ELECTRON-CURRENT GAIN IN DISCHARGE IN A TRANSVERSE MAGNETIC FIELD

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The Townsend discharge in which ionization electron multiplication occurs is well known. In this, the electron current exponentially grows from the cathode to the anode and the relation between the anode current j_a and the cathode current j_k is described by the formula:

$$j_a = j_k \exp(L) \tag{1}$$

$$L = \int_0^l (\nu_i / \upsilon_x) dx = \int_{\varphi_k}^{\varphi_a} \frac{d\varphi}{\varepsilon} \tag{2}$$

(ν_i / υ_x) is the first Townsend coefficient (α);

ν_i is the ionization frequency;

υ_x is the mean electron velocity along the x-axis (in particular, $\upsilon_x = b_e \cdot d\varphi / dx$, b_e is the mobility);

$\varepsilon = (\upsilon_x \cdot d\varphi / dx) / \nu_i$ is the ionization value, i.e., the potential difference an electron passes through between two ionization events);

l is the discharge-gap length;

φ_k, φ_a are the plasma potentials near the cathode and near the anode, respectively.

In the general case, the cathode current j_k is equal to the sum of the external electron current j_0 (e.g., the thermoionic current) and the secondary-emission current under the action of ion bombardment:

$$j_k = j_0 + \Gamma j_k (\exp L - 1) \tag{3}$$

The second term in (3) determines the secondary-electron current; Γ is the effective secondary-emission coefficient. In the self-maintained discharge $j_0=0$ and the gain

$$K_0 = j_a/j_k = \exp L_0 = (1 + \Gamma)/\Gamma \tag{4}$$

For the non-self-maintained discharge with $j_k=Nj_0$, from (3) it follows that

$$K = j_a/j_0 = [N(1+\Gamma) - 1]/\Gamma \tag{5}$$

With N=2, we obtain $K = (1+2\Gamma)/\Gamma$. Therefore, with $\Gamma \ll 1$, the admissible gain of the non-self-maintained discharge (K) controlled by the external current j_0 practically coincides with the gain of the self-maintained discharge (K_0).

A magnetic field parallel to the cathode surface makes all the γ-electrons (i.e., the secondary electrons) which have not experienced gas-atom scattering go back to the cathode and thus provides a many-fold decrease of the secondary-electron emission so that $\Gamma = \gamma \cdot W \ll \gamma$, where γ is the secondary-electron emission factor under the action of ions, W is the probability of γ-electron capture in the plasma due to scattering over its path length equal toapproximately one-half the Larmour circumference. Thus, the feasibility of conditions with high gain values ($K \approx 1/\Gamma$) controlled by modulating very weak external current j_0 is an attractive feature of discharge in the transverse magnetic field.

The discharge-current density is bounded from above either by the space charge or by the gas-atom flow in the discharge bulk. (It is evident that the ion flow $j_1 = Pq_0 \le q_0$, q_0 is the gas flow, P is the ionization probability). According to numerous measurements [1, 2], at a pressure of 10^{-3} to 10^{-2} torr, the ion flow is practically equal to the gas flow, and hence, $P \le 0.9$. In this, the ion-current density (or the discharge-current density) is equal to 0.1 to 1 A/cm^2 and no restrictions imposed by the space discharge are observed, which suggests that the major part of the discharge bulk is in the quasi-neutral state.

The above features of the discharge in the transverse magnetic field (high gain and quasi-neutral state) can serve as a basis for the development of instruments of a new type for communication and modulation of sufficiently high currents. Therefore, evaluation of the gain of the self-maintained discharge $K_0 \approx \Gamma^{-1} = (\gamma W)^{-1}$ which reduces to the calculation of the γ-electron capture probability W in the plasma due to elastic scattering is of interest.

