ENGINEERING METHODS OF HOLLOW CATHODE CALCULATION

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Abstract

Engineering methods of local and integral hollow cathode parameters and erosion calculations based on the developed mathematically closed model of hollow cathodes are proposed.

A mathematical model of hollow cathodes is presented in the form of a system of differential and integral equations and relationships describing physical processes occuring in the plasma in the active zone of hollow cathodes, on the plasma-cathode boundary and on the cathode surface. The model permits calculations of the main local and integral parameters of hollow cathodes given the discharge current, plasma-generating gas flow rate, cathode channel geometry, heat removal conditions, physical properties of a plasma- generating gas and cathode material. Mathematical model of hollow cathode erosion is organized together with the model of calculation of plasma parameters and hollow cathode walls heating in modes when products of erosion make a small admixture to the primary gas. Hollow cathode erosion was defined by the processes of sputtering, evaporation, sticking and reflection of cathode material atoms and ions. Results of calculation experiments, which are in good agreement with experimental data, are described.

1. Hollow cathode mathematical model in the arc regime.

Let us consider the hollow cathode (HC) as a cylinder channel with the length L and wall thickness $h = r_e - r_i$, where r_e and r_i are external and internal radii of the cathode channel, respectively. The inlet section of the cathode channel is connected with the system of working gas supply. The rest part of the channel is in a vacuum chamber. A so-called active zone (AZ) appears inside HC in which an efficient generation of a gas discharge plasma occurs due to collision-radiation processes and electron emission from the walls. In this zone, close to the internal surface of the HC channel, a cathode potential drop, being a powerful source for heating of the cathode walls and electrons, occurs. Another source of electron energy in AZ iz Joule heating. Intensive generation of a low-temperature plasma in HC is determined by an effective supply of energy into electron gas at optimal pressure. In case of a hollow cathode with a low emission capacity it is often necessary to heat the cathod using an external source in order to maintain the electron emission on a proper level. For a long channel ($r_e \ll L$; $h \ll r_i$) the initial two-dimensional differential equations describing the balance of discharge, mass, impulse and energy are presented by way of their integration over the channel cross section as a quaziunidimensional approximation in which the boundary conditions are directly included into the differential equations and the equations themselves become unidimensional. The determining parameters in the HC model under consideration in the arc regime are : discharge current, I; gas-flow rate, G_g ; cathode radius, r_i ; HC wall temperature, T ; cathode potential drop, U_k ; degree of plasma ionization, Xi ; electron temperature, Te ; pressure of the plasma-generating gas including AZ, Pg ; ionic current density on the cathode, j_{ik} ; total electron current density on the cathode surface, j_{ek} ; AZ length, Lp .

The initial system of equations for determining cathode temperature and potential drop includes the equation of heat conductivity with Joule heating of walls and with superficial energy sources and the integral law of electric current continuity for HC channel walls and as a quaziunidimensional approximation it can be expressed as follows:

$$\frac{h}{L}\frac{d}{dz}\left(K_m \frac{dT}{dz}\right) + h\sigma_m j_{mz}^2 + q_p - q_{a,\beta} - q_{a,H} - q_{\kappa,H} = 0, \quad (1)$$

where q_p - the density of energy flow to the cathode surface due to the energy exchange with ions and electrons;

$q_{a,\beta}$; $q_{a,H}$ - the density of heat flow on the internal and external cathode surfaces, respectively, due to the heat exchange with atoms;

$q_{\kappa H}$ - the density of radiation flow from the cathode walls;

j_{mz} - the longitudinal current density in the HC channel walls,

$$j_{mz} = -\frac{L}{h}\int_z^1 j_n \, dz \; ; \quad (2)$$

j_n - the current density normally to the cathode surface,

$$j_n = j_{ik} + j_{ek} \; ; \quad (3)$$

σ_m , K_m - specific resistance and termal conductivity of the cathode matherial.

