RADIO WAVE REFRACTION IN EXHAUST PLASMA PLUMES

F.F. Gabdullin, V.I. Garkusha, A.G. Korsun, E.M.Tverdokhlebova
Central Research Institute of Machine-Building, Kaliningrad, Moscow region, Russia

Abstract

Plasma formations arising in vicinity of a spacecraft as a result of electric thrusters operation may present a serious communications impact for certain orientations of exhaust plasma plumes respectively to directional patterns of spacecraft antennas.

In reality sizes of plasma formations are large enough comparing to wave lengths of carrier frequencies. Gyromagnetic effects and radio frequency (RF) absorption are usually insufficient. Therefore for calculation of plasma scattering characteristics the ray approximation may be used.

Ray patterns, relative amplitude changes and phase shifts have been computed using developed computing codes for RF waves being received and transmitted by spacecraft antennas. Computed shapes of radio shadow regions and intensity distributions of scattered radiation strictly depend on the angle of incidence and the pitch angle. Particularly it is shown that even rather short length waves may be effectively scattered by plasma formations when the angle of radiation incidence is small relative to direction of plasma injection.

Nomenclature

Φ	–	eiconal (the phasing function)
ε	–	dielectric permeability
x_i	–	spatial coordinates
s	–	variable parameter
q_1^0 , q_2^0	–	reference ray parameters
$d S_1$, $d S_2$	–	cross section of ray's tube
A_1 , A_2	–	radiation flows across transferring $d S_1$, $d S_2$
σ	–	effective area of backward scattering
α	–	pitch-angle
λ	–	wave's length

Introduction

The potential interaction between the RF signal and the charged particles of the electric thruster plume may present serious communications impact in dependence on mutual orientation of radio waves and exhaust plasma plume.

The scattering of radio waves on plasma plume as important aspect of electromagnetic compatibility is an inherent feature of almost all electric propulsion systems that operate in space.

The role of the refraction/scattering effects had been clearly demonstrated in the space experiments with arcjet EPICURE on Cosmos satellites in 1987 [1].

Therefore refraction/scattering effects in exhausted plasmas should be thoroughly studied before electric thrusters become standard onboard equipment. The investigation will help designers of future electrically propelled spacecrafts to minimize the hazard of communication disruption by thrusters plasma plumes [2].

The objective of the given paper is to present an effective method to calculate the main scattering characteristics of exhausted plasma plumes.

Problem formulation

In real conditions the sizes of plasma formations are large as compared with the length of the wave of radiation incidence. Gyromagnetic effects and RF absorption are usually insufficient. Therefore for calculation of scattering characteristics the ray approximation may be used in the frames of geometric optics.

The base of scattering calculations is the ordinary method of the eiconal equation numerical solution with dielectric permeability being expressed via electron density distributions according to the mathematical models of plasma expansion in space [1].

The main properties of exhausted plasmas depend on thruster type (dense jet or a rarefied beam is exhausted), operation mode (stationary or pulse one) and pitch angle between vectors of the Earth magnetic field B and plasma flow velocity U.

Adjacent to a spacecraft plasma formations may acquire a variety of shapes : stretched along U axis symmetry dense plume of arcjet , stretched along B spokes for rarefied beams at $\alpha = 0^0$ and for single plasmoids of plasma pulse thrusters (PPT), petal shaped rarefied plasma plume lying in the plane of vectors B and U for $\alpha = \pi/2$ and at least a grating of spokes stretched along B for PPT operating with high repetition rate .

The ray approximation.

The corresponding electron density distribution define the right side of the eiconal equation

$$\Phi_i^2 = \varepsilon\left(x_i\right) \; ; \; i = 1, 2, 3 \qquad (1)$$

All refraction/scattering calculation are based on general solution of the subsequent set of characteristic equations:

$$\frac{d x_i}{d s} = 2\Phi_i \quad (2)$$

$$\frac{d \Phi_i}{d s} = \varepsilon_i \quad (3)$$

The solution of equations system (2), (3) under given initial conditions defines parametric spatial curve $x_i = x_i\left(s\right)$ that is treated as trajectory of a ray passing through the scattering volume. Tangent lines to the ray at its every point may be also calculated as $\Phi_i = \Phi_i\left(s\right)$.