1. DISCHARGE WITH A TRUNCATED ANODE LAYER.

While passing through the cathode layer with the voltage drop φ_k, a secondary electron with the initial energy T_γ has the kinetic energy $\varphi_k + T_\gamma$ and its path in the plasma is equal to one-half the length of the Larmour circumference $\pi \cdot \rho_e(\varphi_k + T_\gamma)$. With $T_\gamma / \varphi_k \ll 1$, the scattered-electron capture condition has the following form:

$$\Theta^2 \ge \Theta^2_{min} \approx T_\gamma / \varphi_k \qquad (6)$$

where Θ is the angle of electron departure from the normal towards the cathode surface. For the Gaussian angular scattered-electron distribution,

$$f(\Theta)d\Theta^2 = d\Theta / \bar{\Theta}^2 \cdot \exp(-\Theta / \bar{\Theta}^2) \qquad (7)$$

the probability of scattering at the angle $\Theta > \Theta_{min}$ is equal to:

$$W(\Theta_{min}) \approx \int_{\Theta}^{\infty} f(\Theta)d\Theta^2 = \exp\left(\frac{-T_\gamma}{\varphi_k \bar{\Theta}^2}\right) \qquad (8)$$

where

$$\bar{\Theta}^2 = 2 \cdot n_g \cdot \sigma_1 \cdot \pi \cdot \rho_e(\varphi_k) \qquad (9)$$

is the mean square value of the scattering angle over the path $\pi \cdot \rho_e(\varphi_k)$; σ_1 is the transport section; n_g is the gas concentration.

The effective secondary-electron emission factor

$$\Gamma = \gamma \cdot (1 + \beta \cdot \varphi_k / I) \cdot W_p \cdot W(\Theta_{min})$$

The multiplier $(1 + \beta \cdot \varphi_k / I)$ takes into account that with $\beta \cdot \varphi_k / I > 1$ ($\beta < 1$, I is the ionization potential), a γ-electron captured by the plasma executes several ionization event in the process of relaxation. For the sake of simplicity, let us assume that $(1 + \beta \cdot \varphi_k / I) \approx 2$. The third multiplier determines the probability of scattering at any angle $0 \le \Theta \le \pi$: $W_p = n_g \cdot \sigma_1 \cdot \pi \cdot \rho_e(\varphi_k)$. Thus, we obtain

$$\Gamma = \gamma \dot{\Theta}^{2} \exp\left(\frac{-T_{\gamma}}{\varphi_k \bar{\Theta}^{2}}\right) \qquad (10)$$

If it is assumed that T=3V, $\gamma_e = 5 \cdot 10^{-2}$ (the potential secondary-electron emission [3]), and $\sigma_1 = 5 \cdot 10^{-16}$ cm^2, then

$$\left.\begin{aligned} \Gamma &\approx 2 \cdot 10^{-5} (p/H) \cdot \sqrt{\varphi_k} \cdot \exp\left[-7500H/\left(p\varphi_k\sqrt{\varphi_k}\right)\right] \\ K_0 &= \Gamma^{-1} = \left(5 \cdot 10^{4} H/p\sqrt{\varphi_k}\right) \cdot \exp\left[-7500H/\left(p\varphi_k\sqrt{\varphi_k}\right)\right] \end{aligned}\right\} \qquad (10')$$

With allowance for (2), the condition of discharge existence can be represented in the following form:

$$\varphi_a - \varphi_k = <\varepsilon> \ln K_0 \qquad (11)$$

where $<\varepsilon>$ is the mean ε-value in the anode layer. To evaluate $<\varepsilon>$, let us assume that the velocity of electrons υ_x is determined by their transverse mobility under the conditions of high magnetization, i.e., $\omega \ll \nu_0$, and then:

$$\varepsilon = (\nu_0 / \nu_i) \frac{e}{m} \left(\varphi'/\omega\right)^2 = \left(\nu_0 / \nu_i\right) W_{dr} \qquad (12)$$

(W_{dr} is the kinetic electron-drift energy).

According to (12), the ionization value ε is not explicitly dependent on the gas pressure p. It is evident that $\varepsilon > I$ and is always several times as great as the ionization potential I. Hereinafter, while qualitatively analyzing the existence condition (11), let us assume that $<\varepsilon> = 50$ V.

Substitution of K_0 from (10) into (11) gives the anode voltage φ_a as a function of φ_k:

$$У_a = 18{,}4 + 2\ln(H/p) + 120/У\sqrt{У} \cdot (H/p) + У - \ln У \qquad (13)$$

where $У = \varphi_k/25$; $У_a = \varphi_a/25$.

The anode voltage is minimum for $У = У^*$ which is described by the following equation:

$$У \cdot \sqrt{У} \cdot (У^{-1}) = 180 \cdot H/P \qquad (14)$$

Elimination of H/p from (13) using (14) gives $(y_a)_{min} \equiv y_a^*$ as a function of y^*:

$$У_a^* = 17.73 + 5/3 \cdot У^* + 2\ln\left[У^*\left(У^* - 1\right)/180\right] \qquad (15)$$

Fig. 1 shows $(\varphi_a)_{min}$ versus H/p calculated from formulae (14), (15) and the relation $K_0(H/p)$ corresponding to the minimum anode voltage (i.e., the discharge-quenching voltage).