The coordinate axis z is directed along the flow with a starting point in the inlet section at $z = z' / L$ and $0 \le z \le 1$.

Given the current I, from equation (2) and the boundary condition $j_{mz}(0) = I / S_M$ there follows the integral law of current continuity

$$\frac{I}{S_M} = \frac{L}{h}\int_0^1 j_n \, dz \; . \quad (4)$$

where S_M is a cross-sectional area of HC channel walls.

Integral equation (4) determines the relationship between the discharge current and local values of charge flows to the cathode wall.

The specific contens of the quantities included in equation (1-4) can be found in [1]. It should be noted that in the calculations given below the electron current on the cathode was determined by thermoemission and the reverse electron current, and the current of secondary emission were not taken into account. However, electron emission from external surface of the HC channel was taken into consideration.

For the equation of heat conductivity in the HC channel the following boundary conditions are given :

at the inlet of the channel :

$$T = T_0 \quad \text{at} \quad Z = 0 \; ; \quad (5)$$

at the outlet of the channel :

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$$-\frac{K_n}{L}\frac{dT}{dz} = a_{ef}\left(T(L)\right)^4 - T^4(L) + q_{pL} \quad \text{at } z = 1 \ (?$$

where $T(L)$ – channel wall temperature at the outlet;

T_0 – inlet temperature;

a_{ef} – surface blackness coefficient;

q_{pL} – the density of energy flow at the end of the channel due to the energy exchange with plasma.

The method of determination of the T and U values consists in searching for a joint solution of equations (1), (5), (6) and the integral equation (4) with a specified j_{iK} value.

The j_{iK} value is determined, in its turn, from a system of equations describing processes occurring in the plasma of the internal positive discharge column but already with specified U_K and T.

In this work we have restricted ourselves to the consideration of hollow cathodes with inert gases as plasma-generating substances. In this case the ion flow on the cathode surface is practically completely neutralized.

The state of the plasma in the HC active zone is determined by the following system of udependent equations describing the balance of electrons and ions :

$$\frac{1}{L}\frac{dj_{eZ}}{dz} = -\frac{2\pi r_i}{S} j_{eK} - eW_e \quad . \qquad (7)$$

$$\frac{1}{L}\frac{dj_{iZ}}{dz} = -\frac{2\pi r_i}{S} j_{iK} + eW_e \quad . \qquad (8)$$

where j_{eZ} ; j_{iZ} – the longitudinal components of the electron and ion current densities averaged over the HC channel section;

W_e – a resulting rate of ionization and volume recombination;

$S = \pi r_i^2$. $j_Z = j_{eZ} + j_{iZ}$.

Since in the HC active zone $j_{iZ} \ll j_{eZ}$ let us assume that $j_Z = j_{eZ}$. It should be noted that it is equation (8) that determines in hollow cathodes a substantial difference between the degree of plasma ionization in AZ and its equilibrium value found at the same Te and Pg on the basis of Sakha equation.

Estimates show that the rate of triple recombination in the HC active zone for inert gases is significantly lower than the rate of recombination on the HC wall. Therefore in this work is assumed to be equal to the recombination rate on the HC channel wall :

$$eW_e = \frac{2\pi r_i}{S} j_{iK} \quad .$$

As a quaziunidimensional approximation, the equation of electron energy balance in the HC active zone can be presented in the following form:

$$\frac{dq_{eZ}}{dz} = j_{eZ} E_Z + \frac{2\pi r_i}{S} j_{eK} U_K - AeU_i W_e \quad . \qquad (9)$$

where E_Z is the longitudinal component of the electric field tension and A is a relative rate of the loss of electron energy as a result of collision-radiation processes (energy losses upon elastic collisions in HC AZ are negligibly low).

Further on, the condition of approximate similarity of the density of electron energy flow and the density of electron current is taken into account .

$$q_{eZ} = -\frac{K}{e} C^* Te\, j_{eZ} \quad . \qquad (10)$$

where $C^* = 3.2$.