The most of important electro-dynamical characteristics of inhomogeneous plasma such as the effective area of backward scattering, shape and orientation of caustic surfaces, intensity distribution in the observation region etc. as a rule can be determined if a set of neighbor rays with different initial parameters (not a single ray) are used.

Problem linearization.

For many practically important cases the single reference ray could be cosily defined. Then to define the set of neighbor rays instead equations (2), (3) the linear system much more convenient for solution may be formulated. If the direction of rays entrance into the scattering volume is known then the solution of equation (1) may be reduced to solution of two arbitrary chosen parameters (initial data). Let it be known for reference ray:

$$x_i = x_i^0\left(s; q_1^0; q_2^0\right) \qquad (4)$$

$$\Phi_i = \Phi_i^0\left(s; q_1^0; q_2^0\right) \qquad (5)$$

index "0" denotes the reference ray. To define the set of rays neighbor to the reference one for initial values $q_1 = q_1^0 + \delta q_1$ and $q_2 = q_2^0 + \delta q_2$ as a result of relations (4) and (5) expansion in terms of small values δq_1 and δq_2 one may obtain

$$x_i\left(s; q_1^0; q_2^0\right) = x_i^0\left(s; q_1^0; q_2^0\right) + \left(\frac{d x_i}{d q_1}\right)^0 \delta q_1 + \left(\frac{d x_i}{d q_2}\right)^0 \delta q_2 + \ldots \qquad (6.1)$$

$$\Phi_i\left(s; q_1^0; q_2^0\right) = \Phi_i^0\left(s; q_1^0; q_2^0\right) + \left(\frac{d \Phi_i}{d q_1}\right)^0 \delta q_1 + \left(\frac{d \Phi_i}{d q_2}\right)^0 \delta q_2 + \ldots \qquad (6.2)$$

By substitution of these expressions into the system (2), (3) and expansion of its right sides one may put down

$$\varepsilon_j\left(x_i\right) = \varepsilon_j\left(x_i^0\right) + \left(\frac{d \varepsilon_j}{d x_k}\right)^0 \left(\frac{d x_k}{d q_1}\right)^0 \delta q_1 + \left(\frac{d \varepsilon_j}{d x_k}\right)^0 \left(\frac{d x_k}{d q_2}\right)^0 \delta q_2 + \ldots \qquad (7)$$

taking into account that relations (4), (5) are the solutions of system (2)

$$\frac{d \xi_{i\kappa}}{d s} = 2\,\Psi_{i\kappa}; \; \frac{d \Psi_{i\kappa}}{d s} = \varepsilon_{ij}^0 \xi_{j\kappa}, \; \kappa = 1, 2 \qquad (8)$$

"j" is repeating index to sum up the terms;

$$\xi_{i\kappa} = \left(\frac{d x_i}{d q_\kappa}\right)^0 \quad \Psi_{i\kappa} = \left(\frac{d \Phi_i}{d q_\kappa}\right)^0 \qquad (9)$$

$$\varepsilon_{ij}^0 = \left(\frac{d^2 \varepsilon}{d x_i \, d x_j}\right)^0$$

are the second order derivatives of dielectric permeability where spatial coordinates x_i are substituted with solutions of equation (4).

After the substitutions mentioned above the second order derivatives contain only one independent variable s. Therefore the system (8) occur to be linear with terms of $\xi_{i\kappa}$, $\Psi_{i\kappa}$. While formulating the system (8) and the initial values problem solving or definition of the initial data a

function Φ is usually determined at certain surface that is given in parametric form:

$$x_i = x_i^d(q_1 ; q_2), \tag{10}$$

where x_i^d are known functions and

$$\Phi = \Phi^d(q_1 ; q_2) \tag{11}$$

is also known function has been determined at the surface x_i^d.