The discharge conditions with various φ_k- and K_0-values are feasible by varying H/p or φ_a, the discharge gap being constant. Indeed, if $\varphi_a \gg \varphi_k$, condition (11) gives:

$$\varphi_a \approx <\varepsilon> \ln K_0 \qquad (16)$$

Application of (12) gives:

$$<\varepsilon>=<\nu_0/\nu_i>\frac{e}{m}\frac{\varphi_a^2}{\omega^2 l^2} \qquad (17)$$

where $<\nu_0/\nu_i>$ is the mean value.

From (16), (17) it follows that

$$\varphi_a=\frac{m(\omega l)^2}{e<\nu_0/\nu_i>\ln K_0}\equiv\varphi_H \qquad (18)$$

Since $K_0 \gg 1$, $<\nu_0/\nu_i>\ln K_0 \approx$ const and equality (18) describes the voltage φ_H at which the thickness of the anode layer itself, i.e., its characteristic thickness, l_0, coincides with the discharge gap l. With $<\nu_0/\nu_i>\ln K_0=18$, we can evaluate

$$\varphi_H \approx 10^4 (Hl)_B^2 \qquad (19)$$

If $\varphi_a > \varphi_H$, in a way similar to (19), the characteristic anode-layer thickness l_0 is determined: $\varphi_a=10^4(Hl_0)^2$. In this, the cathode appears to be within the anode layer, i.e.,

$$l/l_0=\sqrt{\varphi_H/\varphi_a}<1 \qquad (20)$$

With the potential distribution in the anode layer, $\varphi(x)=\varphi_a\cdot F(x/l_0)$, known and l prescribed, the cathode drop is equal to

$$\varphi_k=\varphi_a\cdot F(l/l_0)=\varphi_a\cdot F\left(\sqrt{\varphi_H/\varphi_a}\right)$$

It is evident that the function $F(x/l_0)$ monotonically decreases as x increases and reaches its minimum value when $x/l_0 \to 1$. Therefore, with $\varphi_a \to \varphi_H$, the truncated discharge with an incomplete anode layer, minimum cathode drop and maximum gain, $(K_0)_{max}=\exp[\varphi_H(1-F(1))/<\varepsilon>]$, is obtained.

It should be noted that since Eq. 1 refers to the truncated discharge, its applicability is limited by inequality (20), i.e., $HL \gg \sqrt{\varphi_{a\,min}}$.

Fig. 2 showing a family of curves $\varphi_a(K_0)$ for three H/p-values gives a more comprehensive idea of the practical discharge properties. From here it is evident that for a voltage of $300V<\varphi_a<700V$ the gain is higher than 10^3.

2. DISCHARGE WITH A FREE ANODE LAYER.

With the anode voltage decreasing down to $\varphi_a<\varphi_H$ when $l/l_0<1$, the truncated discharge is transformed into a discharge with a free anode layer. In the region of $l_0<x<l$, there appears a quasi-neutral plasma jet, i.e., a quasi-neutral flow of fast ions born in the anode layer and moving practically free with almost constant mean velocity $<\upsilon_i>$ along the x-axis. Under real conditions, the jet height (in the magnetic-field direction H_z) is always bounded by the distance 2h between the cathode surface normal to the magnetic field (i.e., the end-face cathodes) and the jet interacts with them absorbing and entraining some part of γ-electrons into the anode layer. When the jet is long enough, the end-face γ-electron current becomes predominant. Therefore, we can suggest that with $\Delta l=l-l_0$ increasing, the unidimensional discharge with a truncated anode layer is transformed into a two-dimensional discharge with a free anode layer.

As a consequence of electronic ionization and resonant recharge of fast ions, slow ions with the initial velocity of the order of the thermal velocity of atoms are born in the plasma. In case the electric field E_x in the plasma jet is not high, slow ions die away predominantly at the end-face cathodes and cause secondary-electron emission provided the ionization potential is higher than twice the work function (the potential secondary-electron emission [3]).