In addition, the generalized Ohm's law should be taken into consideration .

In equations (9), (10) the electron current longitudinal component is determined by the following integral equation

$$j_{eZ} = -L \int_0^Z \left(\frac{2\pi r_i}{S} j_{eK} + eW_e \right) dz \quad . \qquad (11)$$

at $j_{eZ}(0) = 0$ and $j_{eZ}(1) = I / S$. Thus, E_Z is determined on the basis of equations (11) and Ohm's law.

As shown by the analysis of experimental data and the equations for the balance of atomic and ionic energies in the absence of the magnetic field, it can be assumed that $Ti \simeq Ta \simeq T$ [2]. On the other hand, experimental investigations indicate that the temperature of electrons in HC AZ Te is significantly higher than the temperature of heavy perticles (Te >> T).

In such a case, when introducting the degree of ionization Xi = Ni/(Ni+Na) and the pressure of ionized gas Pg, the concentration of particles can be expressed through the determining parameters as follows :

$$Ne = \frac{Pg}{kT}\,\frac{Xi}{\frac{Te}{T} + Xi} \quad ; \quad Na = \frac{Pg}{kT}\,\frac{1 - Xi}{\frac{Te}{T} + Xi} \quad ; \qquad (12)$$

where Ne and Na are the concentrations of electrons and atoms.

Taking into account the defenition of an average-mass rate of ionized gas and the integral equation of continuity for a stationary flow the plasma flow rate can be also expressed through the determining parameters as follows :

$$v_z = \frac{RT}{S\, Ma\, Pg} Gg \left(1 + \frac{Te}{T} Xi \right) . \qquad (13)$$

where Ma – a molar mass of working gas atoms;

R – a universal gas constant;

Gg – a mass flow rate of the working gas.

The distribution of pressure in the HC channel is found from the equation of ionized gas movement. As a quaziunidimensional approximation, it can be presented for a laminar flow in the following form:

$$\frac{dPg}{dz} = C_{eK} \frac{a^* RT \eta}{MaS\, 2\pi r_i} Gg \left(1 + \frac{Te}{T} Xi \right) . \qquad (14)$$

where η – the coefficient of ionized gas viscosity;

$a^* = 64\pi^2$ – a constant for a laminar flow in the channel of circular section;

$C_{eK} = 1 / (1 + P_{eK}/Pg)$ [1] – a coefficient taking into account the effect of slipping.

Under conditions of a gas flow into space with a sufficiently high vacuum let us take as a boundary condition at the channel outlet the condition of equality of the laminar flow and sound velocities. This makes it possible to determine the plasma pressure at the channel outlet :

$$Pg(L) = f \frac{RT(L)}{\gamma\, Ma} \frac{Gg}{S} \left(1 + \frac{Te(L)}{T(L)} Xi(L) \right)^{1/2} , \qquad (15)$$

where γ – an adiabatic index;

f – a formfactor taking into account the

rate profile assuming the condition of equality of a maximal flow velocity to sound velocity, f = 2, for an average velocity, f = 1.

Thus, the integration of equation (14) with account for condition (15) allows, in particular, the inlet pressure to be determined.

As seen from equations (14), (15), with a significant rate of plasma ionization and with the condition that Te >> T, due to a multiplier (1 + Xi Te/T), reflecting the role of plasma non-isotermicity, a considerable increase in the plasma pressure gradient as compared to changes in the neutral gas pressure is to occur in the HC active zone [2].

The construction of a model of the internal positive discharge column is completed by the determination of j_{iK}, We and A . The specific contens of these quantities can be found in [1].

The integration of equations (8), (9) and (14) with specified (found in the first part of the model) T and U_K with account for the formulated relations makes it possible to find local values of Xi, Te and Pg, and therefore to calculate Ne, Na, vz, j_{iK} and other parameters.

