IEPC-95-207

THE MODIFIED BALANCE METHOD OF NEAR-CATHODE PROCESSES CALCULATION CHARACTERISTICS

Y.V.BRUEVICH*, I.P.NASARENKO*, I.G.PANEVIN*

Abstract

The modified balance method of calculation such near-cathode processes characteristics as the cathode potential drop U_c and the heat flux q_0 which transfers from plasma to cathode surface is discussed. This method which keeps the small expenditure on calculation procedure as in usual balance method takes into account the peculiarities revealed with help of exact calculation method which is founded on the differential equations system solution. It is carry out the comparison of the values U_c and q_0 calculated by exact and modified balance methods for arc discharge burning in argon ($P=10^5$ Pa) on the tungsten cathode. It is shown that the calculation results coincide reasonably and the modified balance method may be used for the calculation such characteristics of near-cathode processes as U_c and q_0.

Introduction

The arcjet thrusters and MPD-thrusters lifetime is defined in many cases the cathode efficiency which depends from the thermal condition and the kind of the plasma contact with its surface (the spot or the diffuse discharge). In the diffuse discharge case the cathode thermal condition is connected with its geometry, the cooling wise and the plasma parameters in the near-cathode region of arc discharge. Therefore the problem of plasma parameters definition in the arc near-cathode region with diffuse discharge presents the obvious interest.

The physical model of arc near-cathode region.

Mostly the two-layer model of arc near-cathode region [1-3] is used for high pressure discharge ($P \sim 10^5$ Pa). This model as it is shown in fig.1 includes the collisionalless layer I and the collisional layer II . The collisionalless layer (the space charge layer) contacts with the cathode surface. Usually it is assumed that its thickness δ_0 is smaller than the electron free path length and the interactions between plasma particles within layer I are absent. Moreover it is considered that in the layer I the positive volume charge is concentrated in the main and the great part of the cathode potential drop U_c is realised. The electrical field which exists in the layer I accelerates the emission electrons moving from the cathode surface toward the discharge plasma, and also the plasma ions moving to the cathode surface. But the electric field in this layer brakes

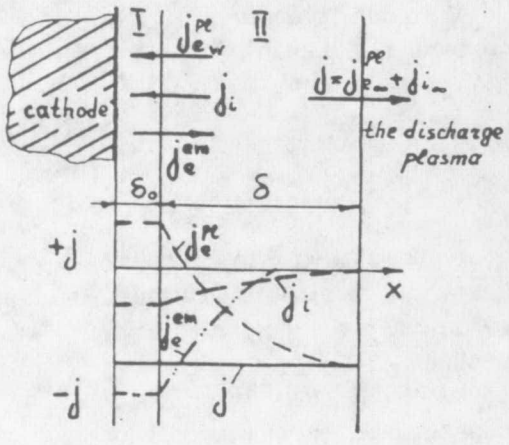

fig 1

* Moscow Aviation Institute, Moscow, Russia

the plasma electrons which direct to the cathode surface, and only the fast electrons which energy is enough to get over the braking barrier U_c are able to reach the cathode surface. The total current density $j = j_e^{em} + j_i + j_e^{pl}$ and its components (the plasma ion current density j_i, the plasma electron current density j_e^{pl} and the emission electron current density j_e^{em}) are invariable within the collisionalless layer. The emission electrons which are accelerated in layer I have on the boundary between layers I and II the energy $(eU_c + \frac{5}{2}KT_w)$. Here T_w - the cathode surface temperature, K - Boltzmann's constant, e - the electron charge. As for the arc discharge in the arcjet $eU_c \gg KT_w$, it is possible to consider the emission electrons as a monoenergetic stream with energy eU_c. This stream brings in the collisional layer the impulse stream $n_e^{em} \cdot V_e^{em} \cdot m_e V_e^{em}$ and the energy stream $n_e^{em} \cdot V_e^{em} \cdot \frac{m_e V_e^{em^2}}{2}$, where n_e^{em} and V_e^{em} - the emission electrons density and their velocity on the boundary between layers I and II, m_e - the electron mass. These impulse and energy streams pass to the plasma particles in the layer II.

The collisional (ionisation) layer thickness δ is much bigger than the free path length of the plasma particles. Within the layer II the ion current forms, the emission electron stream relaxes and the heavy particles temperature changes from the temperature T_w on the boundary with layer I to the temperature value upon the electron temperature on the boundary with discharge plasma $T_{e\infty}$. Because of the difference between the heavy particles and electron temperatures and also because of the ionisation equilibrium displacement the plasma composition in the layer II differs from the equilibrium composition.