The probability of end-face γ-electron capture in the plasma is described by the following formula similar to (10) but different in the scattering length alone: 2h instead of $\pi \cdot \rho_e(\varphi_k)$:

$$\Gamma_t = (1+\beta\varphi_k / I)\cdot\gamma\cdot\sigma_1 n_g \cdot 2h \cdot \exp\left[-T_\gamma / \left(\varphi_k \cdot 4h \cdot n_g \cdot \sigma_1\right)\right]$$

As before, we will assume that $1+\beta\varphi_k / I \approx 2$. Then,

$$\Gamma_t = \gamma \cdot \bar{\Theta}^2 \cdot \exp(-T_\gamma / \varphi_k \bar{\Theta}^2) \qquad (21)$$

$$\bar{\Theta}^2 = 4h \cdot n_g \cdot \sigma_1 \qquad (22)$$

The electron current j_x in the plasma jet is described by the following set of equations:

$$\frac{dj_x}{dx} = (\nu_\gamma + \nu_i)\cdot n_e - \frac{n_e \upsilon_e}{4h}\cdot\exp(-\varphi / T_e) \qquad (23)$$

$$\frac{dj_x}{dx} \approx b_e n_e \frac{d^2\varphi}{dx^2} \qquad (24)$$

The first term in the continuity equation (23) is equal to the rate of electron emergence due to ionization with the frequency ν_i and due to γ-electron capture with the frequency ν_γ described by the following equality:

$$\nu_\gamma \cdot n_e = \Gamma_t\left[\sigma_\Pi \cdot n_g \cdot j_i + \nu_i \cdot n_e\right]$$

σ_Π - is the resonant recharge section;

j_i - is the fast-ion current density.

The frequency ν_γ can be expressed by the formula:

$$\nu_\gamma = \Gamma_t(\nu_n + \nu_e) \qquad (25)$$

where

$$\nu_n = \sigma_n \cdot n_g < \upsilon_i > \cdot n_i / n_e \qquad (26)$$

is the recharge frequency.

The second term in (23) is equal to the velocity of electron departure from the plasma jet through the potential barrier φ in the cathode layer. It is assumed that captured γ-electrons lose their energy in excitation and ionization and relax to the Maxwellian distribution with the temperature T_e. It is also assumed in (23), (24) that the change in the jet potential is reasonably small, i.e., $\Delta\varphi \ll \varphi_a < \varphi_k$, and hence, n_i=const, n_e=const, ν=const, and the current is determined by the transverse mobility: $j_x = b_e n_e d\varphi/dx$. Simple transformations reduce (23)-(24) to the equation for the plasma-jet potential:

$$\frac{d^2\eta}{ds^2} = 1 - \exp(-\eta) \qquad (27)$$

$$\eta = (\varphi - \varphi_p) / T_e \qquad (28)$$

φ_p - is the equilibrium jet potential for which $dj_x/dx=0$:

$$\varphi_p = T_e \cdot \ln\left[\upsilon_e / 4h\left(\nu_\gamma + \nu_i\right)\right] \qquad (29)$$

s - is the dimensionless coordinate:

$$s = (x-l_0)/l_0 \tag{30}$$

x_0 is the characteristic length defined by the equality

$$x_0^2 = \frac{b_e T_e}{(\nu_\gamma + \nu_i)} \tag{31}$$

The electron current coming into the anode layer through the surface s=0 is equal to:

$$j_k = b_e \cdot T_e \cdot n_e / x_0 \cdot (d\eta / ds)_{s=0} \tag{32}$$

Integration of (27) gives

$$\frac{d\eta}{ds} = -\sqrt{2}\sqrt{\eta + \exp(-\eta) - 1} \tag{33}$$

Here it is assumed that $\eta \to 0$; $d\eta/ds \to 0$ with $s \to \infty$. From (32), (33) it is evident that the current j_k is expressed in terms of the potential at the anode-layer boundary $\eta_0 = \eta(s=0)$.

With $\eta \ll 1$, Eq. (27) has an exponential solution:

$$\eta \approx \eta_0 \exp(-s) \tag{34}$$

$$d\eta / ds = -\eta \tag{35}$$

Circumventing cumbersome formulae and accepting corresponding inaccuracy, we can use more simple approximate solution (34), (35).

In the section x=1, either the cathode surface absorbing ions is located or the magnetic field disappears. Therefore, ions reaching the surface x=1 cannot arrive at the end-face cathodes.