Thus, the formulated equations and relations permit all determining HC parameters to be found : T, U_K , Xi, Te and Pg.

2. Hollow cathode erosion theory.

In quaziunidimensional approximation, used in the present paper, when relative variation of cathode radius by means of erosion is small we have :

admixture ions mass balance equation :

$$\frac{S}{L}\frac{d}{2\pi r_i}\frac{}{dz}(v_z N_{iK} m_{iK}) = \dot m_{isp} + \dot m_{iev} - \dot m_{iz} + \sum_{n}^{i,a} R_{ni}\dot m_{nz} + \frac{S}{2\pi r_i} m_{iK} W_{eK} \quad (16)$$

admixture atoms mass balance equation :

$$\frac{S}{L}\frac{d}{2\pi r}\frac{}{dz}(v_z N_{aK} m_{aK}) = \dot m_{asp} + \dot m_{aev} - \dot m_{az} + \sum_{n}^{i,a} R_{na}\dot m_{nr} - \frac{S}{2\pi r_i} m_{aK} W_{eK} \quad (17)$$

admixture ions and atoms mass concervation laws on the surface :

$$a_a + R_{ai} + R_{aa} = 1 \; ; \; a_i + R_{ii} + R_{ia} = 1 \; ; \quad (18)$$

where N_{aK}, N_{iK} - admixture atoms and ions concentrations;

$m_{iK} = m_{aK}$ - admixture atoms and ions mass;

$\dot m_{asp}, \dot m_{isp}$ - mass sputtering rate by atoms and ions;

$\dot m_{aev}, \dot m_{iev}$ - mass evaporation rate by atoms and ions;

$\dot m_{ar}, \dot m_{ir}$ - mass flow densities of admixture atoms and ions to the cathode surface from plasma;

a_a, a_i - sticking coefficients of admixture atoms and ions ;

$R_{ai}, R_{ia}, R_{ii}, R_{aa}$ - reflection coefficients of atoms and ions;

W_{eK} - admixture ionization rate.

Therefore, in the discussed theory of HC erosion recycling of the cathode material was taking into account. Further, we consider regimes when admixture atoms and ions concentrations are much less than propellant atoms and ions concentrations. As calculations showed this is true for most of HC working in thermoemission regime.

In this case plasma ionization degree Xi approximately defined by expression :

$$Xi = \frac{Ne}{Ne + Na}$$

and relative component concentrations are correspondingly :

$$Xa = \frac{Na}{Ne + Na} \; ; \quad Xak = \frac{Nak}{Ne + Na} \; ; \quad Xik = \frac{Nik}{Ne + Na} \; .$$

In the discussed approximation plasma velocity is determined by equation (13).

In equations (16) and (17) values of $\dot m_{sp}$ and $\dot m_{ev}$ may be represented in the following form [3]:

$$\dot m_{sp} = 1{,}044\cdot 10^{-4} A_{zK} Y j_{iK} \; , \; kg/(m^2 \cdot s) \; ;$$

$$\dot m_{ev} = c_{ev} T^{-1/2} \exp(-eE_{ev}/kT) \; , \; kg/(m^2 \cdot s) \; ,$$

where A_{zK} - admixture atom mass ;

c_{ev} - empiric constant;

E_{ev} - evaporation energy;

Y - sputtering coefficient.

Output of erosion products mass by plasma flow is carried out due to convectional transfere along the channel with rate :

$$\dot M = m_{iK} Gg(Xik + Xak)/m_i \; . \quad (19)$$

Specific HC erosion (mass loss coefficient) by definition is equal to

$$g = m_{iK} Gg(Xik + Xak)/m_i I \; . \quad (20)$$

In relations (19) and (20) values Xik and Xak are taking at z = 1. Local recycling rate is determined by sticking of admixture atoms and ions to the cathode wall :

$$\dot m_z = a_a \dot m_{ar} + a_i \dot m_{ir} \; . \quad (21)$$

Local erosion rate of HC channel wall is defined by relations :

mass - $\dot m_w = \dot m_z - (\dot m_{sp} + \dot m_{ev})$;

linear - $\dot r = m_w / \rho$.

where ρ - cathode material density.

