta}(-\bar{j}_r)\dot{m}}{\alpha_1\bar{\phi}} = \frac{\alpha_1 a_{hall}}{a_f}(\omega_e\tau_e)_{B=B_c}J\bar{\phi}B_c r_a \quad (28)$$

$$T_{swirl,max} = (2\alpha_2'')^{\frac{1}{2}} J B_c r_a \tag{29}$$

$$B_c = (\frac{\dot{m}\psi_c}{\bar{\sigma}})^{\frac{1}{2}} \tag{30}$$

$$T_{self,max} = 2\alpha_1 b \frac{\mu_0 J^2}{4\pi} \tag{31}$$

Eq. (22) along with Eqs. (23), (24) and (25) gives the general form of the thrust formula for an applied-field MPD thruster.

4. Estimations of the Thrust Components

In the absence of the applied magnetic field, Eq. (22) is to be the same as the well known self-magnetic thrust formula given by Jahn.[3] Therefore,

$$\alpha_1 = \frac{1}{2} \tag{32}$$

When $\psi \ll \psi_c$, i.e. at a weak applied magnetic field, Eq. (24) becomes

$$T_{swirl} = (2\alpha_2'')^{\frac{1}{2}} J B r_a \tag{33}$$

In order for Eq. (33) to be equal to the Fradkin's formula,[2] it is needed that

$$\alpha_2 = \frac{1}{2} \tag{34}$$

From the radial component of Eq. (2),

$$\bar{j}_r \simeq \bar{\sigma}(\bar{E}_r + \bar{\beta}\frac{\bar{dp}_e}{dr}) \equiv \bar{\sigma}\bar{E}_r'. \tag{35}$$

Hence, the exhaust velocity calculated from Eq. (28) is approximately given by

$$u_{hall,max} \equiv \frac{T_{hall,max}}{\dot{m}} \simeq \frac{2|\bar{E}_{hall,\theta}'|}{\bar{B}_r} \tag{36}$$

$$\bar{E}_{hall,\theta}' = \bar{\omega_e\tau_e}\bar{E}_r' \tag{37}$$

Eq. (36) implies that the exhaust velocity only due to the generalized Hall acceleration is twice as high as $\bar{E}_{hall,\theta}' \times \bar{B}_r$ drift velocity.

The dependence of $\bar{E}_r'$ on B is determined both by the impedance of an MPD thruster and by the characteristics of the power supply for the arc discharge. Generally, an applied-field MPD thruster is connected with a power supply with a constant-current profile. In this case, $\bar{E}_r'$ varies with B. However, for example, if the power supply has a constant-voltage profile, the thruster operates as an electrostatic acceleration thruster, being identical with a 'Hall-current accelerator' or a 'closed-drift thruster.'[5]

To the generalized Hall acceleration, the exhaust velocity is an important parameter. As is seen from Eqs. (23) and (28), it is expressed just by the characteristic parameter $\psi = \bar{\sigma}B^2/\dot{m}$. The ratio ψ/ψ_c can be a criterion whether the effect of the generalized Hall acceleration is large or not. Under a typical condition that $\psi_c' = 2\psi_c$ (e.g. from the data of Fig. 1, $r_c/r_a = 0.38$, $a_{hall} = 0.58$ and $a_f = 0.44$), T_{hall} becomes maximum at $\psi = 0.73\psi_c$.

5. Operation Regimes

Table 1 shows the typical operation parameters input in experiments and the respective thrust components which are calculated by the present theory in the cases of some applied-field MPD thrusters.

In the thruster operation regime performed by Sasoh and Arakawa (University of Tokyo),[1],[6] it is seen that the effect of the generalized Hall acceleration is dominant. In this case, $T/\dot{m}$, which is equivalent to the average exhaust velocity, is expressed by the characteristic parameter ψ (see Fig. 1). In order to obtain the exhaust velocity as the general function of the characteristic parameter which is applicable for different kinds of propellant species, $B^2/\dot{m}$ – the characteristic parameter in Ref. [1] – needs to be multiplied by the electrical conductivity. In the characteristic parameter ψ, to what extent the azimuthal current is induced is estimated by $\bar{\sigma}B$, which scales with the electron Hall parameter. The thruster operation is made at relatively high-B, low-$\dot{m}$ levels (compare the ψ with the others). The ratio ψ/ψ_c is 0.69. The effect of the swirl acceleration is not large here – about 10 % thrust increase. The effect of the self-magnetic acceleration is almost negligible.