Therefore, for the discharge to exist, the potential distribution in the jet $\varphi(x)$ should meet the condition of "magnetic cut-off" of ions:

$$\varphi(x) - \varphi(l) \le \frac{M \cdot \omega_i^2}{2l}(l-x)^2 \tag{36}$$

With (36) satisfied, ions born at the point x have a return point $x_B \le 1$, and hence, practically all slow ions will leave the plasma jet for the end-face cathodes.

This inequality can be conveniently represented in the following form:

$$\eta(s) - \eta(s_e) \le \eta_M (1 - s/s_e)^2 \tag{36'}$$

$$s_e = l - l_0 / x_0 = \Delta l / x_0 \tag{37}$$

$$\eta_M = M\omega_i^2 \Delta l^2 / 2lT_e \approx 50(H\Delta l)^2 / MT_e \tag{38}$$

For a sufficiently long jet, η_M is high and the cut-off condition always holds. If $\eta(s_e) \ll \eta_0$, (36) is simplified:

$$\eta_0 \le \eta_M \tag{39}$$

The cut-off condition is most essential for heavy ions. For example, with M=200, we obtain $\eta_M = (H\Delta l)^2 / 4T_e < (16T_0)^{-1} \ll 1$ at $H\Delta l = 0.5$

With the cut-off condition assumed to be satisfied, let us consider the region of free-discharge existence and evaluate the gain. It can be shown that in the casse of resonant recharge $\nu_n \gg \nu_i$, and moreover, $\nu = \Gamma_i \cdot \nu_n \gg \nu_i$, $T_e < 1.5$ V.

Therefore, plasma-electron ionization will be neglected.

Application of the expressions for j_k, Γ_i and x_0 gives

$$j_k = j_i \left.\frac{d\eta}{ds}\right|_{s=0} \cdot \frac{\rho_e(T_e)}{4h} \cdot \left(\frac{\upsilon_e}{2<\upsilon_i>} \cdot \gamma \cdot \frac{\sigma_n}{\sigma_1}\right)^{\frac{1}{2}} \left(\bar{\Theta}^2\right)^{\frac{3}{2}} \exp\left(\frac{-T_\gamma}{2\varphi_k \bar{\Theta}^2}\right)$$

In this,

$$\frac{\upsilon_e}{<\upsilon_i>} = \sqrt{\frac{M}{m}}\sqrt{\frac{2T_e}{\varphi_a}} = 20;\ \frac{\sigma_n}{\sigma_1} = 10;\ T_\gamma = 3;\ \gamma = 5\cdot 10^{-2};\ \left.\frac{d\eta}{ds}\right|_{s=0} = -\eta$$

Then, for K_i we obtain

$$K_i = -j_i / j_k \approx 536 \cdot Hh / \eta_0\sqrt{T_0} \cdot \left(\bar{\Theta}^2\right)^{3/2} \exp\left[3/\left(2\bar{\Theta}^2 \cdot \varphi_k\right)\right] \qquad (40)$$

From the condition of discharge existence $\exp[(\varphi_a - \varphi_k)/<\varepsilon>] = K_i$, the expression for the anode voltage follows:

$$\varphi_a = \varphi_k + <\varepsilon>\left\{3/\left(\bar{\Theta}^2 \cdot \varphi_k\right) + \ln\left[536 \cdot Hh \Big/ \eta_0\sqrt{T_0} \cdot \left(\bar{\Theta}^2\right)^{3/2}\right]\right\} \qquad (41)$$

The minimum voltage $(\varphi_a)_{min}$ takes place when

$$\varphi_{k*} = \sqrt{3/2 \cdot <\varepsilon> / \bar{\Theta}^2} \qquad (42)$$

and is equal to

$$(\varphi_a)_{min} = \sqrt{6<\varepsilon>}\Big/\bar{\Theta}^2 + <\varepsilon> \cdot \ln\left[536Hh \Big/ \eta\sqrt{T}\left(\bar{\Theta}^2\right)^{3/2}\right] \qquad (43)$$

With $\eta_0\sqrt{T_0} \approx 1$; $<\varepsilon> = 50V$; $\bar{\Theta}^2 = 7\cdot 10^{-2}$ ph, we obtain

$$\varphi_a = 514 + 50\ln(H/p) - 25\ln(ph) + \varphi_k + 1070/ph\varphi_k \qquad (41')$$

$$\varphi_{k*} = 32.7/\sqrt{ph} \qquad (42')$$

$$(\varphi_a) = 514 + 50\ln(H/p) - 25\ln(ph) + 65.46/\sqrt{ph} \qquad (43')$$

The dependence of $(\varphi_a)_{min}$ on H/p for the free discharge is similar to that for the truncated one (see 14, 15 and Fig. 1) but differs from it in that the blanking voltage decreases noticeably as ph increases. It should be noted that (43') agrees satisfactory with that measured in [4].