To determine $\dot m_{iz}$ and $\dot m_{ar}$ a system of five-component partly ionized plasma in hydrodinamical approximation in projection to HC channel radius-vector was considered [1]. The system of equation was solved in relation of diffusion velocities of all components. The solution was carried out with assumption that Xik, Xak << Xi . It was supposed so that Ta,Ti << Te. Finally, for admixture mass flow densities follwing relations were found :

$$\dot m_{ir} = \alpha_{ir} Xik \; ; \; \dot m_{ar} = \alpha_{ar} Xak \; . \quad (22)$$

where proportion coefficients within taking approximation is undependent on Xik and Xak and are complex functions of plasma parameters and effective particle collision cross sections. Approximate estimation of this values may be done according to relations :

$$\alpha_{ir} = \frac{Nak}{F}\frac{j_{iK}}{Xi} \; ; \quad \alpha_{ar} = \frac{Nak}{F}\frac{j_{iK}}{1 + Xi} \; .$$

where j_{iK} - ion current density at the cathode;

Xi - degree of plasma ionization; Mak - admixture atom molar mass, F - Faradey number.

As result of cathode surface bombardment by relatively slow ions (with energy less than 100 eV for arcs regimes) a flow of cathode atoms appears. Atom mass flow density is determined by the follwing relation :

$$\dot{m}_{sp} = \frac{Mak}{F} Y(E,T)\, j_{ik} \quad . \qquad (23)$$

Equation (23) permits calculation of local cathode surface sputtering rate for given local ion current donsity, bombarding ions energy, which may be assumed equal to the cathode potential drop in considering of ion charge, and cathode temperature.

In [3] we formulated half-empirical methode of calculation of sputtering and ion kinetic energy accomodation coefficients for given values of ion energy and surface temperature. In the same place values of sputtering and ion kinetic energy accomodation coefficients for argon ions are presented.

Take the following relation for admixture atoms ionization rate [4]:

$$W_{ex} = \alpha_{ex} Nak\, Ne \quad . \qquad (24)$$

Including Xak into equation (24) one can found :

$$W_{ex} = \alpha_{ex} Xak\, Ne(Na + Ne) \quad .$$

Thus, admixture atoms ionization rate is proportional to its concentration.

Taking that at the beginning of the HC active zone admixture component concentrations are equal to zero :

$$Xak(0) = Xik(0) = 0 \quad . \qquad (25)$$

Thus, by means of solution of linear differential equation system (16) and (17) with boundary condition (25) and with given reflection and sticking coefficients of cathode material ions and atoms for determining parameters of HC regime, one can define admixture atoms and ions concentrations (Xak,Xik), which permits calculation of HC specific erosion, local mass and linear cathode erosion rates.

3. Results of investigations by the method of mathematical modelling of HC parameters.

On the basis of the developed mathematical HC model several calculation experiments have been carried out for tantalum cylinder cathodes with Lk = 0.1 m, h = $2\cdot10^{-4}$ m, r = $2.8\cdot10^{-3}$ m and without a magnetic field. The working gas - argon.

These parameters meet the conditions of experimental studies described in [5]. A specific feature of this work is that using the probe technique the authors have managed to determine, although not in the whole HC active zone, the concentration of electrons,their temperature and plasma potential relative to the cathode from which it is possible to judge about the extent of a cathode potential drop. The calculations were carried out at a discharge current 10, 15 and 20 A and in argon flow rate $1.2\cdot10^{-6}$ kg/s and $3.5\cdot10^{-6}$ kg/s.