In practice the direction of flat wave incidence on the scattering volume is often known or for more general case the field of directions is determined at certain surface. Then the initial conditions acquire the following form:

$$\Phi_i = \Phi_i^d(q_1 ; q_2) \tag{12}$$

Since while formulating the problem the absolute value of vector (12) has not been given the latter is to be chosen so that equation (1) is satisfied at every point of considered surface (10). In this case by differentiation of (10) and (12) wit respect to q_κ one may obtain:

$$\Psi_{i\kappa}^d = \frac{d\,\Phi_i^d}{d\,q_\kappa} \tag{13}$$

$$\xi_{i\kappa}^d = \frac{d\,x_i^d}{d\,q_\kappa} \tag{14}$$

It is assumed that $s = 0$ at the given surface (10). Then the initial data for system (8) solution may be defined using relations (13) and (14) at $s = 0$.

The system (8) may be used for calculation the mast important characteristics of radiation scattering in inhomogeneous plasma formations.

Effective area of Backward Scattering.

The geometric optics assumes that radiation energy spreading within the given elementary ray tube is concerned i.e. the following equation is always satisfied:

$$A_1\,d\,S_1 = A_2\,d\,S_2, \tag{15}$$

If vectors S, q_1, q_2 form the ortogonal coordinate system then the initial cross sectional area of an elementary ray tube may be expressed as $d\,S_1 = d\,q_1 \cdot d\,q_2$. Let us assume that the second cross section outstanding from the scattering volume at a distance much more great than characteristic dimensions of this volume. Then the elementary area of the second cross section may be found out if the solution of system (8) is known. The value of effective area of backward scattering can be expressed as

$$\sigma = \frac{4\pi}{\sqrt{\begin{vmatrix}\Psi_{11} & \Psi_{12}\\ \Psi_{21} & \Psi_{22}\end{vmatrix}^2 + \begin{vmatrix}\Psi_{21} & \Psi_{22}\\ \Psi_{31} & \Psi_{32}\end{vmatrix}^2 + \begin{vmatrix}\Psi_{31} & \Psi_{32}\\ \Psi_{11} & \Psi_{12}\end{vmatrix}^2}} \tag{16}$$

Caustic surface.

Using the known solution of system (8) one may obtain the equation of caustic surface. The caustic of ray set (4) is its envelope. To calculate the envelope of family of surfaces

$$f(x_i ; q) = 0 \tag{17}$$

depending on the single parameter q the latter must be excluded from Eq.(16) using condition

$$\frac{d\,f(x_i ; q)}{d\,q} = 0 \tag{18}$$

Expression (4) defines the general solution of system (2), (3) under the assuming that q_1^0 and q_2^0 are variables. Then it may be shown that equations (4) determine the uniparametric set of surfaces (17). To get to this end the values of q_1^0 and q_2^0 may be derived from the former two equations (4) (then these values occurs to be functions of spatial coordinates and parameter s) and substituted in the third equation (4). Differentiation of the obtained correlation with respect to s gives the expression similar to Eq. (18). Differentiation of correlations (4) with respect to parameter s while q_1^0 and q_2^0 depends on s results in system of equations

$$\frac{d\,x_i}{d\,s} + \frac{d\,x_i}{d\,q_\kappa^0}\cdot\frac{d\,q_\kappa^0}{d\,s} = 0 \tag{19}$$

This system is compatible if its determinant is equal to zero. Taking into account (5), (9) one may obtain

$$\begin{vmatrix}\Phi_1^0 & \xi_{11} & \xi_{12}\\ \Phi_2^0 & \xi_{21} & \xi_{22}\\ \Phi_3^0 & \xi_{31} & \xi_{32}\end{vmatrix} = 0 \tag{20}$$

Using this expression for calculation of parameter s and excluding it from Eqs.(4) we find the caustic surface equation in parametric form. If the parameters q_1^0 and q_2^0 are fixed then equation (20)

defines a touch point on the caustic surface for the reference ray.