The total current density j also is invariable within the collisional layer. However as it is followed from fig.1 the current densities j_e^{em} and j_i within the layer II change their values. and the current density j_e^{pl} changes the value and the sign: on the boundary with layer I the current density j_e^{pl} is provided with the fast plasma electrons and is directed to the cathode surface, on the boundary with discharge plasma the current density j_e^{pl} is caused by the motion of all plasma electrons and is directed to the discharge plasma.

The energy losses with the streams of the plasma particles which leave the layer II through the boundaries are compensated by the energy supply with the stream of emission electrons.

Mostly it is used two calculation method for the definition of the near-cathode region characteristics. The first method which we shall call the exact method founds on the differential equations system solution and allows to receive the information about the cathode potential drop U_c, the heat flux q_o which passes from the near-cathode plasma to the cathode surface and also the information about the distributions over the collisional layer thickness the temperatures and the concentrations of the particles, the electric field strength E, the plasma potential φ and so on.

The second method which is used for the estimation of U_c and q_o values proceeds from the integral balance correlations. This method we shall call the balance method. Let us dwell on these methods essence in detail.

The exact calculation method.

As the collisional layer thickness δ in the diffuse discharge is much smaller than the radial dimension of the discharge, the equations system for the layer II is written in the onedimensional approach. Besides it is suggested that the emission electrons current density may be written with the help of Richardson-Deshman's equation with the Shottky's correction.

The equations system which is taken into account the processes of relaxation of emission electrons stream, the ion current forming, the ionisation equilibrium displacement and so on was suggested in [3]

$$\frac{5}{2}Kn_eU\frac{dT_e}{dx} + \frac{5}{2}Kn_eV_e\frac{dT_e}{dx} = \frac{d}{dx}\left(\lambda_e\frac{dT_e}{dx}\right) + j_e^{pl}\cdot E - \dot{n}_e\left(e\chi_i + \frac{5}{2}KT_e\right) - W_e - \\ - \frac{5}{2}KT_e\cdot\frac{dj_e^{em}}{dx} + \frac{d(j_e^{em}\varepsilon)}{dx}, \qquad (1)$$

$$\frac{5}{2}K\left[n_a(u+V_a)+n_i(u+V_i)\right]\frac{dT}{dx} = \frac{d}{dx}\left(\lambda\frac{dT}{dx}\right)+W_e+j_iE\ , \qquad (2)$$

$$W_e = 3\frac{m_e}{M_i}Kn_e\left(\nu_{ea}+\nu_{ei}\right)\left(T_e-T\right)\ , \qquad (3)$$

$$-en_eE-\frac{dP_e}{dx}-\frac{m_en_e\nu_{ea}}{K^{\sigma}_{add}\cdot K^{\sigma}_{ea}}\left(V_e-V_a\right)-\frac{m_en_e\nu_{ei}}{K^{\sigma}_{add}\cdot K^{\sigma}_{ei}}\left(V_e-V_i\right)=0\ , \qquad (4)$$

$$en_iE-\frac{dP_i}{dx}+\frac{m_en_e\nu_{ei}}{K^{\sigma}_{add}\cdot K^{\sigma}_{ei}}\left(V_e-V_i\right)-\frac{M}{2}n_i\nu_{ia}\left(V_i-V_a\right)=0\ , \qquad (5)$$

$$m_en_eV_e+M_in_iV_i+M_an_aV_a=0\ , \qquad (6)$$

$$n_e=n_i\ ;\ \left(n^{em}_e\ll n_e\right), \qquad (7) \qquad P=K\left(n_eT_e+n_iT+n_aT\right)=const, \qquad (8)$$

$$P=Kn_eT_e\ , \qquad (9) \qquad P_i=Kn_iT\ , \qquad (10)$$

$$\dot{n}_e=\beta(T_e)n_en_a-\alpha(T_e)n_e^3\ , \qquad (11)$$

$$\dot{n}_e=\frac{1}{e}\frac{dj_i}{dx}=\frac{d(n_iV_i)}{dx}\ , \qquad (12)$$

$$j^{pl}_e=-en_eV_e\ , \qquad (13) \qquad j_i=en_iV_i\ , \qquad (14)$$

$$j^{em}_e=-en_eV^{em}_e\ , \qquad (15) \qquad j=j^{pl}_e+j+j=const\ , \qquad (16)$$