Fig. 1 presents the calculated local values of plasma pressure and wall temperature for two flow rate values. In work [5] the values of inlet gas pressure for these flow rates and currents were given. It is seen that the deviation of the calculated data from the measured ones does not exceed 20 %. The cathode temperature agrees satisfactorily with the experiment both by the character of distribution and by its value.

Fig. 2 shows the values of electron concentration in the active zone, electron temperature and potential drop. It should be noted that Fig. 2 contains the values of plasma potential which differs from the cathode potential drop. These parameters are close only at the beginning of the active zone. A satisfactory agreement is observed between the values of electron temperature and the values of their concentration. In a region with z > 0.65 the probe caused noticeable disturbances.

Fig. 3 presents volt-amperic characteristics and the discharge flow rate. Our HC model makes it possible to calculate the power supplied to the cathode :

$$W_K = \int_0^L (j_n U_k + j_{e2} E_z + q_w r_H^2)\, dz \quad .$$

This parameter and the cathode potential drop at the outlet are given in Fig. 3.

Fig.4. and Fig.5. show results of erosion parameters calculation for cylindrical (r = 2.8 mm) thin-walled (h = 0.2 mm) tantalum hollow cathode.

Fig.4. shows the relations of specific erosion, cathode material atoms and ions concentrations at HC exit and characteristic time, necessary for forming of cavern with depth equal to half of the wall width of HC channel

$$t(h/2) = h/2\dot{h} \quad .$$

The influence of cathode material atoms and ions sticking, as can be seen, on HC erosion parameters is insignificant. This in turn allows to judge about possible values of sticking coefficient for given case by means of comparision with corresponding experimental data not only for specific erosion, but for erosion products concentrations also. Specific erosion for indicated HC geometry and regime with $a_i = a_a = 0.75 - 0.95$ is about 10^{-10} kg/C, which does not conflict with experimental data for discussed HC type.

Fig.5 shows local HC erosion for given regime. While considering erosion before, one should pay attension to significantly increased scale of surface thickness in respect of longitudinal scale. Corresponding distributions of wall temperature, non-dimensional ion current density $\bar{j}_{ik} = j_{ik}/j_{k0}$, $j_{k0} = 0.65\cdot10^{9}$ A/m^2 and non-dimensional cathode potential drop $\bar{U}_k = U_k/Ui$, Ui = 15.75β along HC channel are given on the same figure.

Cavern formation, its depth, evidently, define cathode life-time. Besides that, variation of channel wall thickness and, possible, its physical properties may lead to HC erosion acceleration. We think that a special attention should be payed to the research of caverns formation on the cathode surface in connection with the problem of MPD thrusters life-time.

In conclusion we shall note that the formulated mathematical HC model makes it possible to study the influence of different factors on HC parameters by the method of calculation experiment and after experimental substantiation of the reliability of calculation experiments to carry out

prognostication of working parameters of real cathodes.

References.

1. Vaulin E.P. In : Rocket engines and energetic devices. M.: NIITP, 1991, pp. 108-134.
2. Delcroix J.L., Trindade A.R. Hollow cathode arcs. Adv. Electron and Electron. Phys., vol. 35 , New-York-London, 1974, p. 87-190.
3. Vaulin E.P., Georgieva N.E., Martynenko T.P. Transfere processes in near-wall layers of ionized gases. Coll. of scientific works - M.: MAI, 1989, p. 54-65.
4. Zeldovich Ya.B., Raizer Yu.P. Physics of shock waves and high-temperature hydrodynamic events.- M.: Nauka, 1966, 686 p.
5. Brunet A. De'charge a cathode creuse : regime d'arc avec colonne positive interiure et e'coulement d'argon. Note Technique, N 1975-3. Office national d'etudes et de recherches, 55 p.