To provide a more comprehensive idea, Fig. 3 gives a family of curves $\varphi_a(K_i)$ similar to that shown in Fig. 2. We notice that in the same interval, i.e., 300 V < φ_a < 700 V, the gain $K_i > 10^3$.

The function $\varphi_a(K_0)$ for the truncated discharge (Fig. 2) and $\varphi_a(K_i)$ for the free discharge (Fig. 3) have a minimum $(\varphi_a)_{min}$. Therefore, with $\varphi_a > \varphi_{a\,min}$, there exist two equilibrium states considerably different in gain. The outlined theory does not take into account many factors, it corresponds to the horizontal volt-ampere characteristic (φ_a=const for any current) and does not make it possible to decide which of the two states is stable.

However, it would appear reasonable that $dj_a/dK > 0$. With this assumption made, the characteristics $\varphi_a(K)$ prove to be similar to the volt-ampere ones, and hence, the falling branches of the curves should be considered as unstable

and practically infeasible under steady-state conditions. With allowance for this assumption made, straight lines showing the loading characteristics of the external circuit are given in Fig. 2.3.

In conclusion, it should be noted that free discharge seems to be most attractive for practical application since it is more convenient and more tolerable with respect to the parameters.

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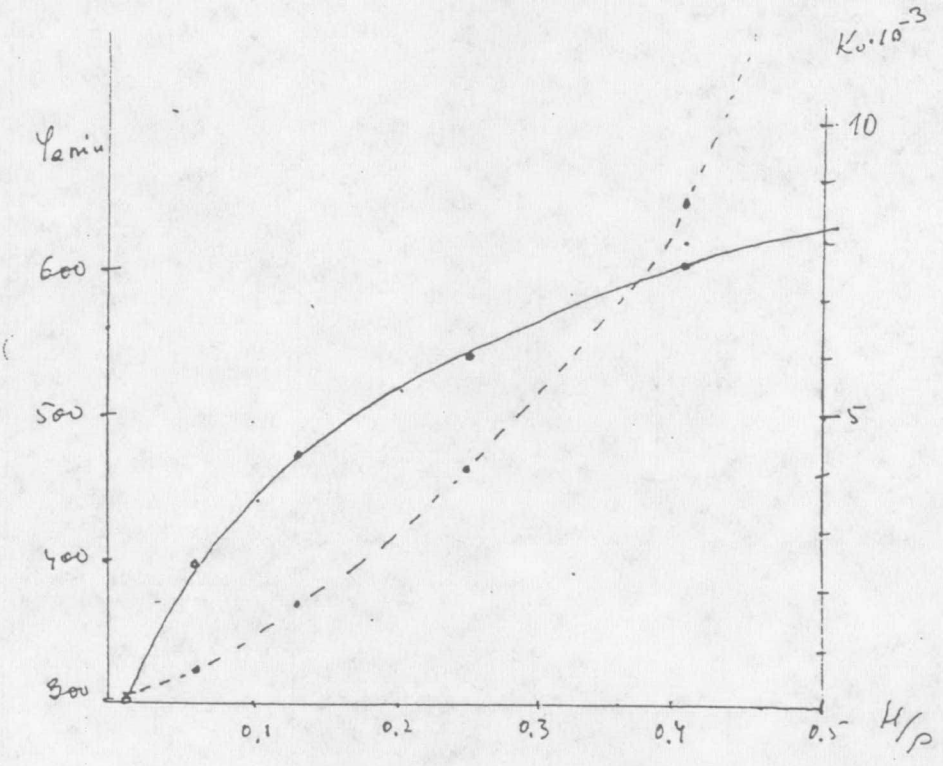
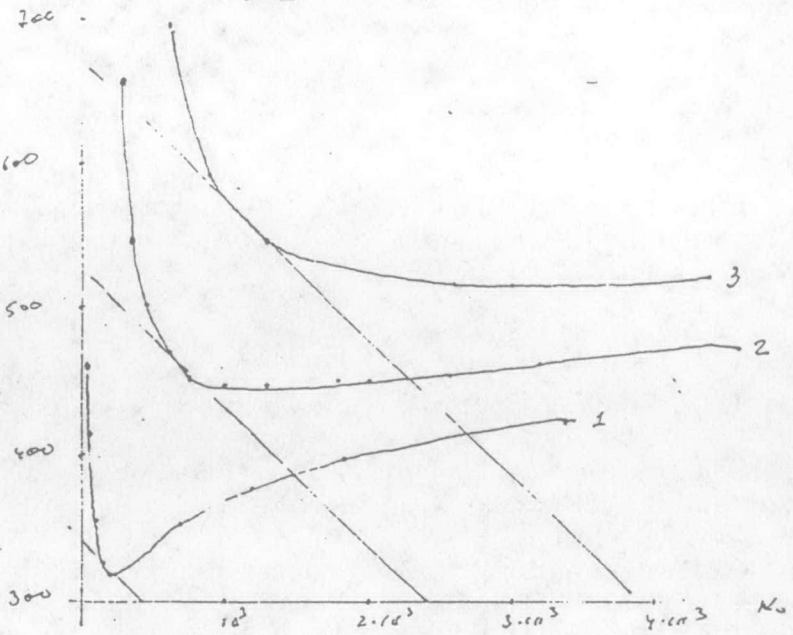