$$j^{em}_e=j^{em}_w\cdot\exp\left[-\int_0^x\left(n_a\langle Q^{1,1}_{ea}\rangle+n_i\langle Q^{1,1}_{ei}\rangle+\frac{1}{2}n_e\langle Q^{1,1}_{ee}\rangle\right)dx\right]\ , \qquad (17)$$

$$\rho^{pl}=m_en_e+M_in_i+M_an_a\ , \qquad (18)$$

$$d\left(\rho^{pl}\cdot u^2\right)=d\left(m_en^{em}_e\cdot V^{em^2}_e\right)\ , \qquad (19)$$

$$E=-\frac{d\varphi}{dx}\ , \qquad (20) \qquad \frac{d\varepsilon}{dx}=\frac{1}{2}\left(\varepsilon-\frac{3}{2}KT_e\right)n_e\langle Q^{1,1}_{ee}\rangle, \qquad (21)$$

$$\frac{dV^{em}_e}{dx}=\frac{1}{2}\left(V^{em}_e-V_e\right)n_e\langle Q^{1,1}_{ee}\rangle+\left(V^{em}_e-V_i\right)n_i\langle Q^{1,1}_{ei}\rangle\ , \qquad (22)$$

Here T_e and T - the electron and heavy particles temperatures; λ_e and λ - the electron and heavy particles heat conductivities; n_e . n_i . n_a - the electron, ion and atom concentration; n^{em}_e - the emission electrons density; V_e . V_i . V_a - the directed velocities of the electron. ion and atom motion; V^{em}_e - the emission electron velocity; $\varepsilon=\frac{m_eV^{em^2}_e}{2}$ - the kinetic energy of emission electrons; M_i and M_a - the atom and ion masses; φ - the plasma electric potential; χ_i - the ionisation potential; U - the directed velocity of motion of plasma as whole in the layer II because the transfer of the emission electrons impulse stream to the plasma particles; $\dot{n}_e$ - the net electron (ion) generation rate; ν_{ea} . ν_{ei} . ν_{ia} - the frequencies of electron - atom, electron - ion and ion - atom elastic collisions; $\langle Q^{1,1}_{ea}\rangle$. $\langle Q^{1,1}_{ei}\rangle$. $\langle Q^{1,1}_{ee}\rangle$ - the average cross sections of impulse transfer by elastic collisions of particles; $\beta(T_e)$ and $\alpha(T_e)$ - the ionisation and recombination constant; K^{σ}_{ea} . K^{σ}_{ei} . K^{σ}_{add} - kinetic coefficients; P_e . P_i - the partial pressures of electrons and ions.

The influence of the collisionalless layer on the system of equations (1)-(22) is taken into account in the boundary conditions: as the cathode potential drop it is understood the potential drop in the layer I - U_c. Besides because of the small thickness of the collisionalless layer δ_0 the parameters on the boundary between the layers I and II and the cathode surface are identified.

The boundary conditions for the system (1)-(22) are written the next manner:

a) on the boundary between the layers I and II (x=0)

$$T = T_w ; \quad \varphi = U_c ; \quad U = 0 ; \quad \varepsilon = e U_c ; \quad j_i = j_{iw} ;$$
$$\frac{dT}{dx} = \frac{dT}{dx}\Big|_w ; \quad \lambda_e \frac{dT_e}{dx}\Big|_w = -j_e^{pe}\Big|_w \cdot U_c ;$$
$$j_e^{pe}\Big|_w = j_{e_{ch}} \cdot \exp\left(-\frac{eU_c}{KT_{e_w}}\right) ; \quad j_{e_w}^{em} = AT_w^2 \exp\left(-\frac{e\varphi_{ef}}{KT_w}\right) ; \qquad (23)$$

b) on the boundary with the discharge plasma (x=δ)

$$j_e^{em} = 0 ; \quad \frac{dT_e}{dx}\Big|_\infty = \frac{dT}{dx}\Big|_\infty = C_1 ; \quad \frac{dn_e}{dx}\Big|_\infty = C_2 \qquad (24)$$

Here A - the Richardson's constant, φ_{ef} - the effective material work function, subscriptions "w" and "∞" - mean the conditions on the boundary between the layers I and II and on the boundary with the discharge plasma accordingly.