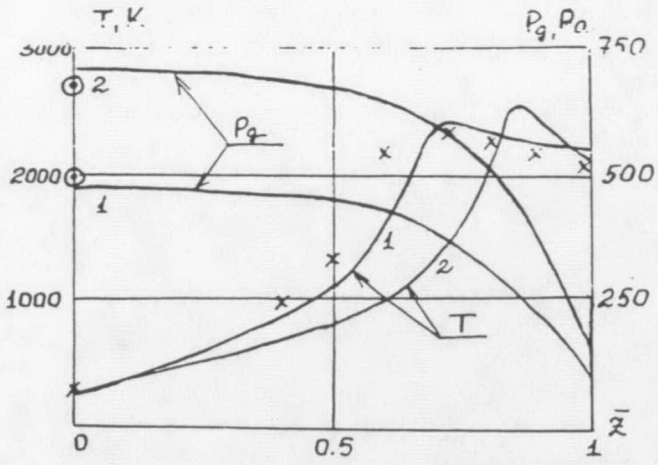

Fig.1. Plasma pressure and HC channel walls temperature,
⊙, x - experiment,
—— - calcalculation.
I = 10 A, 1. Gg = $1{,}2\cdot10^{-6}$ kg/s;
2. Gg = $3{,}5\cdot10^{-6}$ kg/s.

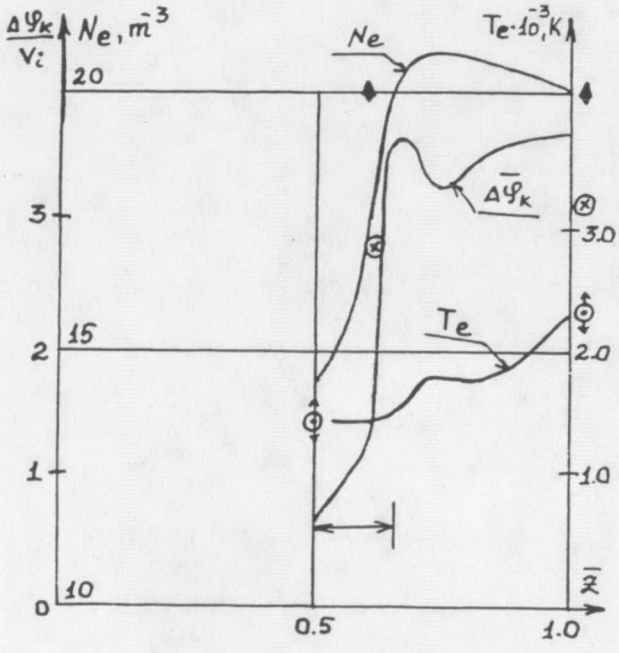

Fig.2. Distribution of cathode potential drop ($\Delta\varphi_k$), electron concentration (N_e) and temperature (T_e)
⊗, ♦, ⊙, - experiment,
$\Delta\varphi_k$, N_e, T_e, respectively;
—— - calculation,
I = 10 A, Gg = $1{,}2\cdot10^{-6}$ kg/s

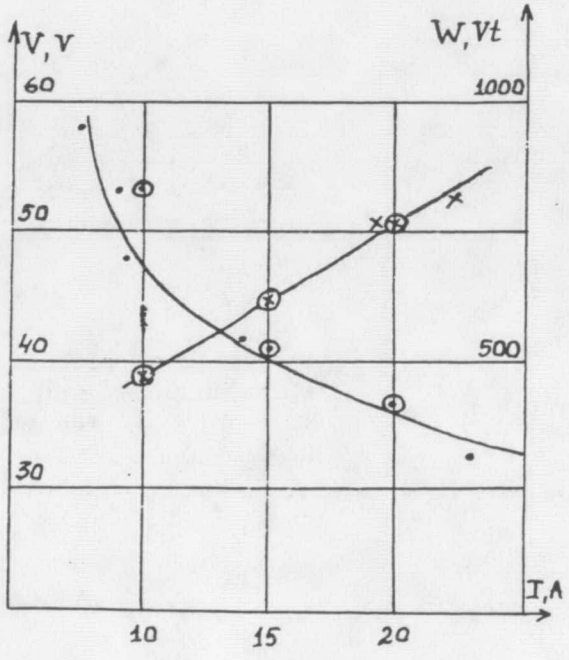

Fig.3. Current voltage characteristic and HC discharge power,
·, x - experiment;
⊙, ⊗ - calculation,
Gg = $2\cdot10^{-6}$ kg/s.