The combined solution of equations (2), (3) for calculation of the reference ray and Eq. (8) for its variations allows to save run time and increase accuracy of calculation comparing to the ordinary algorithm based on step by step solution of systems (2), (3) with variation of the initial data. In this case the required calculation accuracy is achieved using method of proper chaise of initial data variation i.e. in the way of solution the same equations (2), (3). Since on arbitrary chosen ray may serve as reference one, the equation (16) allows to calculate the angular dependence of scattered signal intensity in the region of observation.

Samples of realization and discussion.

The methodology presented in previous issues is use at the elaborating set of PC codes for numerical calculation of the principal scattering characteristics of exhaust plasma plumes namely ray path patterns, spatial intensity distribution in the observation region, caustic surfaces and radio shadow regions geometries for the mathematical models of plasma expansion in space presented in Ref.[1].

The developed PC codes being realized on Turbo Pascal and optimized as to criteria of minimal running time and required accuracy. The resultant PC software has been successfully used for prediction and analysis of the space experiment results.

The typical ray path patterns and spatial signal intensity distributions including interference patterns at the radio shadow boundaries for the space experiment geometry are presented in Fig. 4 and 5 of Ref.[1] where the experiment results are discussed in details. It is shown that computed and experimental data are in a good agreement.

Fig. 1a,b shows typical ray path pattern for the single plasma spoke and for intensity distribution of scattered radiation for lattice of spokes arising as a result of plasma impulse thruster operation in space. It should be noted that in this case the plasmoid does not contain overdense core.

Fig. 2a shows the ray path pattern for flat wave incident at low angle relative to plane the symmetry of petal shaped plasma plume. Note that the radiation scattering results in formation of the region of lowered intensity. The intensity distributions are shown in Fig. 2b. It should be noticed that dimension of this lowered intensity region is much greater than the characteristic size of overdense plasma flow core. This suggests that communications with spacecraft should be essentially suppressed if its receiving antennas fall down in the region of lowered intensity which may be treated as the zone of "half-shadow". It also means that even rather rarefied exhaust plasma plumes and beams may effectively scatter radiation in practically important frequency ranges under certain conditions.

The caustic's shapes, arising when spherical wave is scattered on axis-symmetrical plasma plume, are shown on Fig. 3: a) the along section caustic's surface, b) cross section. The ray path pattern and angular intensity distribution of field, when spherical wave on petal shaped plasma plume is scattered, are shown on Fig. 4.

Fig. 5a shows the petal shaped plasma plume flattened by the Earth magnetic field B action. The vector B is directed along OZ axis. For this geometry Fig. 5b shows the curves of equal values of effective area of backward scattering (EABS) in direction cosines coordinates. Parameters of circles correspond to the values of $\cos\Theta_y$ and indexes at curves denote EABS values as $lg\sigma$. Values of σ are normalized relative to square of wave number ($\lambda^2/4\pi^2$). Fig. 5b demonstrates that EABS is extremely low for small angles of radiation incidence relative to plane of symmetry of plasma petal.

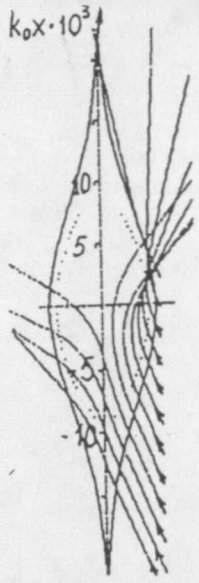
On the other hand at the certain angles of radiation incidence the "mirror" regions appear on flat surface of the plasma petal i.e. the latter may serve as an effective reflector.

This reflection effect was checked and reveal in the space experiments with arcjet EPICURE as the observed fact of communications restoration within intervals of plasma injection when the satellite was flying on local radio horizon i.e. out of signal range.