The value of the heat flux q_0 which is passed into the cathode body is defined from the energy balance on the cathode surface

$$j_e^{pe}\Big|_w \cdot \left(\frac{5}{2}\frac{KT_{e_w}}{e} + \varphi_{ef}\right) + j_{iw}\left(U_c + \gamma_i - \varphi_{ef}\right) + \lambda \frac{dT}{dx} = q_0 + \varepsilon_w \sigma T_w^4 + j_e^{em}\left(\frac{5}{2}\frac{KT_e}{e} + \varphi_{ef}\right), \qquad (25)$$

where ε_w - the blackness degree of the cathode surface, σ - the Stephan - Boltzmann's constant.

As it is shown in [3] the solution of the equations system (1)-(22) is the enough labour-consuming procedure and requires the large expenditure of the calculation time. Therefore the set of the received solutions is limited: it was calculated the near-cathode processes characteristics for the arc discharge burning in argon ($P=10^5$ Pa) on the tungsten cathode for the difference values of T_{e_w} when the values T_w or q_0 remain invariable.

The balance calculation method.

The balance calculation method which is used usually is estimational and is not taken into account all completeness of the collisional layer. In this method it is assumed that the electron and heavy particles temperatures have the constant value in layer II. Besides it is suggested that the energy flux $j_e^{em} \cdot U_c$ carried in the layer II by the emission electrons is spent on the compensation of energy losses with the ion current $j_{iw}\left(\gamma_i + \frac{5}{2}\frac{KT_w}{e}\right)$, with the fast plasma electrons current $j_{e_w}^{pe}\left(U_c + \frac{5}{2}\frac{KT_e}{e}\right)$ and with total current on the boundary with the discharge plasma $j \cdot \frac{5}{2}\frac{KT_e}{e}$. The equations system in the balance method which is used usually has the next form [4]

$$j = j_e^{em} + j_i + j_{e_w}^{pe} , \quad (26) \qquad j_e^{em} = AT_w^2 \cdot \exp\left(-\frac{e\varphi_{ef}}{KT_w}\right) , \quad (27)$$
$$\varphi_{ef} = \varphi_w - \Delta\varphi , \quad (28) \qquad \Delta\varphi = 3{,}75 \cdot 10^{-4} E_w^{1/2} , \quad (29)$$

$$E_w = 5700\,(A_i)^{1/4}\,U_c^{1/4}\,j_i^{1/2}\,, \qquad (30)$$

$$j_{e_w}^{pe} = j_{e_{ch}} \cdot \exp\left(-\frac{eU_c}{KT_{e_w}}\right) = \frac{en_e\bar{v}_e}{4}\exp\left(-\frac{eU_c}{KT_{e_w}}\right), \qquad (31)$$

$$j_i = j_{i_{ch}} = \frac{en_i\bar{v}_i}{4}\,, \qquad (32)$$

$$p = K\left(n_eT_e + n_iT + n_aT_a\right)\,, \qquad (33)$$

$$n_e = n_i\,, \quad (34) \qquad \beta(T_e)\,n_e n_a = \alpha(T_e)\,n_e^3 \qquad (35)$$

where A_i - the atom weight. $\bar{v}_e = \sqrt{\frac{8KT_e}{\pi m_e}}$. $\bar{v}_i = \sqrt{\frac{8KT}{\pi m_i}}$. φ_w - the cathode material work function.

The energy balance equations on the cathode surface and for the collisional layer as a whole are solved in common with the equations (26)-(35)

$$q_o + \varepsilon_w\sigma T_w^4 + j_e^{em}\left(\frac{5}{2}\frac{KT_w}{e} + \varphi_{ef}\right) = j_i\left(U_c + \chi_i - \varphi_{ef}\right) + j_e^{pe}\frac{5}{2}\frac{KT_e}{e}\,, \qquad (36)$$

$$j_e^{em}\left(U_c + \frac{5}{2}\frac{KT_w}{e}\right) = j_i\left(\chi_i + \frac{5}{2}\frac{KT}{e}\right) + j_e^{pe}\left(U_c + \frac{5}{2}\frac{KT_e}{e}\right) + j\cdot\frac{5}{2}\frac{KT_e}{e}\,, \qquad (37)$$

The equations system (26)-(37) was solved for such conditions as a system (1)-(22). The comparison of the calculation results which was obtained the exact and balance methods in [3] showed that the cathode potential drop values U_c calculated with the help of the balance method exceed on 1-2 V the values of the cathode potential drop which was defined by the exact method. The difference of the values of the heat flux q_o which is passed into the cathode body is more essential. The values q_o which was defined by the balance method can be twice as much than the values q_o calculated by the exact method.

Since the balance method possesses essential lesser labour-consuming than the exact method. it is imagined expedient the modification of this method by which it is necessary to remain small labour-consuming of calculation procedure and to rise the precision of the values U_c and q_o determination.