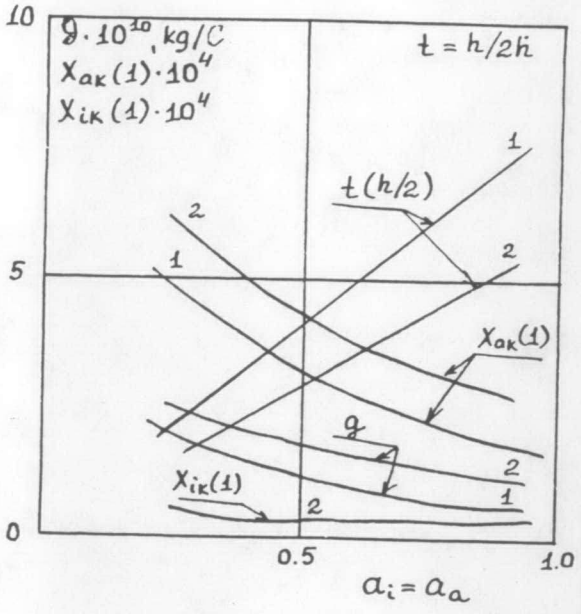

Fig.4. Influence of cathode material atoms and ions sticking on HC erosion parameters :
1. I = 20 A 2. I = 15 A;
Gg = $1.2 \cdot 10^{-6}$ kg/s,
r_K = $2.8 \cdot 10^{-3}$ m,

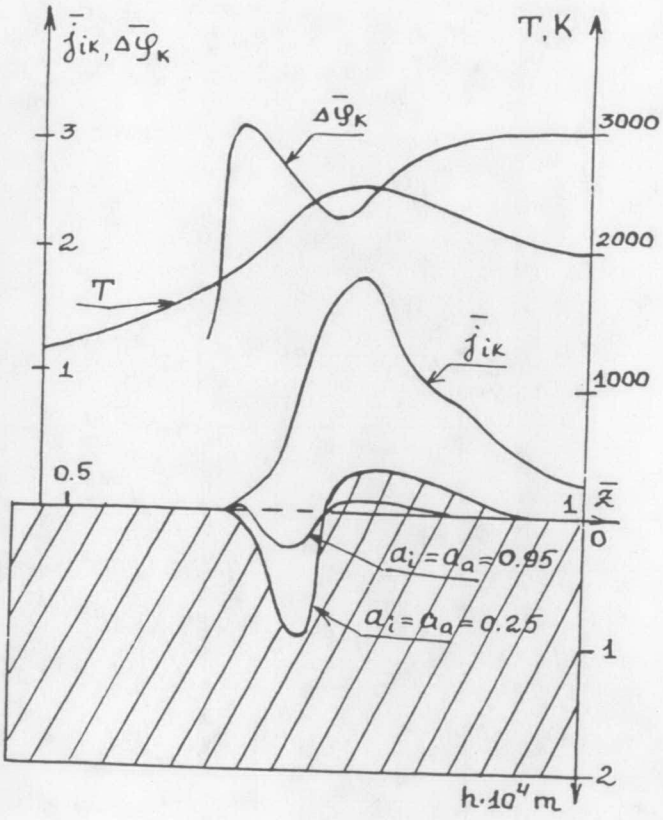

Fig.5. Local HC erosion,
I = 15 A,
Gg = $1.2 \cdot 10^{-6}$ kg/s,
r_K = $2.8 \cdot 10^{-3}$ m,
exposition time 1,5 houers.