Fig. 1.

$\varphi_{a\,min} = f(H/p)$

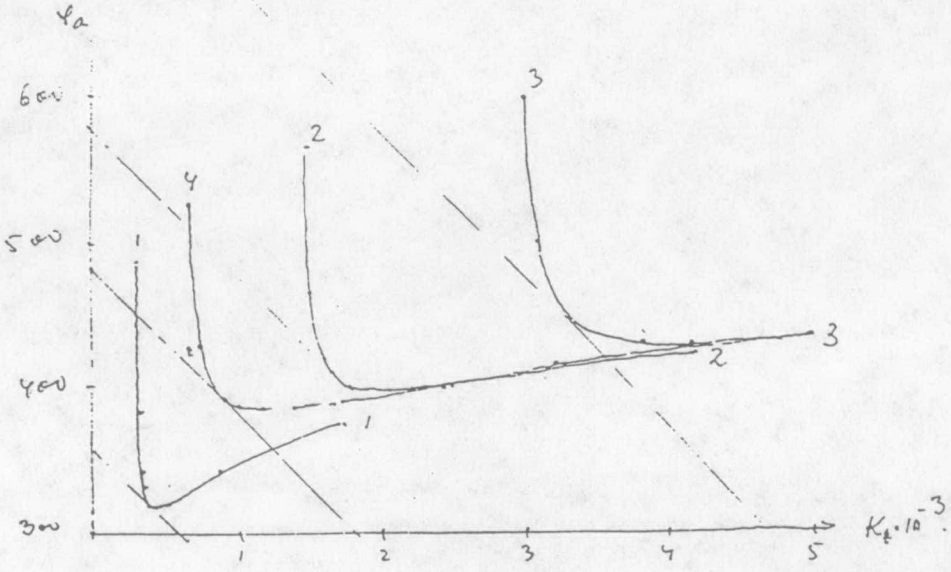
—— $\varphi_{a\,min} = f(H/p)$

- - - - $K_0 = f(H/p)$ при $\varphi_{a\,min}$



1. $H/D = 0,02$; $\varphi_{a\,min} = 318$ В
2. $H/D = 0,1$; $\varphi_{a\,min} = 446$ В
3. $H/D = 0,2$; $\varphi_{a\,min} = 517$ В.

Fig. 2



$Hk = 0,08$ 1. $\{H/D = 0,02$; $ph = 4.\}$ $\varphi_a^{\vee} = 16,03$; $\varphi_a^{\wedge} = 317$

$Hk = 0,4$ 2. $\{H/D = 0,1$; $ph = 4.\}$ $\varphi_a^{\vee} = 16,35$ $\varphi_a^{\wedge} = 347$

$Hk = 0,8$ 3. $\{H/D = 0,2$ $ph = 4.\}$ $\varphi_a^{\vee} = 16,35$; $\varphi_a^{\wedge} = 432$

$Hk = 0,02$ 4. $\{H/D = 0,2$ $ph = 1.\}$ $\varphi_a^{\vee} = 34$ $\varphi_a^{\wedge} = 384$

Fig. 3