In general the developed codes occurred to be rather effective and convenient at work by tool for calculations of radiation refraction / scattering effect in plasma.

Conclusion

1. Linearization of eiconal characteristic system of equations for thin ray tubes within the frames of geometric optics allowed to develop effective algorithms and PC codes for calculation of the main scattering characteristics of exhaust plasma plumes - ray path patterns, spatial intensity distributions, caustic surfaces geometries and effective areas of backward scattering.



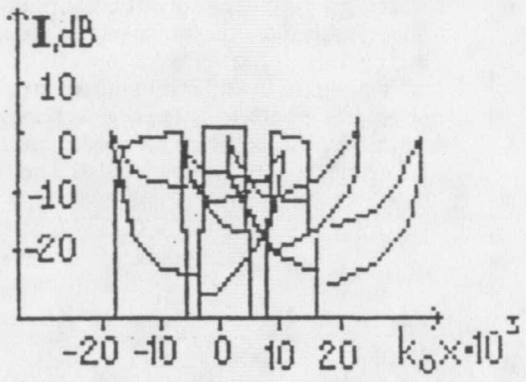
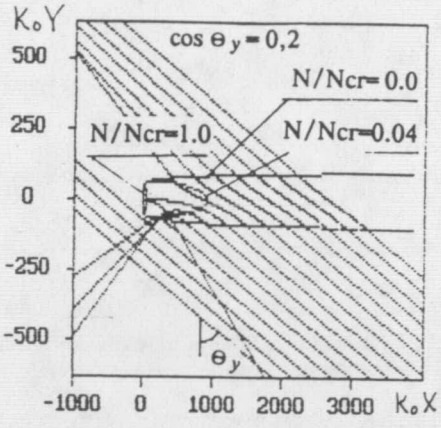


Fig.1

a) Plane wave scattering in plasmoid

b) Intensity distribution of field at lattice of plasmoids produced by impulce plasma thruter



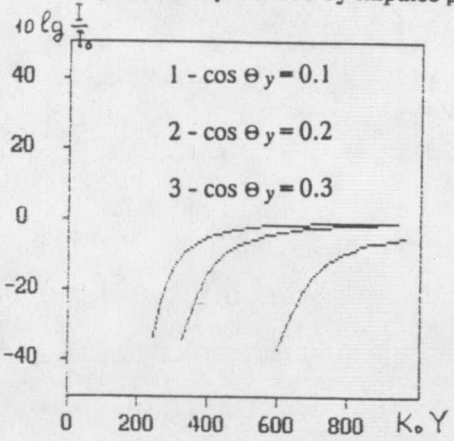
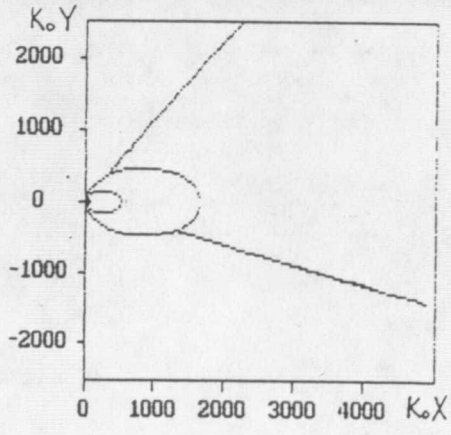


Fig.2

a) Ray path pattern for plane wave scattered in plasma petal

b) Intensity distributions for low angles of inidence



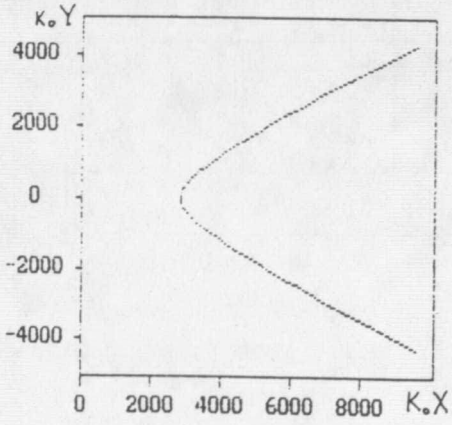


Fig. 3.