In the operation regime performed by Fradkin et al. (Los Alamos Scientific Laboratory),[2] the effect of the swirl acceleration is found dominant. The maximum value of ψ set in the experiment is only 4 % of that in the first case. In other words, the applied magnetic field at the input propellant mass flow rate is weak in comparison with the former case.

Under the condition that $\psi = \xi\psi_c$ and $\psi_c' = 2\psi_c$, the ratio of T_{hall} to T_{swirl} is calculated from Eqs. (23) and (24) as follows:

$$\frac{T_{hall}}{T_{swirl}} = c_6(\xi)(\frac{\bar{\sigma}\bar{\rho}u_{ex}\bar{\beta}^2}{l})^{\frac{1}{2}} = c_6(\xi)\frac{\bar{\omega_e\tau_e}M_A}{R_m^{\frac{1}{2}}} \tag{38}$$

$$c_6(\xi) = (\frac{\xi}{\xi+1})^{\frac{1}{2}}(1-\frac{\xi}{2})\frac{1}{a_f}(\frac{a_{hall}}{8\alpha_2''})^{\frac{1}{2}} \tag{39}$$

$$M_A \equiv \frac{u_{ex}}{B/(\mu_0\bar{\rho})^{\frac{1}{2}}} \tag{40}$$

$$R_m \equiv \bar{\sigma}\mu_0 u_{ex} l \tag{41}$$

At $\xi = 0.5$, $c_6(\xi)$ is maximum and is of the order of 0.5. As is seen from Eq. (38), the ratio increases with increasing exhaust velocity. Moreover, when the propellant mass flow rate is decreased, both $\bar{\sigma}$ and $\bar{\rho}\bar{\beta}^2$ ($= m_i/(e^2 n_e)$ for a fully ionized plasma) will increase, thereby enhancing the effect of the generalized Hall acceleration. Also, the axial length of the acceleration region is to be small so that the contribution of the generalized Hall acceleration is large.

Tahara et al. (Osaka University)[4] combines a self-field MPD thruster with an applied magnetic field coil connected in series with the discharge circuit. In this case, the applied magnetic field is the function of the discharge current. Since the external magnetic field is applied so that it becomes as strong as the self-induced magnetic field, the T_{self} and T_{swirl} are almost equal. From Eqs. (24) and (25), the ratio of T_{self} to T_{swirl} under the condition that $\psi \ll \psi_c$ is calculated;

$$\frac{T_{self}}{T_{swirl}} = c_7\frac{B_{\theta,r_a}}{B} \quad (\psi \ll \psi_c) \tag{42}$$

are almost equal. From Eqs. (24) and (25), the ratio of T_{self} to T_{swirl} under the condition that $\psi \ll \psi_c$ is calculated;

$$\frac{T_{self}}{T_{swirl}} = c_7 \frac{B_{\theta,r_a}}{B} \quad (\psi \ll \psi_c) \tag{42}$$

$$B_{\theta,r_a} = \frac{\mu_0 J}{2\pi r_a} \tag{43}$$

$$c_7 = \frac{b}{2^{3/2}\alpha_2^{n1/2}} \tag{44}$$

The value of c_7 is of the order of unity. Therefore, it is found from Eq. (42) that the ratio of these components approximately equals the ratio of the self-induced to applied magnetic fields.

In the last case, the effect of the generalized Hall acceleration is negligible; the maximum value of the characteristic parameter of this case is five orders of magnitude smaller than that of the first case.