The modified balance method.

The considerable difference for the values q_o calculated by the different methods which was peculiarities set in the collisional layer of near-cathode region of the arc discharge which is not taken account by the balance method calculations. For the arc discharge burning in argon ($P \sim 10^5$ Pa) on the tungsten cathode these peculiarities may be written in the following way:

1) the electron temperature near the cathode surface T_{e_w} exceeds the electron temperature on the discharge plasma boundary T_{e_∞} approximately by 2000 K;

2) the heavy particles temperature T near the cathode surface is equal to the cathode surface temperature T_w and on the discharge plasma boundary $T \approx T_{e_\infty}$:

3) the heat flux $\lambda\frac{dT}{dx}$ which is determinated by the heavy particles heat conductivity changes very weakly and is equal approximately 4×10^3 W/cm^2;

4) the current density on the cathode surface j_{i_w} is defined by the diffusion in the near-cathode layer and may by written as $j_{i_w} = B(T_{e_w}) \cdot j_{i_{ch}}$ where $B(T_e)$ - the coefficient which depends on temperature T_{e_w}. $j_{i_{ch}} = \frac{e n_i \bar{v}_i}{4}$ - the chaotic ion current density;

5) the near-cathode plasma composition is non-equilibrium, the net electron generation rate $\dot{n}_e$ values by the temperatures T_{e_w} and T_{e_∞} make up about 0,95 from value $\dot{n}_{e\,max}$ which is the maximum value by the corresponding temperatures T and T_e ;

6) under the emission electrons action the plasma motion as a whole in the collisional layer is arised towards the discharge plasma and through the boundary with the discharge plasma the energy flux $U \cdot \frac{5}{2} K T_e (n_{e_\infty} + n_{i_\infty} + n_a)$ goes out.

With account of these peculiarities the equations system for the modified balance method has the following form:

$$j = j_e^{em} + j_{i_w} + j_{e_w}^{pl}, \tag{38}$$

$$j_e^{em} = A T_w^2 \exp\left(-\frac{e\varphi_{ef}}{K T_w}\right), \tag{39}$$

$$\varphi_{ef} = \varphi_w - \Delta\varphi, \tag{40}$$

$$\Delta\varphi = 3{,}75 \cdot 10^{-4} \cdot E_w^{1/4}, \tag{41}$$

$$E_w = 5700\,(A_i)^{1/4} \cdot U_c^{1/4} \cdot j_i^{1/2}, \tag{42}$$

$$j_{e_w}^{pl} = j_{e_{ch}} \cdot \exp\left(-\frac{eU_c}{K T_{e_w}}\right) = \frac{e n_e \bar{v}_e}{4} \exp\left(-\frac{eU_c}{K T_{e_w}}\right), \tag{43}$$

$$j_{i_w} = B(T_{e_w}) \cdot j_{i_{ch}} = \frac{e n_i \bar{v}_i}{4} \cdot B(T_e), \tag{44}$$

$$P = K(n_e T_e + n_i T + n_a T_a)_w = K(n_e T_e + n_i T + n_a T_a)_\infty, \tag{45}$$

$$T_\infty = T_{e_\infty}, \tag{46}$$

$$n_e = n_i, \tag{47}$$

$$T_{e_\infty} = T_{e_w} - 2000, \tag{48}$$

$$\dot{n}_e = \beta n_e n_a - \alpha n_e^3, \tag{49}$$

$$\dot{n}_e = 0{,}95 \cdot \dot{n}_{e\,max}, \tag{50}$$

$$U = \sqrt{\frac{2eU_c}{m_e}} \cdot \sqrt{\frac{m_e n_e^{em}}{(m_e n_{e_\infty} + M_i n_{i_\infty} + M_a n_{a_\infty})}}, \tag{51}$$

The energy balance equations in the modified balance method may be written as

$$q_0 + \varepsilon_w \sigma T_w^4 + j_e^{em}\left(\frac{5}{2}\frac{K T_w}{e} + \varphi_{ef}\right) = \lambda \frac{dT}{dx} + j_{i_w}\left(U_c + \chi_i - \varphi_{ef} + \frac{5}{2}\frac{K T_w}{e}\right) + j_{e_w}^{pl}\left(\frac{5}{2}\frac{K T_{e_w}}{e} + \varphi_{ef}\right), \tag{52}$$

$$j_e^{em}\left(U_c + \frac{5}{2}\frac{K T_w}{e}\right) = \lambda \frac{dT}{dx} + j_{e_w}^{pl}\left(U_c + \frac{5}{2}\frac{K T_{e_w}}{e}\right) + j_{i_w}\left(\chi_i + \frac{5}{2}\frac{K T_w}{e}\right) + j\,\frac{5}{2}\frac{K T_{e_\infty}}{e} + U \cdot \frac{5}{2} K T_\infty (n_{e_\infty} + n_{i_\infty} + n_{a_\infty}), \tag{53}$$

The results of equations system (38)-(53) calculations for the arc discharge burning in argon ($P = 10^5$ Pa) on the tungsten cathode are presented on fig 2 and fig 3.