a) Along section caustic's surface

b) Cross section caustic's surface

2. The developed set of PC codes together with mathematical models of plasma expansion in space can be used as the software for refraction / scattering effects evolution and communications conditions prognosis for electrically propelled spacecrafts. The validity of this software had been confirmed in space experiments with arcjet EPICURE on Cosmos satellites.

3. The analysis of available refraction / scattering calculation results suggests that even rather rarefied exhaust plasma plumes and beams can effectively scatter radiation with correspondent impacts (under certain conditions) to communication. This means that refraction / scattering effects are the important aspect of electromagnetic compatibility for electric thrusters.

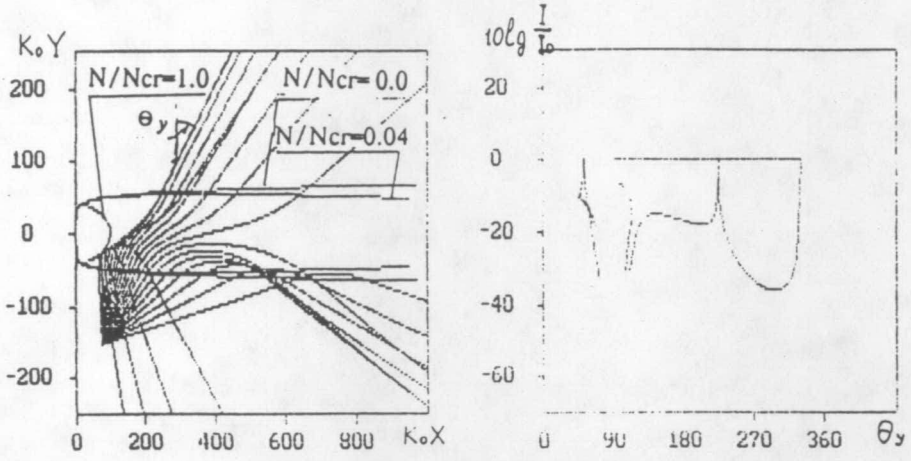


Fig 4.

a) Ray path pattern for spherical wave scattered in plasma petal

b) Angular intensity distribution for upper case

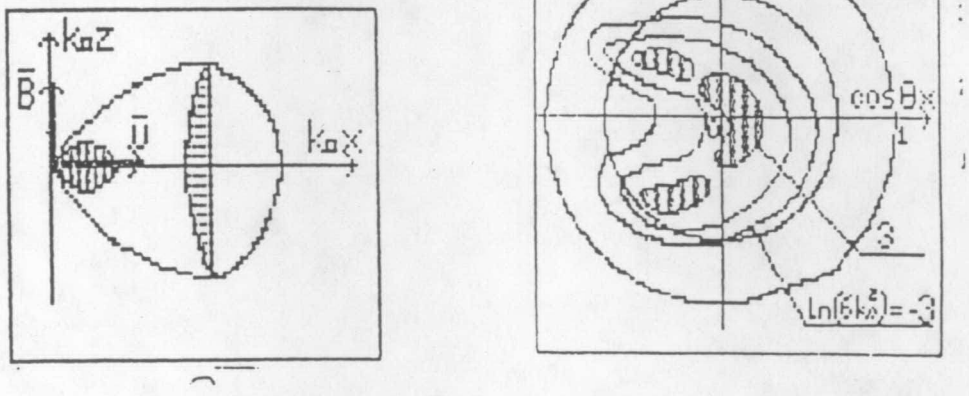


Fig.5

a) Computed shape of plasma petal

b) Angular dependense of effective area of backward scattering in direction cosines coordinates

Reference

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