5. Conclusion

The formula which gives the thrust produced by an applied-field MPD thruster from the controllable thruster operation parameters has theoretically been obtained; the formula is derived from the energy conservation equation of the propellant plasma flow. Using this formula, the relative contributions of the generalized Hall acceleration, swirl acceleration and self-magnetic acceleration can be estimated. The relative contributions of the self-magnetic and swirl accelerations are found to be estimated by the ratio of the self-induced to applied magnetic field strengths. At a moderate applied-field strength, i.e. when ψ is comparable with ψ_c, a large effect of the generalized Hall acceleration is obtained under the conditions that the exhaust velocity is high, the propellant mass flow rate is low and the axial length of the acceleration region is small. It follows from these results that the generalized Hall acceleration is found to be suitable for small-sized, high-I_{sp} thruster operations.

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Table 1 Operation conditions and calculated thrust components of some applied field MPD thrusters.

Thruster	B, T	$\dot{m}$, kg/s	J, A	ψ, m^{-3} (ψ/ψ_c)	T_{hall}, N	T_{swirl}, N	T_{self}, N
Univ. Tokyo (Refs. [1] and [6])	0.10	9×10^{-7}	200	2.4×10^{8} (0.69)	4.4×10^{-2}	1.4×10^{-2}	1×10^{-4}
Los Alamos Sci. Lab. (Ref. [2])	0.19	2.5×10^{-5}	350	9.7×10^{6} (0.31)	1.8×10^{-1}	6.9×10^{-1}	9×10^{-3}
Osaka Univ. * (Ref. [4])	0.075	2.75×10^{-3}	15000	1.4×10^{3} (0.0005)	4×10^{0}	2.2×10^{1}	2.2×10^{1}

* The coil configuration is C1-(3)-type; 5 % of the total input power is assumed to be input to the electron translational energy.

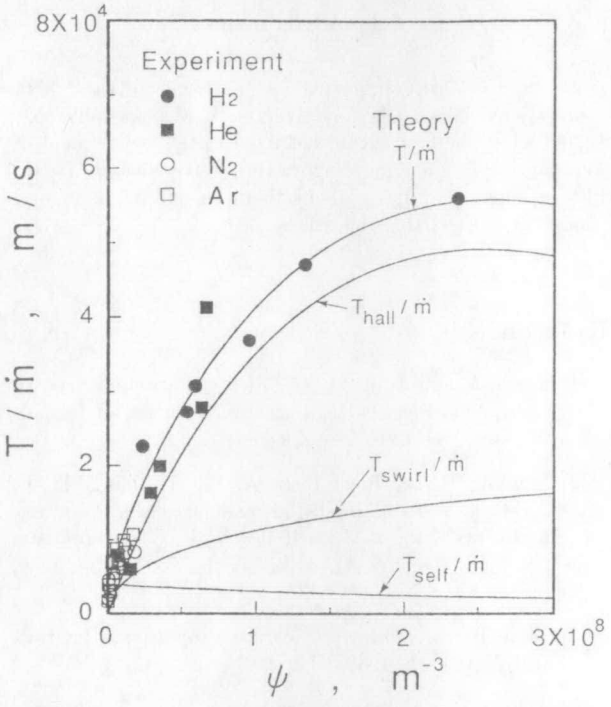

Fig. 1 $T/\dot{m}-\bar{\sigma}B^2/\dot{m}$ relation by the applied-field MPD thruster of Refs. [1] and [6]. The theoretical lines are drawn under the following condition; $\bar{r} = 3.5 \times 10^{-3}$ m, $r_{a,ef} = 8 \times 10^{-3}$m, $r_{c,ef} = 3 \times 10^{-3}$m, $\bar{\phi} = 5 \times 10^{-2}$, $\bar{\sigma}\dot{m} = 2 \times 10^{-2}$kg/($\Omega$ms), $\bar{\beta} = 5 \times 10^{-2}$m^3/C, $l = 2 \times 10^{-2}$m, $\bar{j}_r = c_c J/(2\pi\bar{r}l)$ A/m^2, $c_c = 0.2$, J = 200A.