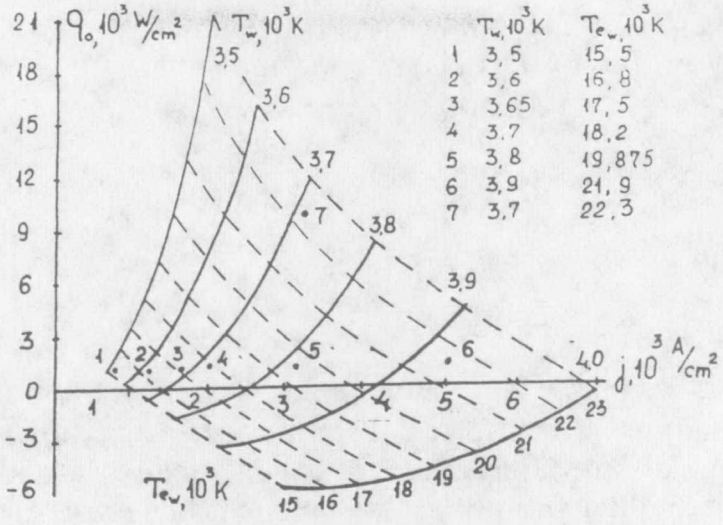

Fig 2

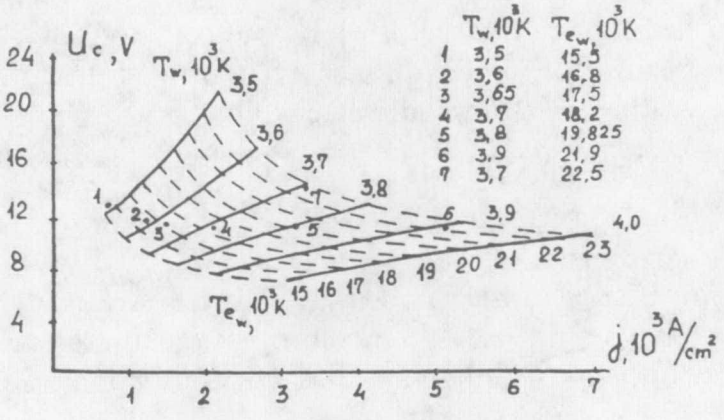

Fig 3

By the calculations the diapason of change of the cathode surface temperature T_w maked up from 3500 K to 4000 K, of the near cathode electron temperature T_{ew} maked up from 15000 K to 23000 K. On fig.2 and fig.3 the dependencies of the cathode potential drop U_c and the heat flux q_0 from the total current density j are presented.

Here it is shown the influence on the values U_c and q_0 such parameters as the temperature T_w and T_{ew}. On fig.2 and fig.3 for the comparison the values U_c and q_0 which were calculated by the exact method are shown. The sufficiently correspondence of the results which were received by the exact and modified balance methods allows to apply this modified balance method for the calculation of the argon near-cathode region characteristics.

The application of the modified balance method for other P, T, T_e diapasons in argon or the arc discharge burning in other gases requires the additional calculations of the exact method.

References

1. T.H.Lee, A.N. Greenwood and W.D.Breingan, "A self consistent model for the cathode region of a high pressure arc". Proc. 7th Intern. Conf. Phenomena Ionized Gases. Beograd.1965. p.670-680

2. K.C. Hsu and E. Pfender, "Analysis of the cathode region on a free-burning high intensity argon arc". J.Appl.Phys.,vol.54,N7,1983,p.1769-1777.

3. А.М.Зимин. И.П.Назаренко. И.Г.Паневин. В.И.Хвесюк. Математическое моделирование катодных процессов. Новосибирск: ВО "Наука", 1993.

4. Приэлектродные процессы и эрозия электродов плазмотронов / Под ред. М.Ф.Жукова. Новосибирск, 1977.